Center for the Advancement of Natural Discoveries using Light Emission

Resonance Properties of Electron Beams Radiation in a Two Layer Waveguide

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Problem Descriptions

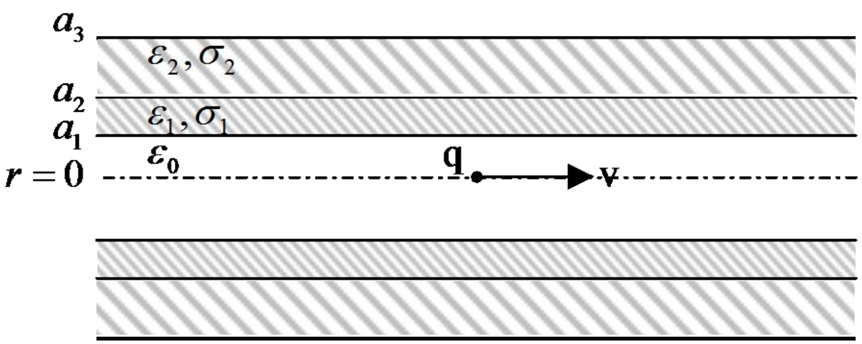


Fig.1 Geometry of two layer tube

a_1 – inner radius

a_3 – outer radius

$d_1 = a_2 - a_1$ – cover tickness

ε_1 – first layer permittivity

σ_1 – first layer conductivity

ε_2 – second layer permittivity

σ_2 – second layer conductivity

q – point-like particle charge

v – constant velocity

Impedances and Wakefields for Ultrarelativistic Particle.

For the range $a_1^{-1} << |\chi_1| << d_1^{-1}$

χ_1-transvere propogation constant

$$Z_{\parallel}^{0}(\omega) = \frac{Z_0 c}{\pi a_1^2} \frac{\alpha\omega^2 + j\omega(\omega^2 - \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + \alpha^2\omega^2}$$

where $\alpha = \frac{2c}{\sqrt{3}a_1}\left(\xi + \xi^{-1}\right), \xi = d_1\sigma_1 Z_0 / \sqrt{3}$

$$W_{\parallel}(s) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\parallel}(\omega) e^{-j\frac{\omega}{c}s} d\omega$$

$$W_{\parallel}(s) = \frac{Z_0 c}{\pi a_1^2}\left(\frac{\alpha}{\beta}\sin\left(\beta\frac{s}{2c}\right) - \cos\left(\beta\frac{s}{2c}\right)\right) e^{-\frac{\alpha s}{2c}}$$

where $\beta = \sqrt{4\omega_0^2 - \alpha^2}$

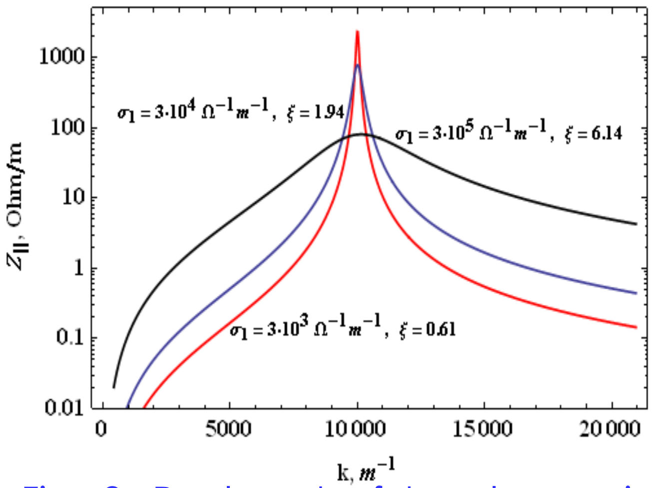


Fig. 2. Real part of two layer pipe longitudinal impedance for various conductivities of inner layer.

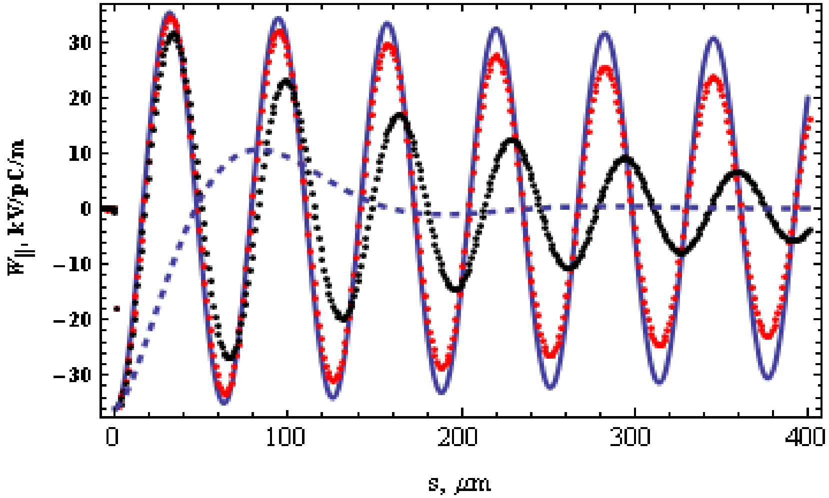


Fig. 3. Wake function, $a_1 = 1mm$, $d_1 = 0{,}2\mu m$, $\sigma_1 = 3 * 10^4\Omega^{-1}m^{-1}$perfectly conducting external layer (blue, solid),finite conducting external layer (black, dotted), single-layer waveguide (blue, dashed).

Connection Between Resonance Frequencies and Synchronous Mode

For the range $a_1^{-1} \ll |\chi_1| \ll d_1^{-1}$ eigenvalue equation is given by

$$\frac{1}{\nu_{0,i} a_1} \frac{J_1(\nu_{0,i} a_1)}{J_0(\nu_{0,i} a_1)} = \frac{1}{k^2 a_1 d_1}$$

Specific solution with $(\nu_{0,i} = 0)$ is $k_0 = \sqrt{2/a_1 d}$.

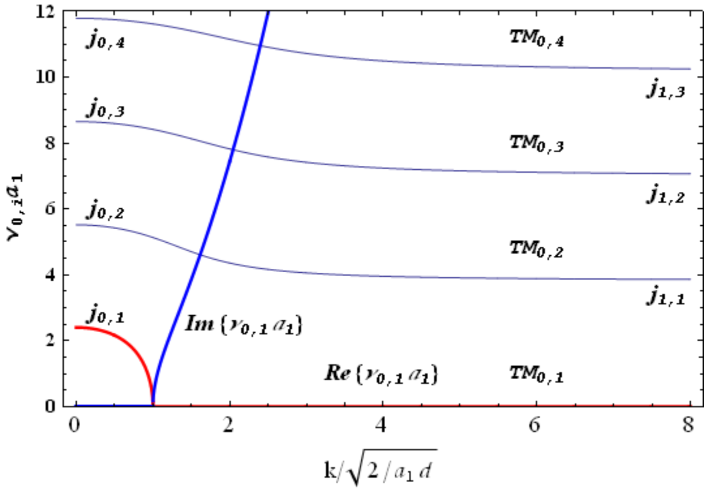


Fig. 4. Real (red) and imaginary (blue) parts of TM_{01} transverse eigenvalues versus frequency. Real transverse eigenvalues for the high axially symmetric TM modes.

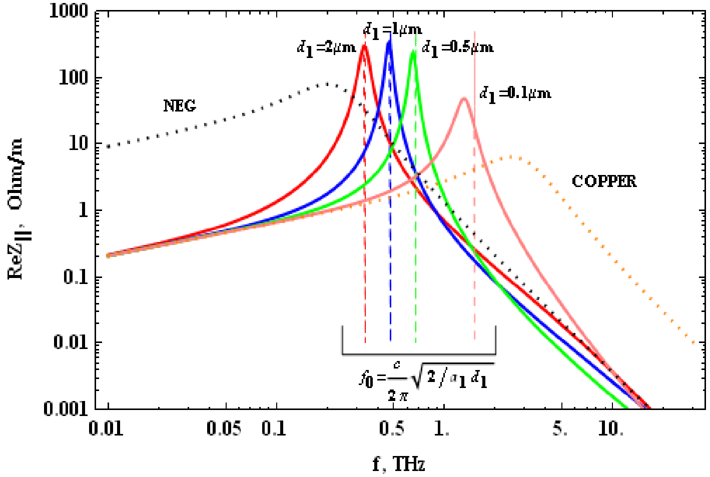


Fig. 5. Real part of copper-NEG unbounded tube impedances for various cover thickness.

Radiation From Open End Waveguide

$$\begin{Bmatrix} E_\phi(\vec{R}) \\ E_\theta(\vec{R}) \end{Bmatrix} = 2qZ_0 \frac{c}{d_1} F(\theta) e^{-\frac{\alpha}{2}(t-R/c)} \frac{e^{j\omega_0(t-R/c)}}{R} \begin{Bmatrix} \cos\theta \sin\phi \\ \cos\phi \end{Bmatrix} \qquad E_R(\vec{R}) = 0$$

The radiation pattern is zero at the principal direction $\theta = 0$ and reaches its maximum value at $\theta = \arcsin(2{,}3\sqrt{d_1/2a_1})$. Meanwhile, at $\frac{d_1}{2a_1} \ll 1$ it is close to the principal direction. The main output power (about 75%) is contained in a cone with a resolution $\Delta\theta = 2\arcsin(\chi\sqrt{d_1/2a_1})$, where χ the first root of the Bessel function J_2 .

$$F(\theta) = \frac{J_2\left(\sqrt{2a_1/d_1}\sin\theta\right)}{\sqrt{2a_1/d_1}\sin\theta}$$

$$P = \frac{\pi q^2 Z_0 \omega_0^2}{2}\left(1 - \frac{d_1}{2a_1}\right) e^{-\beta(t-R/c)}$$

$d_1, \mu m$	$\sigma_1, \Omega^{-1}m^{-1}$	a_1, mm	P_{max}	f_0, THz
0,2	$3 * 10^4$	1	$0{,}5MW$	4,775
0,2	$3 * 10^4$	10	$50kW$	1,510

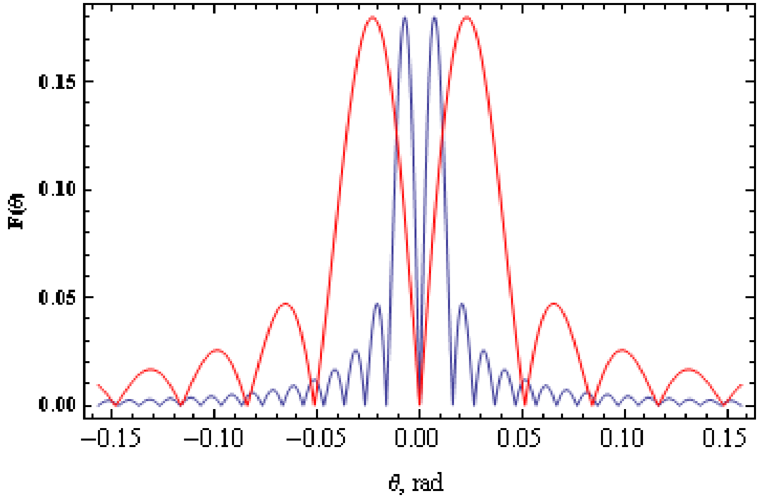


Fig. 5. Radiation pattern $d_1 = 0{,}2\mu m$, $a_1 = 1cm$ (blue), $a_1 = 1mm$ (red).

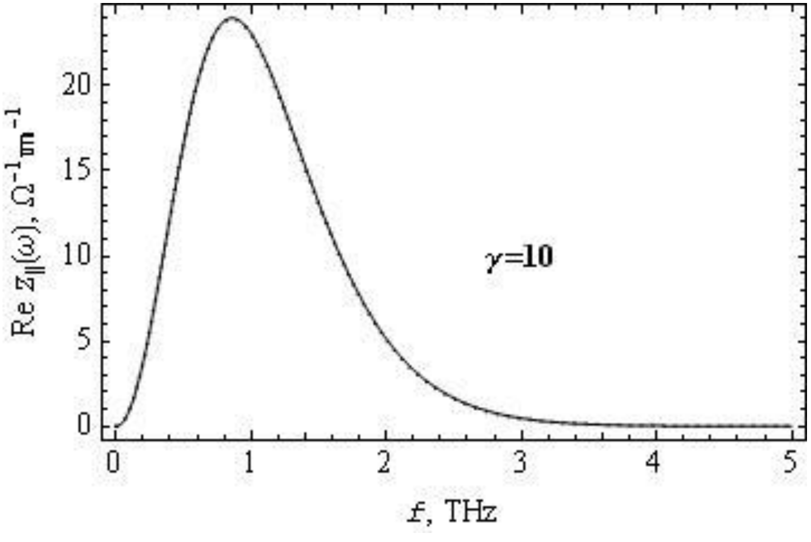
Impedances for Non Relativistic Particle

Impedance is given by

$$Z_{||} = \frac{j\varepsilon_0 \tau \, Z_0}{G(x,y)} I_0(\lambda r)$$

where $G(x,y) = 2\pi a_1 \beta \, I_0(x)(\varepsilon_0 I_1(x) + d_1 \varepsilon_1 k_v \tau \, I_0(x) \, \mathrm{cthy}/y)$

$x = a_1 k_v \tau, \; y = \chi_1 d_1, \; \lambda = k_v \tau, \tau = \gamma^{-1}, \; k_v = \omega/v$



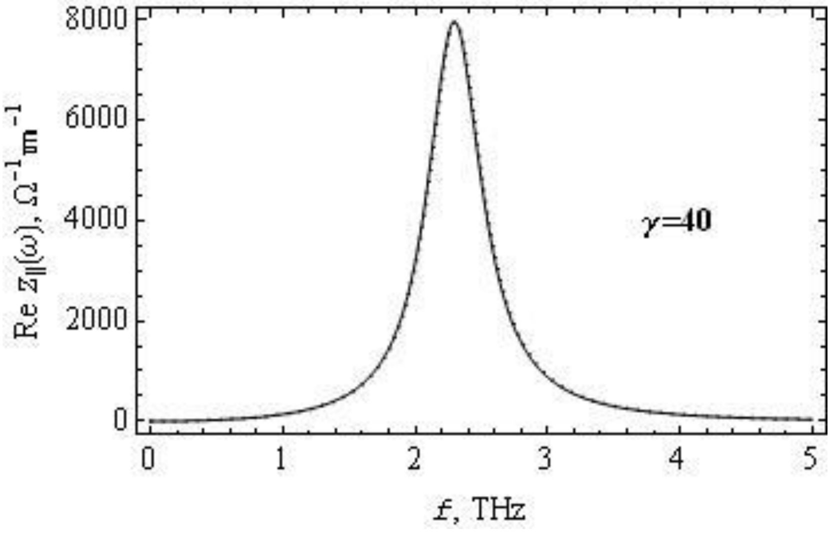


Fig. 6. Real part of two layer pipe longitudinal impedance for various lorentz factor $a_1 = 1mm, d_1 = 0.2\mu m, \sigma_1 = 3*10^4 \Omega^{-1} m^{-1}$.

Wake Functions for Non Relativistic Particle

$$W_{\|}(s) = C\,\mathrm{sign}(s)\sum_{k=1}^{n}\left(1-\mathrm{sign}(s\,\mathrm{Im}\,\omega_k)\right)\frac{j\omega_k - \varepsilon_0\tau^2\omega_k^2/\sigma_1\beta^2}{(a_1\tau/v)^{n-1}\prod_{i=1,\,i\neq k}^{n}(\omega_k-\omega_i)}\,e^{-j\frac{\omega_k}{v}s}$$

Where ω_i is the roots of function $G(x,y)$ $\quad C = 1.12105\times10^{14}\,a_1\sigma_1 Z_0^2/2\pi d_1$

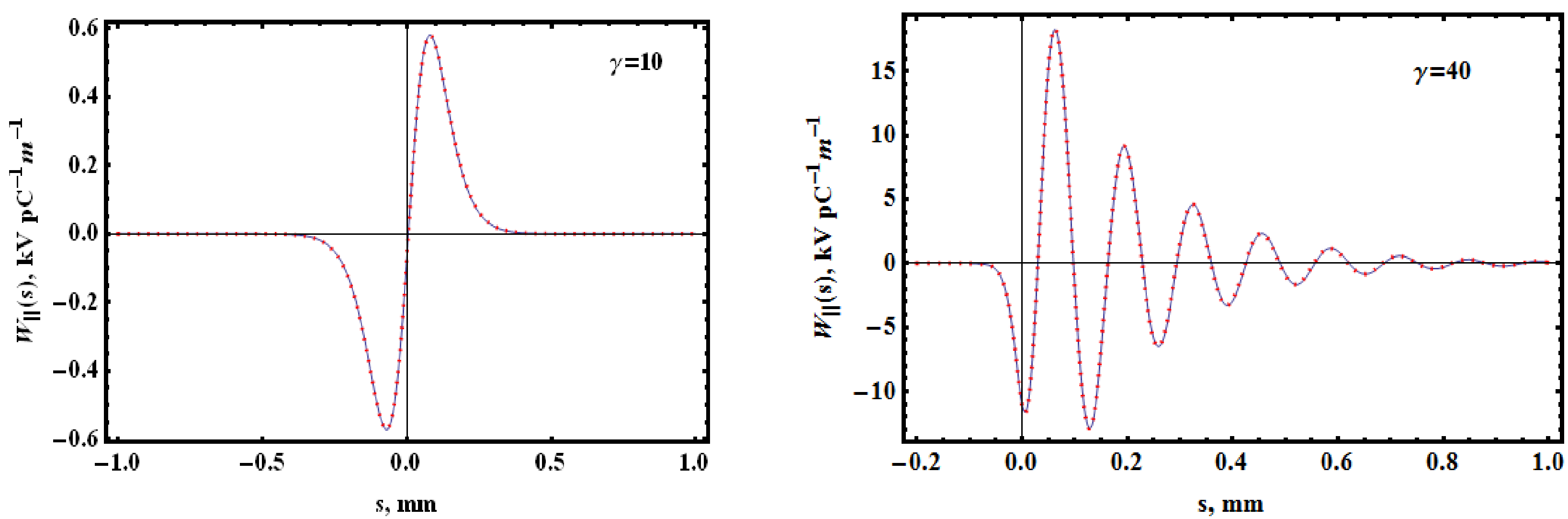


Fig. 7. Wake fields of two layer pipe for various lorentz factor $a_1 = 1mm, d_1 = 0.2\mu m,$ $\sigma_1 = 3*10^4\Omega^{-1}m^{-1}.$

Thank you for attention!!!