Low Emittance Storage Rings 2
(Overview)

A. Sargsyan
Outline

• Introduction

• Low emittance lattice concepts

• Low emittance upgrades

• DA optimization, MOGA

• Summary
Why is it important to achieve low beam emittance in a storage ring?

An important figure of merit \( \textbf{brightness} = \text{photon flux per unit area and per unit solid angle at the source.} \)

\[
B \sim \frac{F(\omega)}{\Sigma_x \Sigma_y (\Delta \omega/\omega)}
\]

spectral brightness or brilliance

\[
\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_r^2}, \quad \Sigma_{x',y'} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'}^2}
\]

Effective source size and angular divergence of radiation determined by the phase space convolution of single electron radiation and electron beam properties.

\[
\sigma_r = \sqrt{\frac{\lambda}{L_u}}, \quad \sigma_{r'} = \sqrt{\frac{\lambda L_u}{4\pi}}
\]

\( \lambda \) – on-axis undulator resonant wavelength
Introduction

Undulator brightness in Gaussian approximation (single electron)

Electron beam Gaussian distribution

\[ f(x, y, x', y') \approx \frac{1}{(2\pi)^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'}} \times \exp \left( - \frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y'^2}{2\sigma_{y'}^2} \right) \]

From convolution theorem

\[ B(x, y, x', y') \approx \frac{N_e F_1(\omega)}{(2\pi)^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \exp \left( - \frac{x^2}{2\Sigma_x^2} - \frac{y^2}{2\Sigma_y^2} - \frac{x'^2}{2\Sigma_{x'}^2} - \frac{y'^2}{2\Sigma_{y'}^2} \right) \]

1) Emittance dominated regime for peak brightness

\[ (\sigma_{x,y} \gg \sigma_r \text{ and } \sigma_{x',y'} \gg \sigma_{r'}) \]

\[ B_0 = \frac{N_e F_1(\omega)}{(2\pi)^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'}} = \frac{F(\omega)}{(2\pi)^2 \varepsilon_x \varepsilon_y} \]

2) Radiation dominated regime for peak brightness

\[ (\sigma_{x,y} \ll \sigma_r \text{ and } \sigma_{x,y'} \ll \sigma_{r'}) \]

\[ B_0 = \frac{F(\omega)}{(2\pi)^2 \sigma_r^2 \sigma_{r'}^2} = \frac{F(\omega)}{(\lambda/2)^2} \]

\[ \varepsilon_{x,y} \ll \frac{\lambda}{4\pi} \]

so-called \textit{diffraction limit}

For X-ray range – from several 10s to several 100s pm
Low emittance lattice concepts

The natural horizontal emittance and energy spread
(balance between the radiation damping and the quantum excitation)

\[ \varepsilon_{x0} = C_q \gamma^2 \cdot \frac{I_5}{I_2 - I_4} \]

\[ \sigma_\delta^2 = C_q \gamma^2 \cdot \frac{I_3}{2I_2 + I_4} \]

\[
\begin{align*}
I_2 &= \int_{bend} \frac{1}{\rho^2} ds \\
I_3 &= \int_{bend} \frac{1}{|\rho|^3} ds \\
I_4 &= \int_{bend} \frac{\eta_x}{\rho} \left( \frac{1}{\rho^2} + 2k_1 \right) ds \\
I_5 &= \int_{bend} \frac{H_x}{|\rho|^3} ds
\end{align*}
\]

\[ \Delta E [\text{keV}] = C_\gamma \gamma^4 I_2 \] - energy loss per turn

\[ \gamma - \text{beam energy} \]

\[ \rho - \text{bending radius} \]

\[ k_1 = \frac{e}{P_0} \frac{\partial B_y}{\partial x} - \text{quad strength} \]

\[ H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2 - \text{“dispersion emittance”} \]

\[ \alpha_x, \beta_x, \gamma_x - \text{Twiss parameters} \]

\[ \eta_x - \text{dispersion} \]

\[ C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \approx 3.832 \times 10^{-13} m \]

\[ C_\gamma = \frac{e^2}{3 \varepsilon_0 (mc^2)^4} \approx 8.846 \times 10^{-5} m / GeV^3 \]
**Low emittance lattice concepts**

Natural emittance in FODO, DBA and TME lattices (app. valid for small dipole bending angle)

<table>
<thead>
<tr>
<th>Lattice type</th>
<th>Minimum Emittance</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°FODO (90°FODO)</td>
<td>$\approx C_q \gamma^2 \theta^3$ ($\approx 2\sqrt{2}C_q \gamma^2 \theta^3$)</td>
<td>$f/L = 1/2$ ($f/L = 1/\sqrt{2}$)</td>
</tr>
<tr>
<td>DBA</td>
<td>$\approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$</td>
<td>$\eta_x = \eta'_x = 0$ $\beta_0 \approx \sqrt{12/5L}$ $\alpha_0 \approx \sqrt{15}$</td>
</tr>
<tr>
<td>TME</td>
<td>$\approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$</td>
<td>$\eta_{x_{\text{min}}}^{(c)} = L\theta/24$ $\beta_{x_{\text{min}}}^{(c)} = L/2\sqrt{15}$</td>
</tr>
</tbody>
</table>

$L$ – dipole length  
$\theta$ – dipole bending angel
Low emittance lattice concepts

Combination of DBA and TME - Multi-bend achromats

- **Double Bend Achromat (DBA)**
- **Triple Bend Achromat (TBA)**
- **Quadruple Bend Achromat (QBA)**

Dipoles with the same bending radius

For M-bend achromat

\[ \varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \frac{M + 1}{M - 1} \theta^3 \]

\( \beta \approx 3^{\frac{1}{3}} \) – value obtained from emittance minimization
Low emittance lattice concepts

As a summary
Achromats have been popular choices for storage ring lattices in third-generation synchrotron light sources for two reasons:
• they provide lower natural emittance than FODO lattices;
• they provide zero-dispersion locations appropriate for insertion devices (wigg. and undul.).

<table>
<thead>
<tr>
<th>Light sources</th>
<th>Lattice type</th>
<th>Emittance</th>
<th>Energy</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPring-8</td>
<td>DBA</td>
<td>3.4 nm rad</td>
<td>8 GeV</td>
<td>1436 m</td>
</tr>
<tr>
<td>ESRF</td>
<td>DBA</td>
<td>4 nm rad</td>
<td>6 GeV</td>
<td>844 m</td>
</tr>
<tr>
<td>DIAMOND</td>
<td>DBA</td>
<td>2.7 nm rad</td>
<td>3 GeV</td>
<td>560 m</td>
</tr>
<tr>
<td>SOLEIL</td>
<td>DBA</td>
<td>3.9 nm rad</td>
<td>2.75 GeV</td>
<td>354 m</td>
</tr>
<tr>
<td>ELETTRA</td>
<td>DBA</td>
<td>7 nm rad</td>
<td>2.4 GeV</td>
<td>259 m</td>
</tr>
<tr>
<td>CANDLE</td>
<td>DBA</td>
<td>8.4 nm rad</td>
<td>3 GeV</td>
<td>216 m</td>
</tr>
<tr>
<td>ALS</td>
<td>TBA</td>
<td>2 nm rad</td>
<td>1.9 GeV</td>
<td>197 m</td>
</tr>
<tr>
<td>SLS</td>
<td>TBA</td>
<td>4.8 nm rad</td>
<td>2.4 GeV</td>
<td>288 m</td>
</tr>
</tbody>
</table>

Increasing the number of bends in a single cell of an achromat("multiple-bend achromats") reduces the emittance, since the lattice functions in the "central" bends can be tuned to conditions for minimum emittance.

"Detuning" an achromat to allow some dispersion in the straights provides the possibility of further reduction in natural emittance, by moving towards the conditions for a theoretical minimum emittance (TME) lattice.
Low emittance lattice concepts

Recent approaches:
- Multi-Bend Achromat concept
- Use of damping wigglers
- Use of vertical focusing bending magnets (and/or Robinson wiggler)
- A round beam by coupling using skew quadrupoles (PETRA III, ALS upgrade)
- Use of dipoles with longitudinal field variation (SIRIUS, ESRF upgrade, SLS upgrade)

<table>
<thead>
<tr>
<th>Name</th>
<th>Lattice type</th>
<th>Energy [GeV]</th>
<th>Circumference [m]</th>
<th>Emittance [pm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX-IV</td>
<td>7BA</td>
<td>3.0</td>
<td>528</td>
<td>326</td>
</tr>
<tr>
<td>SIRIUS</td>
<td>5BA</td>
<td>3.0</td>
<td>518</td>
<td>280</td>
</tr>
<tr>
<td>ESRF upgrade</td>
<td>Hybrid 7BA</td>
<td>6.0</td>
<td>844</td>
<td>132</td>
</tr>
<tr>
<td>APS upgrade</td>
<td>Hybrid 7BA</td>
<td>6.0</td>
<td>1104</td>
<td>65</td>
</tr>
<tr>
<td>SPRING 8 upgrade study</td>
<td>6-10BA</td>
<td>6.0</td>
<td>1436</td>
<td>68</td>
</tr>
<tr>
<td>DIAMOND upgrade</td>
<td>4BA</td>
<td>3.0</td>
<td>562</td>
<td>280</td>
</tr>
<tr>
<td>ANKA upgrade study</td>
<td>4BA</td>
<td>2.5</td>
<td>110</td>
<td>8600</td>
</tr>
<tr>
<td>SLS upgrade studies</td>
<td>Hybrid 7BA</td>
<td>2.4</td>
<td>288</td>
<td>135</td>
</tr>
<tr>
<td>ELETTRA upgrade study</td>
<td>6BA</td>
<td>2.0</td>
<td>260</td>
<td>280</td>
</tr>
<tr>
<td>ILSF project</td>
<td>5BA</td>
<td>3.0</td>
<td>528</td>
<td>477</td>
</tr>
<tr>
<td>CANDLE upgrade study</td>
<td>4BA</td>
<td>3.0</td>
<td>257</td>
<td>933</td>
</tr>
</tbody>
</table>
Low emittance upgrades

MAX-IV: 7-MBA Structure, $E = 3.0$ GeV, $\varepsilon = 326$ pmrad, $N = 20$, $C = 528$ m

$\beta_X / m$  $\beta_Y / m$  $100 \ast \text{DispX} / m$

Machine function / m

s / m
Low emittance upgrades

SIRIUS: 5-MBA Structure, $E = 3.0$ GeV, $\epsilon = 280$ pmrad, $N = 20$, $C = 518$ m
1.) The achromat has at the beginning and end a DBA structure with combined bendings. 2.) The long bendings have a longitudinal gradient. 3.) The phase advance between the sextupole is $\approx n^*\pi$. 
The people at APS made a comparison between the MAX IV lattice (7MBA) and the lattice of the ESRF (HMBA) and found that the HMBA as some advantages (the sextupole strengths are smaller).
Low emittance upgrades

Diamond II: DDBA Structure, $E = 3.0$ GeV, $\epsilon = 280$ pmrad, $N = 24$, $C = 543$ m
Low emittance upgrades

ANKA/LESR: DBA Structure, $E = 2.5$ GeV, $\epsilon = 8.6$ nmrad

$C = 110$ m
Low emittance upgrades

ELETTRA II: 6-MBA Structure, $E = 2.0$ GeV, $\varepsilon = 280$ pmrad, $N = 12$, $C = 259.8$ m
Low emittance upgrades

There are 5 bendings in a unit cell

\( B = -0.3633 \ T \)
\( k = 3.87 \ m^{-2} \)
\( \Phi = -0.78 \ \text{degr.} \)

\( B = 0.70636 \ T \)
\( k = -3.6441 \ m^{-2} \)
\( \Phi = 1.09 \ \text{degr.} \)
Low emittance upgrades

$E = 2.4 \text{ GeV}, \quad \varepsilon = 134 \text{ pm} \cdot \text{rad}, \quad C = 288 \text{ m}, \quad N = 12$

Matching Cell

Unit Cell

Unit Cell

Unit Cell

Unit Cell

Unit Cell

Matching Cell

---

Scaling MAX IV to SLS size and energy gives $\varepsilon \approx 1 \text{ nm}$

LGB/AB-cell based MBA $\Rightarrow \varepsilon \approx 100...200 \text{ pm}$
CANDLE upgrade: 4BA, $E=3\text{GeV}$, $\varepsilon=0.933\text{nm}$, $C=257\text{m}$, $N=16$
Low emittance upgrades

Longitudinal field variation to compensate $H_x$ variation.

\[ I_5 = \int_{bend} \frac{H_x ds}{|\rho|^3} \]

\[ H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2 \]
Low emittance upgrades

Longitudinal field variation to compensate $H_x$ variation.

### Beam dynamics in bending magnet

$$H = \frac{\eta^2 + (\alpha \eta + \beta \eta')^2}{\beta}$$

- Curvature is source of dispersion:
  $$\eta''(s) \sim \frac{1}{\rho(s)} \rightarrow \eta'(s) \rightarrow \eta(s)$$

- Horizontal optics ~ like drift space:
  $$\beta(s) = \beta_0 - 2\alpha_0 s + \frac{1+\alpha_0^2}{\beta_0} s^2$$

Problem:

$$I_5 = \int_L f(\alpha_0, \beta_0, \eta_0, \eta'_0, \rho) \, ds \rightarrow \min$$

- too complicated to solve

### Numerical optimization

- Symmetric half bend in $N$ slices:
  - curvature $\rho_i$, length $\Delta s_i$
- Knobs for minimizer:
  - $\{\rho_i\}$, $\{\Delta s_i\}$, $\beta_0$, $\alpha_0$, $\eta_0, \eta_0'$
- Objective: $I_5$
- Constraints:
  - length: $\Sigma \Delta s_i = L/2$
  - angle: $\Sigma \rho_i \Delta s_i = \Phi/2$
  - [ field: $\rho_i > \rho_{\text{min}}$ ]
  - [ optics: $\beta_0$, $\eta_0$ ]
Low emittance upgrades

LGB editor of OPA

LGBs found in Lattice
none

Half symmetric bend

Angle [deg] 4.000
Length [m] 0.400
Slices 12

Minimum beta [m] 0.060
Maximum B-field [T] 5.00

Optimize field profile

Successful termination.
Create new Name LGB
overwrite

Show field B(s)

Write data

Reduced emittance I5 [1e-6/m] 0.714 0.322
Iso-mag emittance [pm rad] 86.13 27.27
Beta at center [m] 0.1033 0.0741
Disp at center [mm] 4.654 0.655
Iso-mag radiation loss [keV] 15.4 22.0
Iso-mag energy spread [o/oo] 0.3582 0.4919

Exit
1st approach

Need for installation of several families of “harmonic” sextupoles and octupoles to compensate adverse effects from the chromatic sextupoles:

- third order sextupolar resonances
- fourth order octupolar resonances
- amplitude dependent tune shifts
- second order chromaticities

by minimization of analytically calculated NDTs of 1\textsuperscript{st} and 2\textsuperscript{nd} order in sextupole strengths and 1\textsuperscript{st} order in octupole strength (with weights).

2nd approach

A pair of identical sextupoles connected by a minus-identity matrix transformer in ideal case of kick-like magnets cancels all geometrical aberrations and provide infinite dynamic aperture.

Finite length sextupole pair cancels up to the second order geometrical aberrations. Higher orders still exist, but the DA is large (although not infinite).

Testing by dynamic aperture tracking including misalignments, magnet errors, etc.
Application of Multi Objective Genetic Algorithms (MOGA) for DA optimization.

Issues in conventional approach:
- There are numerous NDTs. Which ones are dominating DA? If only single penalty function is used, how to specify weight to each NDT?
- Having small NDTs is an necessary but insufficient condition for having a large DA

A: small NDTs and large DAs
B: small NDTs but small DAs
C: large NDTs and small DAs

Multi-objective optimization is suitable

Genetic Algorithm (GA) mimics the evolution of nature:
- Crossover: generate children from parents.
- Mutation change the children.
- Natural selection: keep only certain number of population.

MOGA
1. Initialize population (first generation, random)
2. Repeat (generation by generation)
   3. crossover: 2 parents generate 2 children.
   4. mutation: change children.
   5. calculate children’s parameters(par. comp.)
   6. natural selection: sorting (non-dominated)
3. Until stop criteria fulfills (find solution)
4. A bunch of candidate solutions available, select the suitable solutions
DA MOGA optimization by varying geometrical sextupoles

1. Choose the number of initial populations and the number of generations

2. Start from random seeds for sextupole configuration

3. For each configuration, calculate NDTs up to 2\textsuperscript{nd} order using the formulae derived by C-X. Wang (> 30 terms)

4. Implement standard MOGA iteration

5. Using DA tracking code to pick the best solution(s) from the last generation

6. If no satisfied DA is found, repeat step 3-4, or some modification on linear optics might be necessary
1. About M in M-bend achromat
   - *M=7 appears to be a good balance between difficulty and performance*

2. About breaking symmetry
   - Need to be very, very careful with nonlinearities

3. About usage of LGBs combined with anti-bends
   - Need to deal with momentum acceptance issues

4. The role of MOGA
   - Less understanding of physics behind the results
Summary

- 1-st Low Emittance Rings Workshop, CERN 12 - 15 January 2010
- 2-nd Low Emittance Rings Workshop, Crete, 3 - 5 October 2011
- 3-rd Low Emittance Rings Workshop, Oxford, 8 -10 July 2013 (supported by EuCARD - 2 Network)
- 4-th Low Emittance Rings Workshop, Frascati, 17-19 September 2014 (supported by EuCARD - 2 Network)
- 1st Workshop on Low Emittance Lattice Design, Barcelona, 23-24 April 2015 (supported by EuCARD - 2 Network)

2nd Workshop on Low Emittance Lattice Design, September 2015, ESRF

EuCARD-2: Enhanced European Coordination for Accelerator R&D