Low Emittance Storage Rings (Overview)

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Outline

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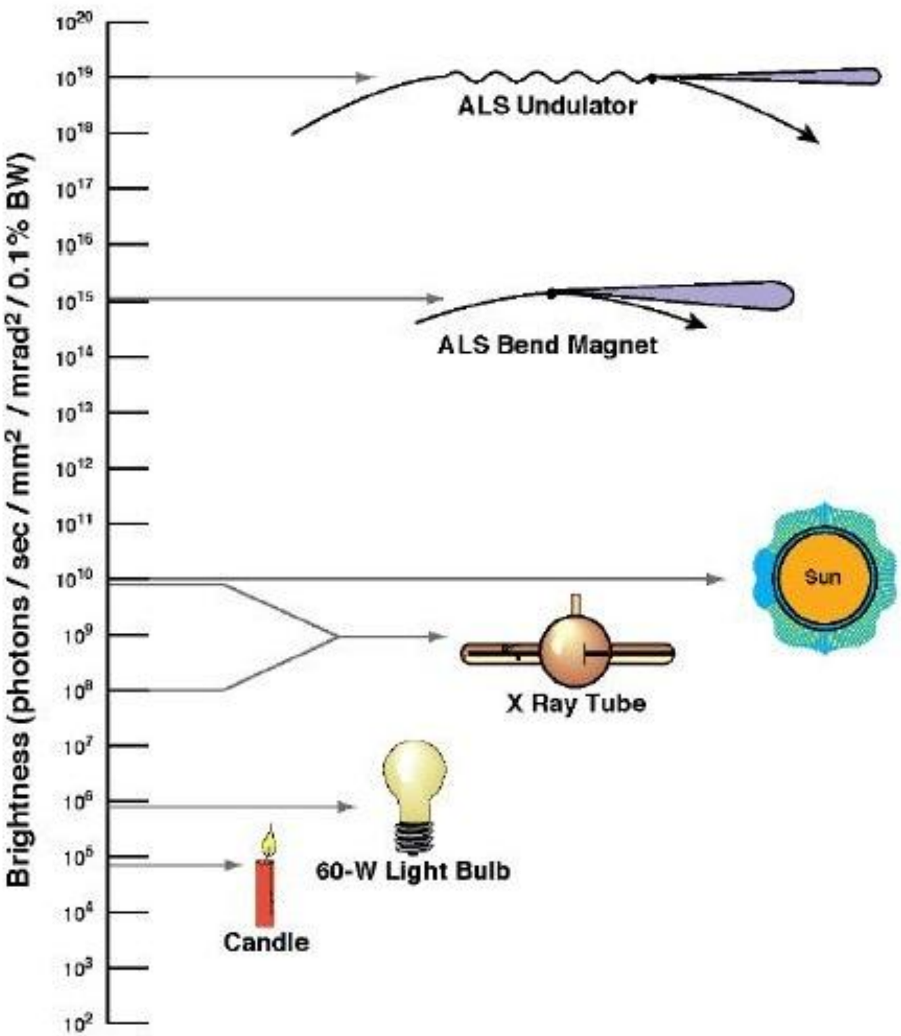
- Chromaticity
- Dynamic aperture optimization
- Summary

Introduction

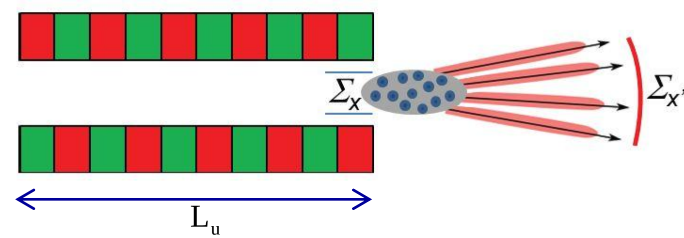
Why is it important to achieve low beam emittance in a storage ring?

An important figure of merit ***brightness*** = *photon flux per unit area and per unit solid angle at the source.*



$$B \sim \frac{F(\omega)}{\Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'} (\Delta\omega/\omega)}$$

spectral brightness or brilliance



$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_r^2}, \quad \Sigma_{x',y'} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'}^2}$$

Effective source size and angular divergence of radiation determined by the phase space convolution of single electron radiation and electron beam properties.

$$\sigma_r = \sqrt{\frac{\lambda}{L_u}}, \quad \sigma_{r'} = \frac{\sqrt{\lambda L_u}}{4\pi}$$

$\lambda -$ on-axis undulator resonant wavelength

Introduction

Undulator brightness in Gaussian approximation (single electron)

$$B_1(r,r') \approx \frac{F_1(\omega)}{2\pi\sigma_r\sigma_{r'}} \exp\left(-\frac{r^2}{2\sigma_r^2} - \frac{r'^2}{2\sigma_{r'}^2}\right)$$

$F_1(\omega)$ – photon flux of single electron within spectral bandwidth

Electron beam Gaussian distribution

$$f(x,y,x',y') \approx \frac{1}{(2\pi)^2 \sigma_x\sigma_y\sigma_{x'}\sigma_{y'}} \times \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y'^2}{2\sigma_{y'}^2}\right)$$

From convolution theorem

$$B(x,y,x',y') \approx \frac{N_e F_1(\omega)}{(2\pi)^2 \Sigma_x\Sigma_{x'}\Sigma_y\Sigma_{y'}} \exp\left(-\frac{x^2}{2\Sigma_x^2} - \frac{y^2}{2\Sigma_y^2} - \frac{x'^2}{2\Sigma_{x'}^2} - \frac{y'^2}{2\Sigma_{y'}^2}\right)$$

1) Emittance dominated regime for peak brightness

$$\left(\sigma_{x,y} >> \sigma_r \ \ and \ \ \sigma_{x',y'} >> \sigma_{r'}\right)$$

$$\boxed{B_0 = \frac{N_e F_1(\omega)}{(2\pi)^2 \sigma_x\sigma_{x'}\sigma_y\sigma_{y'}} = \frac{F(\omega)}{(2\pi)^2 \varepsilon_x\varepsilon_y}}$$

2) Radiation dominated regime for peak brightness

$$\left(\sigma_{x,y} << \sigma_r \ \ and \ \ \sigma_{x',y'} << \sigma_{r'}\right)$$

$$\boxed{B_0 = \frac{F(\omega)}{(2\pi)^2 \sigma_r^2\sigma_{r'}^2} = \frac{F(\omega)}{(\lambda/2)^2}}$$

$$\boxed{\varepsilon_{x,y} << \frac{\lambda}{4\pi}}$$

so-called ***diffraction limit***

For X-ray range – from several 10s to several 100s pm

Low emittance lattice concepts

The natural horizontal emittance and energy spread
(balance between the radiation damping and the quantum excitation)

$$\boxed{\varepsilon_{x0} = C_q \gamma^2 \cdot \frac{I_5}{I_2 - I_4}}$$

$$\boxed{\sigma_\delta^2 = C_q \gamma^2 \cdot \frac{I_3}{2I_2 + I_4}}$$

synchrotron
radiation
integrals

$$I_2 = \oint_{bend} \frac{1}{\rho^2} ds$$

$$I_3 = \oint_{bend} \frac{1}{|\rho|^3} ds$$

$$I_4 = \oint_{bend} \frac{\eta_x}{\rho} \left(\frac{1}{\rho^2} + 2k_1 \right) ds$$

$$I_5 = \oint_{bend} \frac{H_x}{|\rho|^3} ds$$

$\boxed{\Delta E\,[\mathrm{keV}] = C_\gamma \gamma^4 I_2}$ - energy loss per turn

$\gamma -$ beam energy

$\rho -$ bending radius

$k_1 = \frac{e}{P_0} \frac{\partial B_y}{\partial x} -$ quad strength

$H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2 -$ "dispersion emittance"

$\alpha_x\, \beta_x, \gamma_x -$ Twiss parameters

$\eta_x -$ dispersion

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \approx 3.832 \times 10^{-13} m$$

$$C_\gamma = \frac{e^2}{3\varepsilon_0 (mc^2)^4} \approx 8.846 \times 10^{-5} m / GeV^3$$

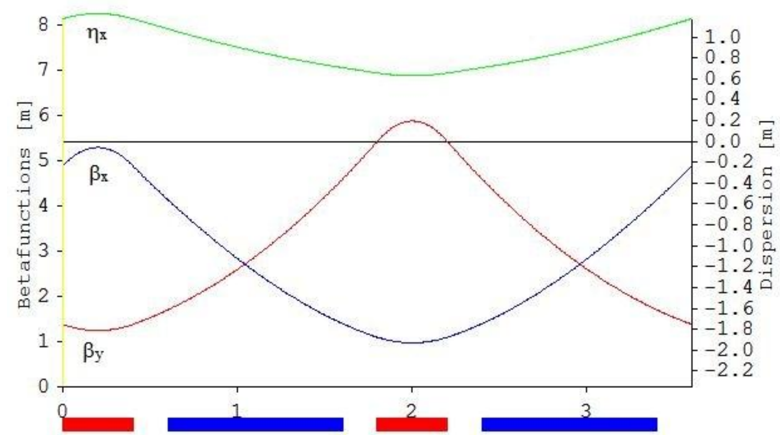
Low emittance lattice concepts

Natural emittance in FODO, DBA and TME lattices (app. valid for small dipole bending angle)

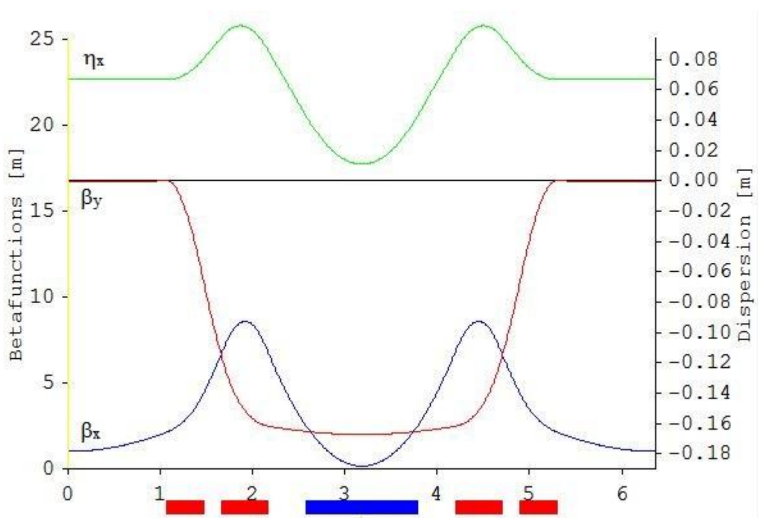
Lattice type	Minimum Emittance	Conditions
180°FODO (90°FODO)	$\approx C_q \gamma^2 \theta^3$ $(\approx 2\sqrt{2} C_q \gamma^2 \theta^3)$	$f/L = 1/2$ $(f/L = 1/\sqrt{2})$
DBA	$\approx \dfrac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_x = \eta'_x = 0$ $\beta_0 \approx \sqrt{12/5} L$ $\alpha_0 \approx \sqrt{15}$
TME	$\approx \dfrac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta^{(c)}_{x\min} = L\theta/24$ $\beta^{(c)}_{x\min} = L/2\sqrt{15}$

$L -$ dipole length

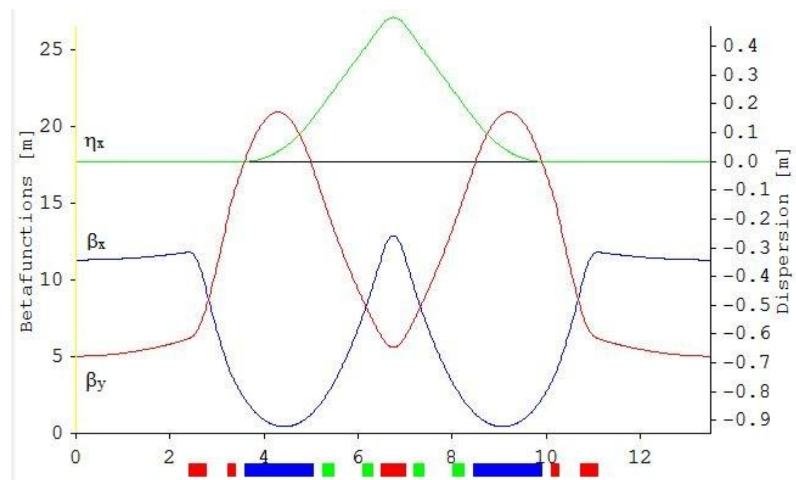
$\theta -$ dipole bending angel



FODO



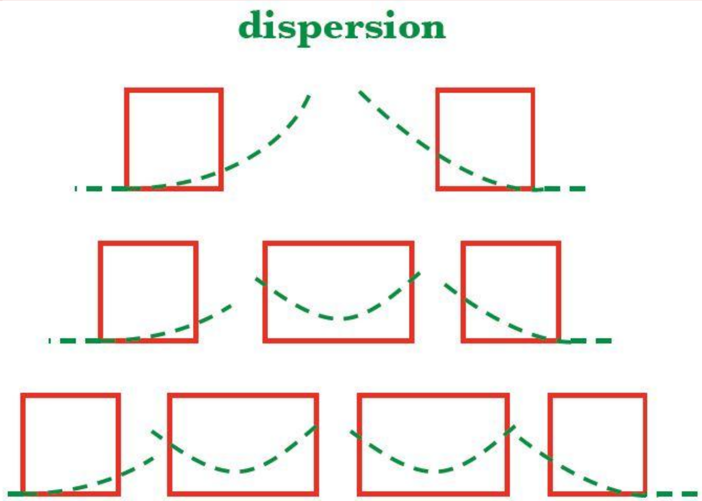
TME



DBA

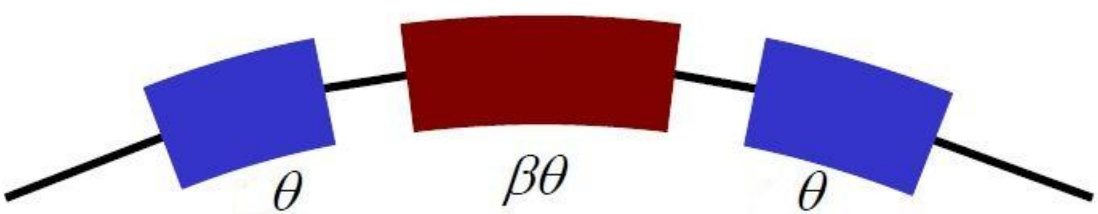
Low emittance lattice concepts

Combination of DBA and TME - Multi-bend achromats



- Double Bend Achromat (DBA)
- Triple Bend Achromat (TBA)
- Quadruple Bend Achromat (QBA)

Dipoles with the same bending radius



For M-bend achromat

$$\boxed{\varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \frac{M+1}{M-1} \theta^3}$$

$\beta \approx \sqrt[3]{3}$ – value obtained from emittance minimization

Low emittance lattice concepts

As a summary

Achromats have been popular choices for storage ring lattices in third-generation synchrotron light sources for two reasons:

- they provide lower natural emittance than FODO lattices;
- they provide zero-dispersion locations appropriate for insertion devices (wigg. and undul.).

Light sources using double and triple-bend achromats

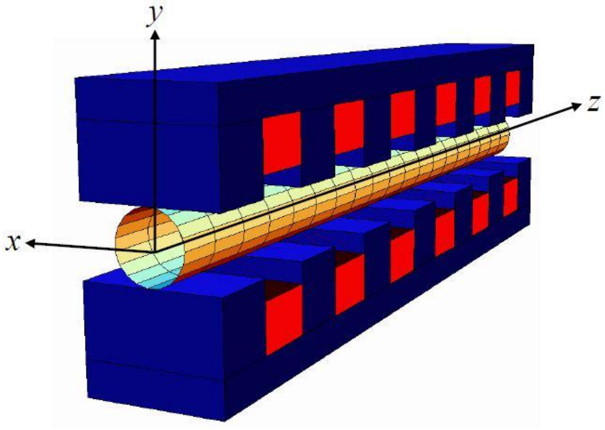
Light sources	Lattice type	Emittance	Energy	Circumference
SPring-8	DBA	3.4 nm rad	8 GeV	1436 m
ESRF	DBA	4 nm rad	6 GeV	844 m
DIAMOND	DBA	2.7 nm rad	3 GeV	560 m
SOLEIL	DBA	3.9 nm rad	2.75 GeV	354 m
ELETTRA	DBA	7 nm rad	2.4 GeV	259 m
CANDLE	DBA	8.4 nm rad	3 GeV	216 m
ALS	TBA	2 nm rad	1.9 GeV	197 m
SLS	TBA	4.8 nm rad	2.4 GeV	288 m

Increasing the number of bends in a single cell of an achromat("multiple-bend achromats") reduces the emittance, since the lattice functions in the "central" bends can be tuned to conditions for minimum emittance.

"Detuning" an achromat to allow some dispersion in the straights provides the possibility of further reduction in natural emittance, by moving towards the conditions for a theoretical minimum emittance (TME) lattice.

Low emittance lattice concepts

Effect of insertion devices on the natural emittance:



$$B_y = B_0 \sin(k_z z)$$

$$\lambda_w = \frac{2\pi}{k_z}$$

Placed in dispersion free straight

$$\frac{\varepsilon_w}{\varepsilon_0} = \frac{1 + 1.21\times 10^{-12} \frac{\beta_x L_w \lambda_w^2 \rho_0 B_w^5}{j_x \varepsilon_0 E^3}}{1 + 7.16\times 10^{-3} \frac{L_w \rho_0 B_w^2}{E^2}}$$

Self-dispersion impact on quantum excitation

Contribution in both radiation damping (I_2) and quantum excitation (I_5)

- short period length
- not weak field, not strong field (due to the *B^5* term in the numerator compared to the B^2 term in the denominator)
- placement at locations with small horizontal beta function

Low emittance lattice concepts

Recent approaches:

- Multi-Bend Achromat concept
- Use of damping wigglers
- Use of vertical focusing bending magnets (and/or Robinson wiggler)
- A round beam by coupling using skew quadrupoles (PETRA III, ALS upgrade)
- Use of dipoles with longitudinal field variation (SIRIUS, ESRF upgrade, SLS upgrade)

Name	Lattice type	Energy [GeV]	Circumference [m]	Emittance [pm]
PETRA-III	FODO/DBA	6.0	2304	4400 → 1000
NSLS-II	DBA	3.0	792	2000 → 660
MAX-IV	7BA	3.0	528	328 → 200
SIRIUS	5BA	3.0	518	280
ESRF upgrade	Hybrid 7BA	6.0	844	147
APS upgrade	Hybrid 7BA	6.0	1104	65
SPRING 8 upgrade study	6-10BA	6.0	1436	68
DIAMOND upgrade	4BA	3.0	562	275
ANKA upgrade study	4BA	2.5	110	6000
PEP-X study	7BA	4.5	2200	29 → 10
ALS upgrade study	6-10BA	2.0	200	100
SLS upgrade studies	Hybrid 7BA	2.4	288	75 / 185
ELETTRA upgrade study	6BA	2.0	260	250
ILSF project	5BA	3.0	528	477
CANDLE upgrade study	4BA	3.0	240	1000

831

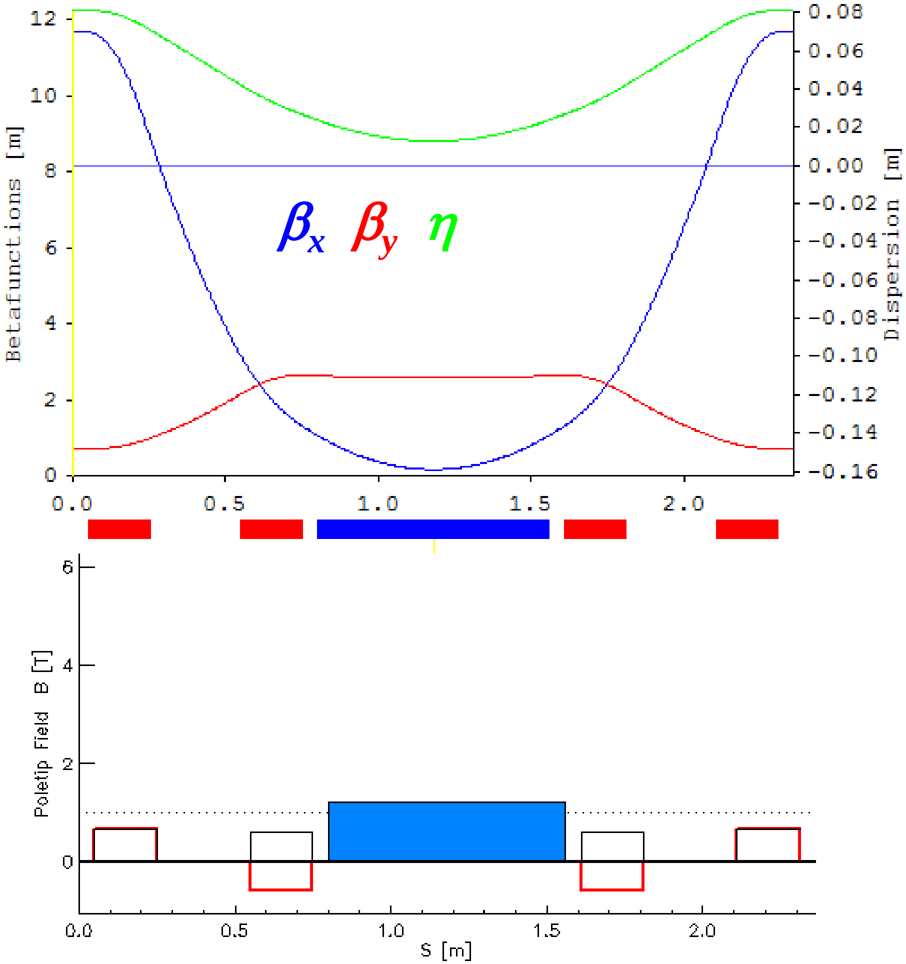
3531

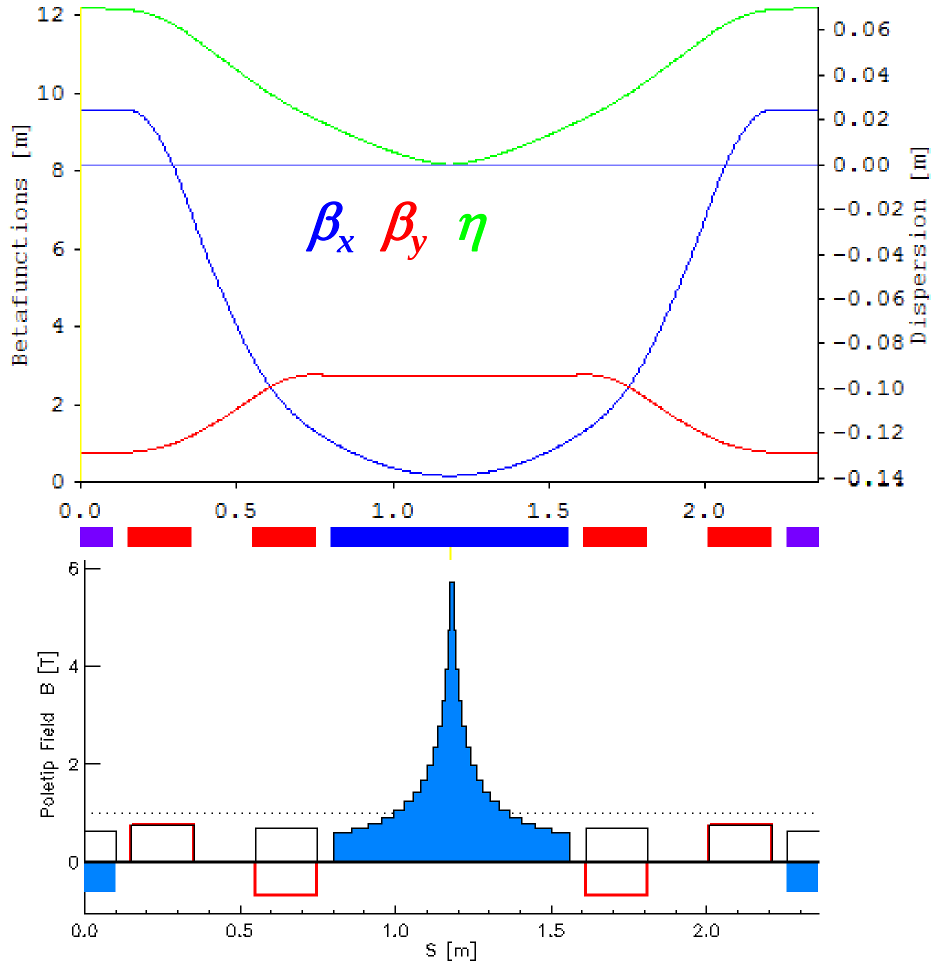
5079

Low emittance lattice concepts

Longitudinal field variation to compensate H_x variation.

$$I_5 = \oint_{bend} \frac{H_x}{|\rho|^3} ds \qquad H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$





Low emittance lattice concepts

Longitudinal field variation to compensate H_x variation.

Beam dynamics in bending magnet	Numerical optimization
$$H = \frac{\eta^2 + (\alpha\eta + \beta\eta')^2}{\beta}$$ • Curvature is source of dispersion: $\eta''(s) \sim 1/\rho(s) \rightarrow \eta'(s) \rightarrow \eta(s)$ • Horizontal optics ~ like drift space: $\beta(s) = \beta_0 - 2\alpha_0 s + \frac{1+\alpha_0^2}{\beta_0} s^2$ Problem: $I_5 = \int_L f(\alpha_0, \beta_0, \eta_0, \eta_0', s, \rho)\, ds \rightarrow \min$ • too complicated to solve	• Symmetric half bend in N slices: curvature ρ_i, length Δs_i • Knobs for minimizer: $\{\rho_i\}, \{\Delta s_i\}, \beta_0, \alpha_0, \eta_0, \eta_0'$ • Objective: I_5 • Constraints: – length: $\Sigma \Delta s_i = L/2$ – angle: $\Sigma \rho_i \Delta s_i = \Phi/2$ – [field: $\rho_i > \rho_{min}$] – [optics: β_0, η_0]

Low emittance lattice concepts

The availability of NEG coating allows the vacuum chamber cross sections to be reduced by about 3 times compared to storage rings with discrete pumps.

Strong horizontal focusing to provide a focus for β_{x}, η_x in dipoles

New generation of MBA lattices with small magnets

Short lattice cells

Low emittance

Large number of cells within given circumference

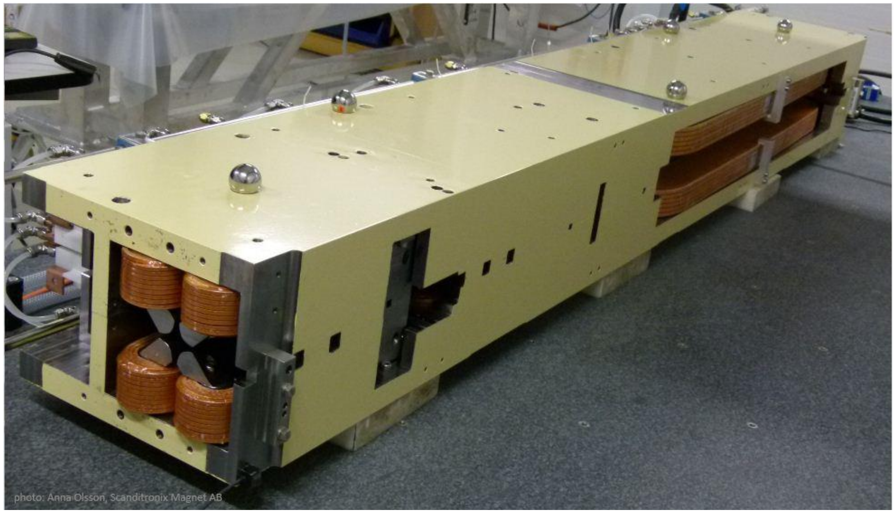
Chromaticity

Profit from the cubic scaling $\varepsilon_0 \sim N_{cell}^{-3}$

Magnets

Diffraction limited emittance requires magnets with unprecedented strength in storage ring.

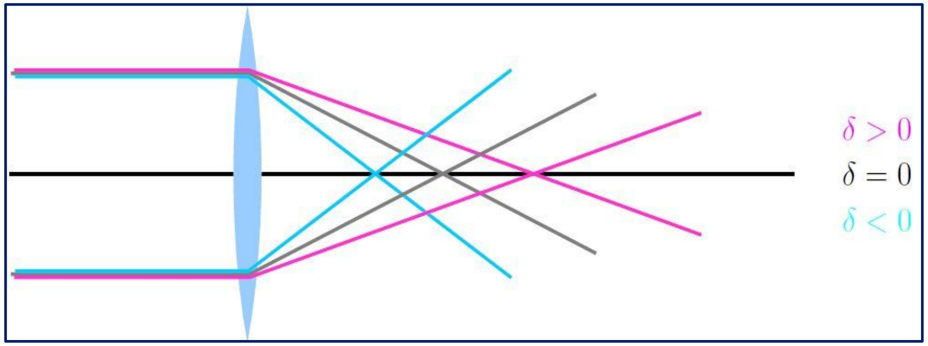
	Field gradient in Dipoles [T/m]	Quad. Gradient [T/m]	Sext. Gradient [T/m2]
MAX IV	9	40	2200
ESRF II	34	85	2200
Diamond II	15	70	2000
CANDLE LE	11	40	1000
CANDLE	3.3	17	350



MAX IV magnet design

Chromaticity

Strong quadrupoles cause large chromaticities – a reason for single bunch head tail instability and several multi bunch instabilities.



Quad strength

$$k = \frac{1}{B\rho}\frac{dB_y}{dx}$$

magnetic rigidity

$$B\rho[Tm] := \frac{p}{e} = 3.3356\,E[GeV]$$

$$k(\delta) = \frac{k_0}{1+\delta} \approx k_0(1-\delta) \qquad \delta = \frac{\Delta p}{p} << 1$$

Chromaticity - the variation of tune with momentum

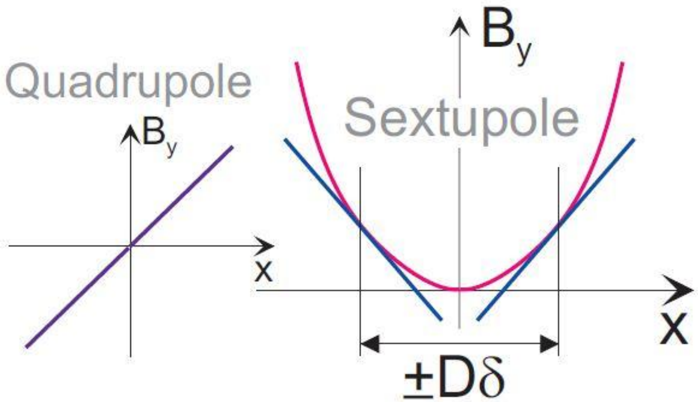
$$\xi_{x/y} = \frac{\Delta Q_{x/y}}{\delta} = \mp\frac{1}{4\pi}\oint k(s)\beta_{x/y}(s)ds < 0$$

- ***negative chromaticity must be avoided for suppression of the head-tail instability***
- ***large absolute value of chromaticity would result in a wide tune spread of the beam halo leading to particle losses at low order resonances***

zero or moderate positive values are mandatory

Sextupoles for chromaticity correction

The parabolic field variation in a sextupole makes it an essentially nonlinear device which causes chaotic or unbounded particle motion beyond some maximum stable amplitude.



$$B_{yQ}(x) = B'_{yQ}x \qquad B_{yS}(x) = \frac{1}{2}B''_{yQ}x^2$$

$$k_Q = \frac{1}{B\rho}\frac{dB_y}{dx} \qquad k_S = \frac{1}{2}\frac{1}{B\rho}\frac{d^2B_y}{dx^2}$$

In a small range around some point *x* it can be considered like a quadrupole with gradient

$$B'_y(x) = B''_y x$$

Combined effect on chromaticity

$$\boxed{\xi_{x/y} = \pm\frac{1}{4\pi}\oint\big(2k_S(s)D(s) - k_Q(s)\big)\beta_{x/y}(s)ds}$$

2 *families* of sextupoles **SF, SD**

Decoupling: SF locations: $D\uparrow$, $\beta_x\uparrow$, $\beta_y\downarrow$

SD locations: $D\uparrow$, $\beta_x\downarrow$, $\beta_y\uparrow$

Additional sextupolar terms cause problems with dynamic aperture

The phase space area enclosing stable particle oscillations **-** ***dynamic acceptance***
Dynamic acceptance's projection onto physical space **-** ***dynamic aperture***

Dynamic aperture optimization

1st approach

Need for installation of several families of "harmonic" sextupoles and octupoles to compensate adverse effects from the chromatic sextupoles:

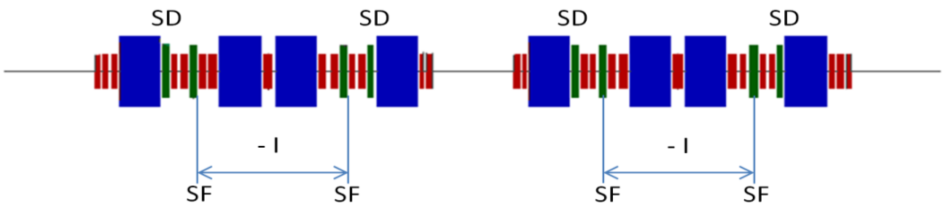
- third order sextupolar resonances
- fourth order octupolar resonances
- amplitude dependent tune shifts
- second order chromaticities

by minimization of analytically calculated NDTs of 1st and 2nd order in sextupole strengths and 1st order in octupole strength (with weights).

Testing by dynamic aperture tracking including misalignments, magnet errors, etc.

2nd approach

A pair of identical sextupoles connected by a minus-identity matrix transformer in ideal case of kick-like magnets cancels all geometrical aberrations and provide infinite dynamic aperture.
Finite length sextupole pair cancels up to the second order geometrical aberrations. Higher orders still exist, but the DA is large (although not infinite).



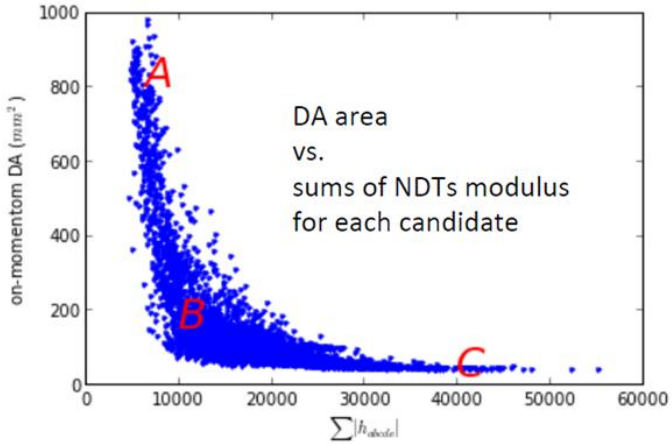
Sextupole locations in the LE ANKA lattice

DA optimization

Application of Multi Objective Genetic Algorithms (MOGA) for DA optimization.

Issues in conventional approach:

- There are numerous NDTs. Which ones are dominating DA? If only single penalty function is used, how to specify weight to each NDT?
- Having small NDTs is an necessary but insufficient condition for having a large DA



A: small NDTs and large DAs

B: small NDTs but small DAs

C: large NDTs and small Das

<u>Multi-objective optimization is suitable</u>

Genetic Algorithm (GA) mimics the evolution of nature:

- Crossover: generate children from parents.
- Mutation change the children.
- Natural selection: keep only certain number of population.

MOGA

1. Initialize population (first generation, random)
2. Repeat (generation by generation)

    3. crossover: 2 parents generate 2 children.
    4. mutation: change children.
    5. calculate children's parameters(par. comp.)
    6. natural selection: sorting (non-dominated)

7. Until stop criteria fulfills (find solution)
8. A bunch of candidate solutions available, select the suitable solutions

Sextupole optimization

DA MOGA optimization by varying geometrical sextupoles

1. Choose the number of initial populations and the number of generations
2. Start from random seeds for sextupole configuration
3. For each configuration, calculate NDTs up to 2nd order using the formulae derived by C-X. Wang (**> 30 terms**)
4. Implement standard MOGA iteration
5. Using DA tracking code to pick the best solution(s) from the last generation
6. If no satisfied DA is found, repeat step 3-4, or some modification on linear optics might be necessary

Summary

- 1-st Low Emittance Rings Workshop, CERN 12 - 15 January 2010
- 2-nd Low Emittance Rings Workshop, Crete, 3 - 5 October 2011
- 3-rd Low Emittance Rings Workshop, Oxford, 8 -10 July 2013 (supported by EuCARD - 2 Network)
- 4-th Low Emittance Rings Workshop, Frascati, 17-19 September 2014 (supported by EuCARD - 2 Network)

1st Workshop on Low Emittance Lattice Design, Barcelona, 23-24 April 2015

(supported by EuCARD - 2 Network)

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