Linearization of the longitudinal phase space without higher harmonic field

Ultrafast Beams and Applications

Benno Zeitler
CFEL, UHH, LAOLA.

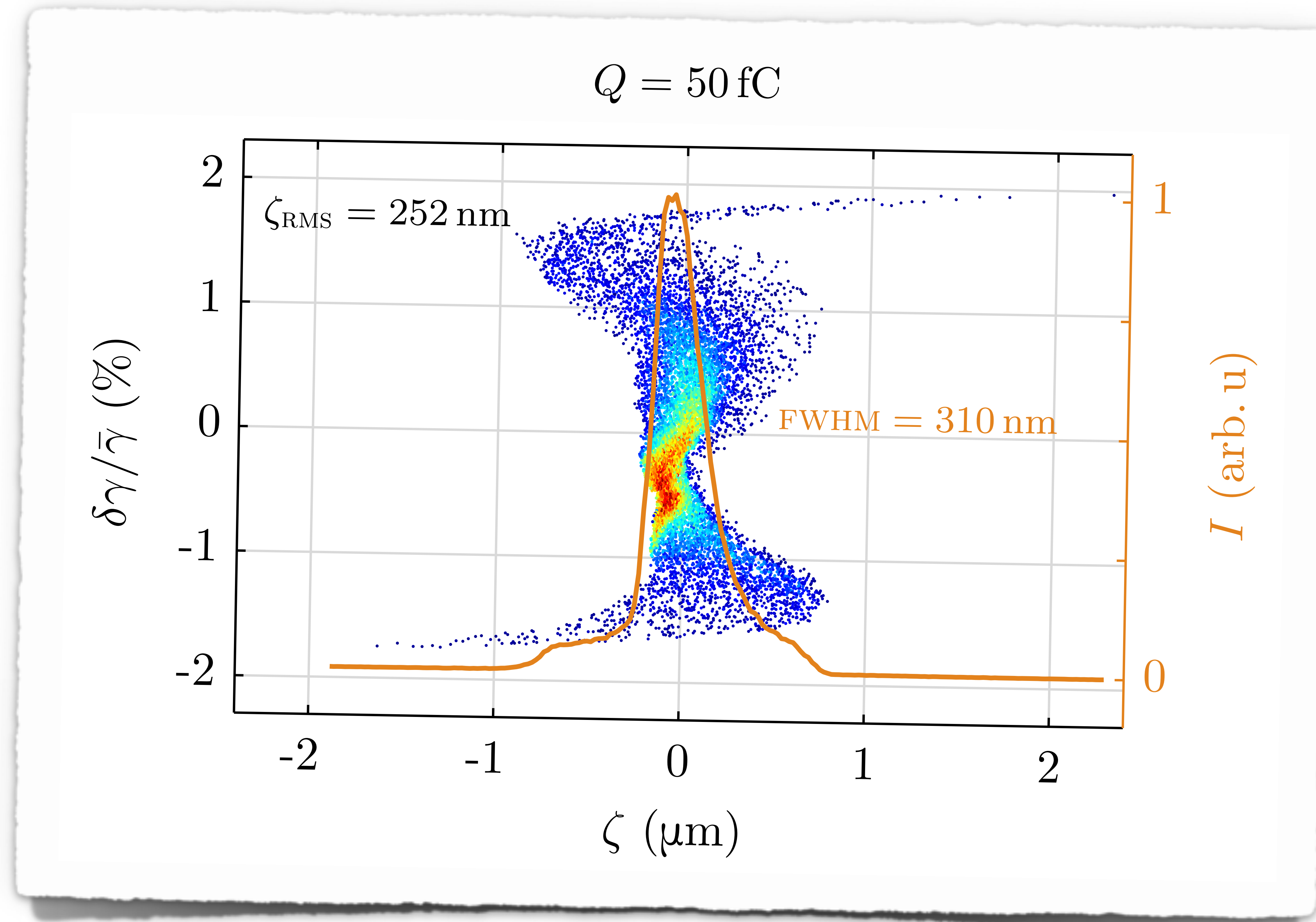
CFEL SCIENCE

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung
05K16GUB

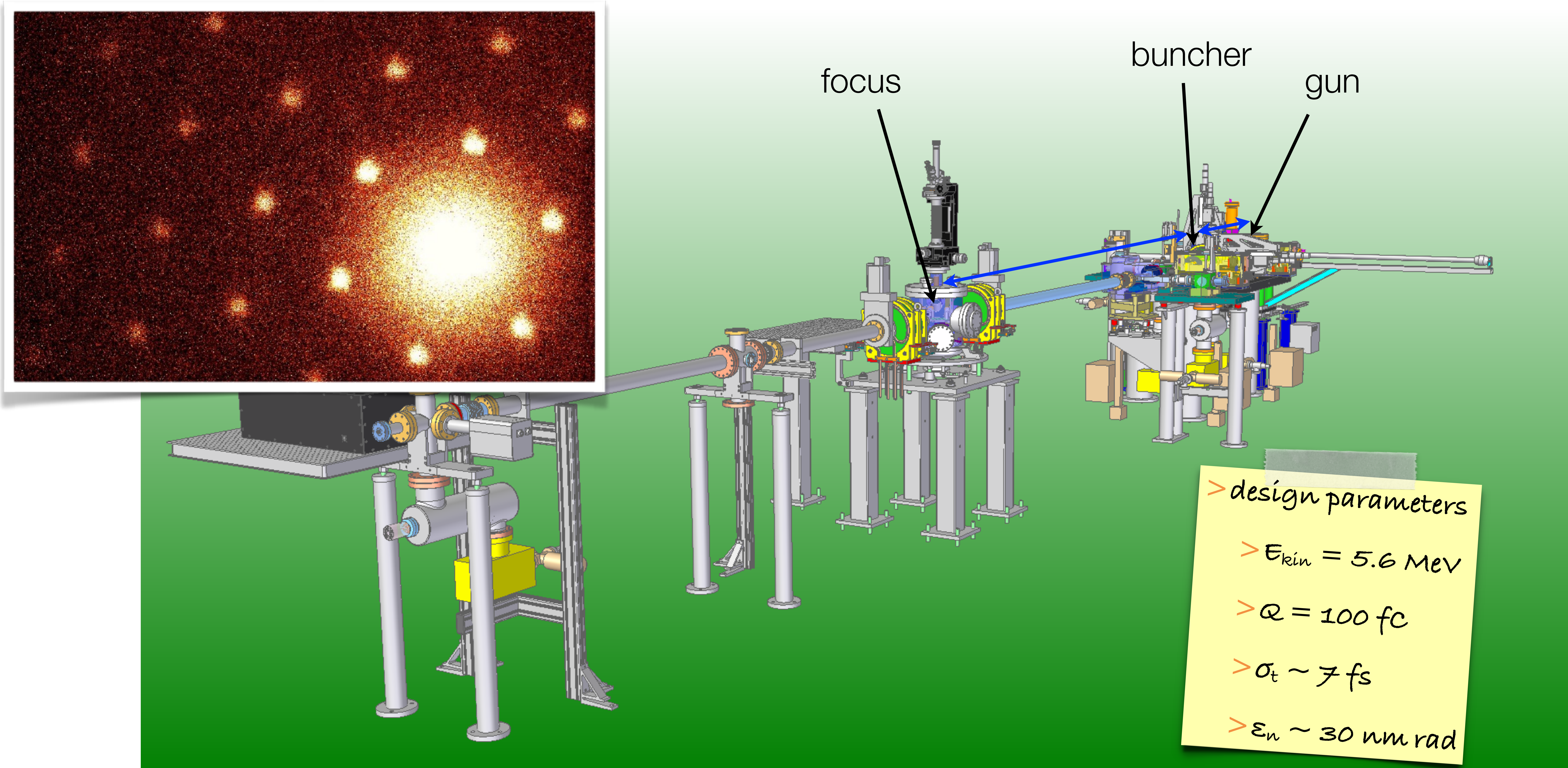
LAOLA. is a collaboration of UHH DESY

Outline

- > REGAE
- > ballistic bunching mechanism
- > phase space curvature
- > linearization concept
- > applications
- > how to measure?



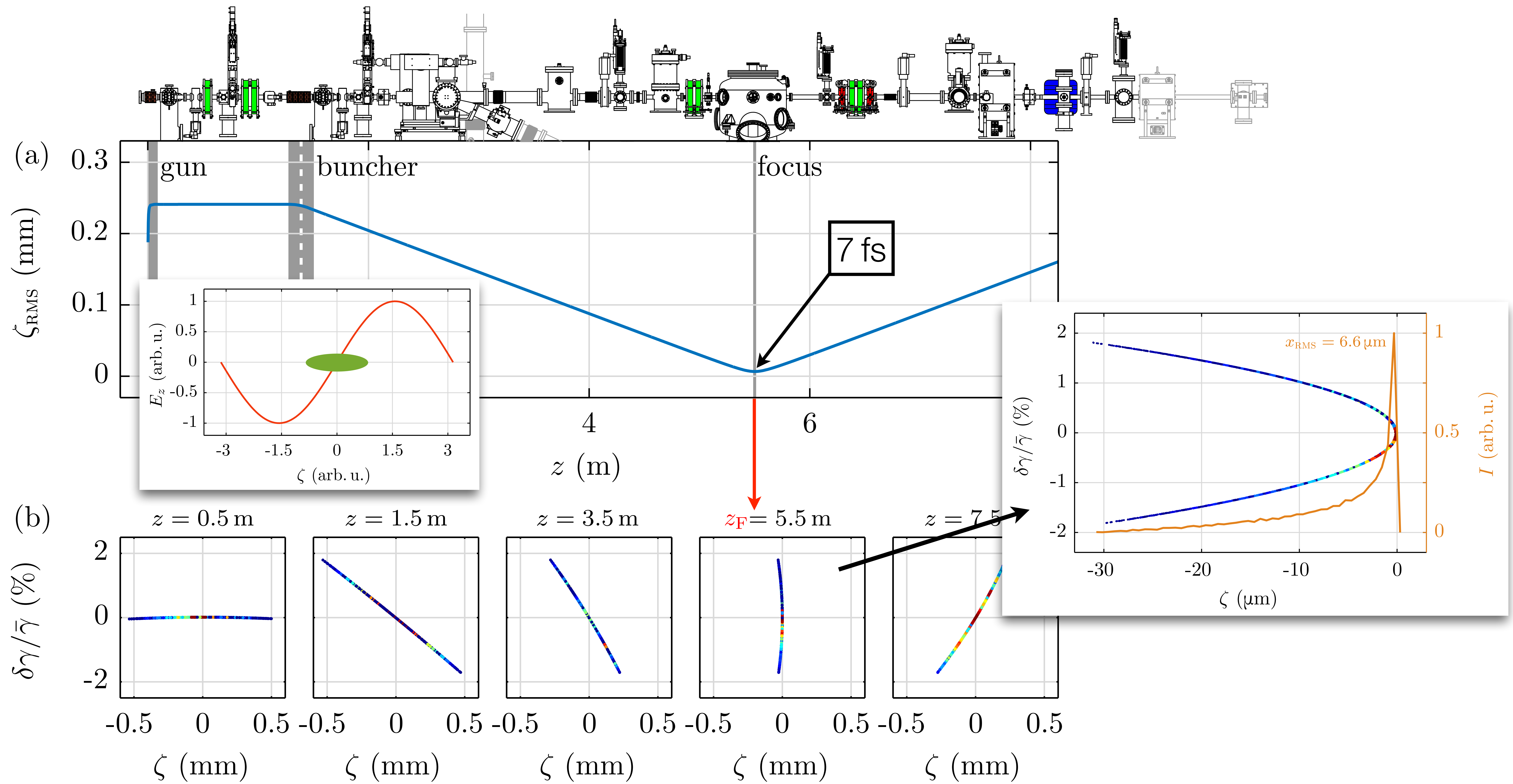
REGAE — Relativistic Electron Gun for Atomic Exploration



> design parameters
> $E_{kin} = 5.6$ MeV
> $Q = 100$ fC
> $\sigma_t \sim 7$ fs
> $\varepsilon_n \sim 30$ nm rad

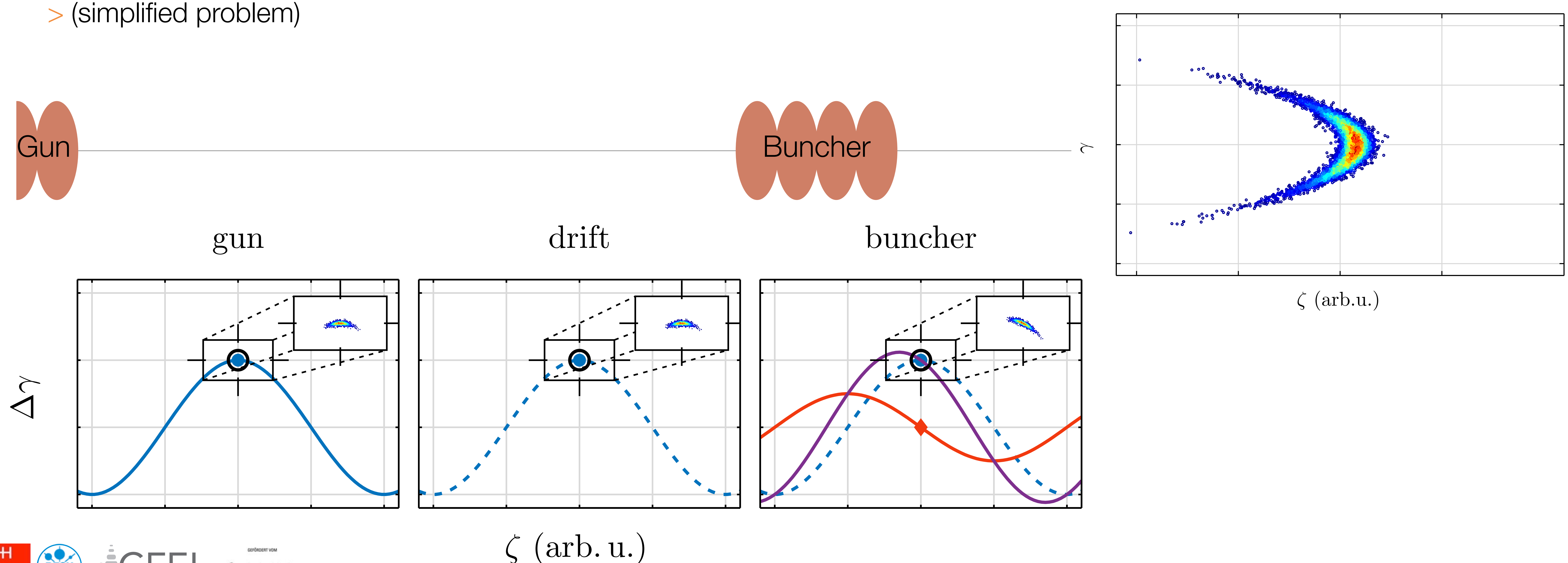
see also talk by K. Floettmann

Ballistic Bunching



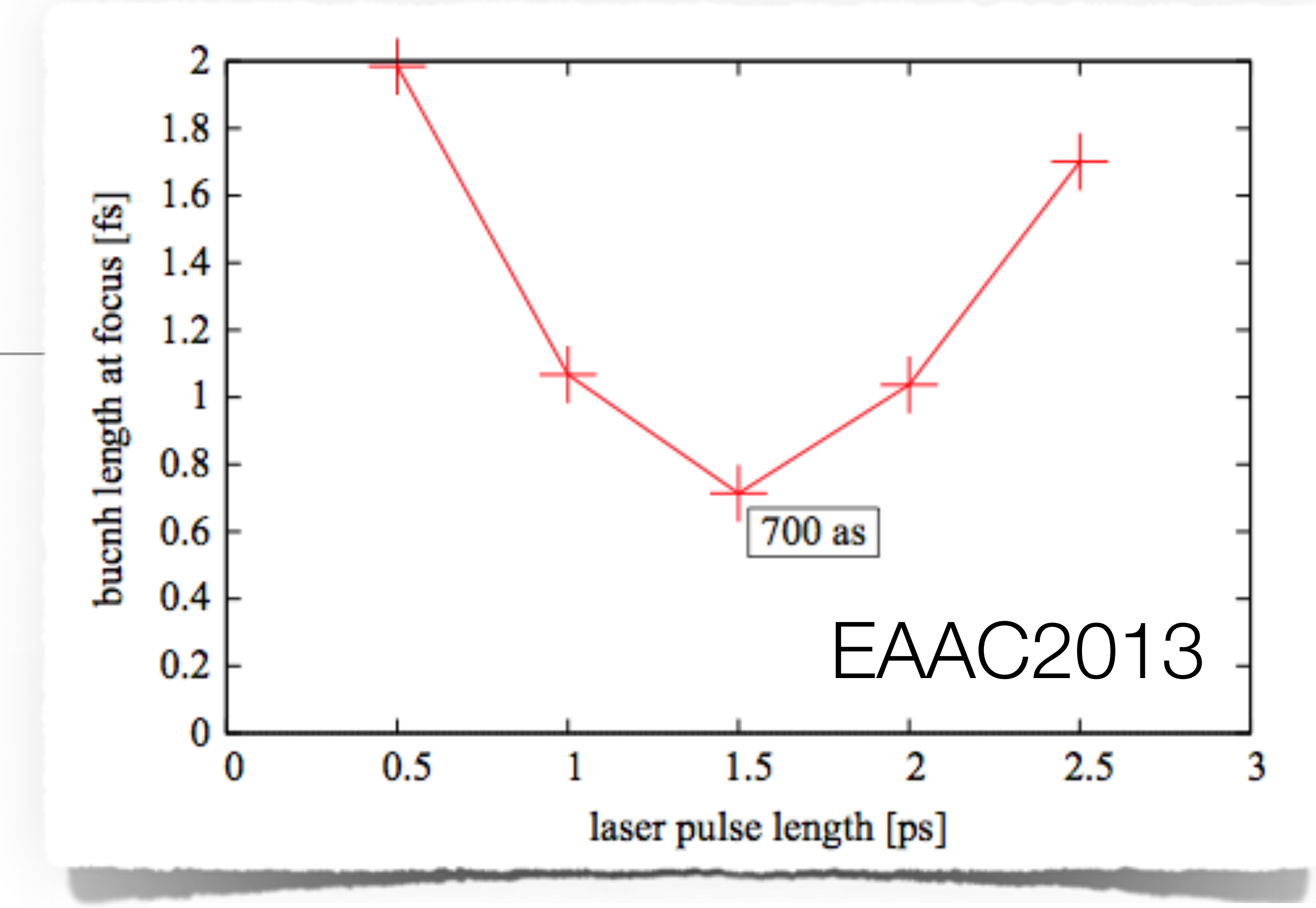
Phase Space Linearization

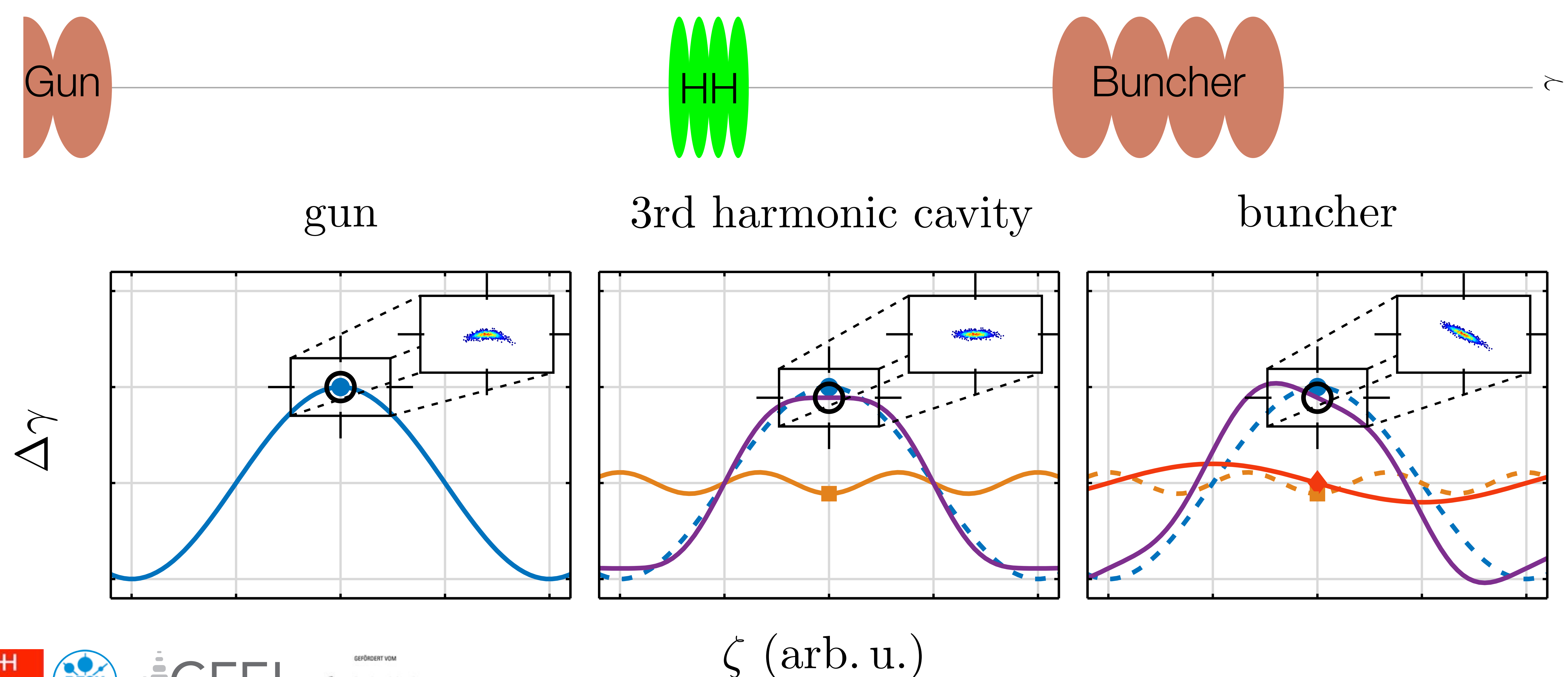
- origin of curvature?
  - curvature of accelerating fields
  - combined with finite bunch length
  - (simplified problem)

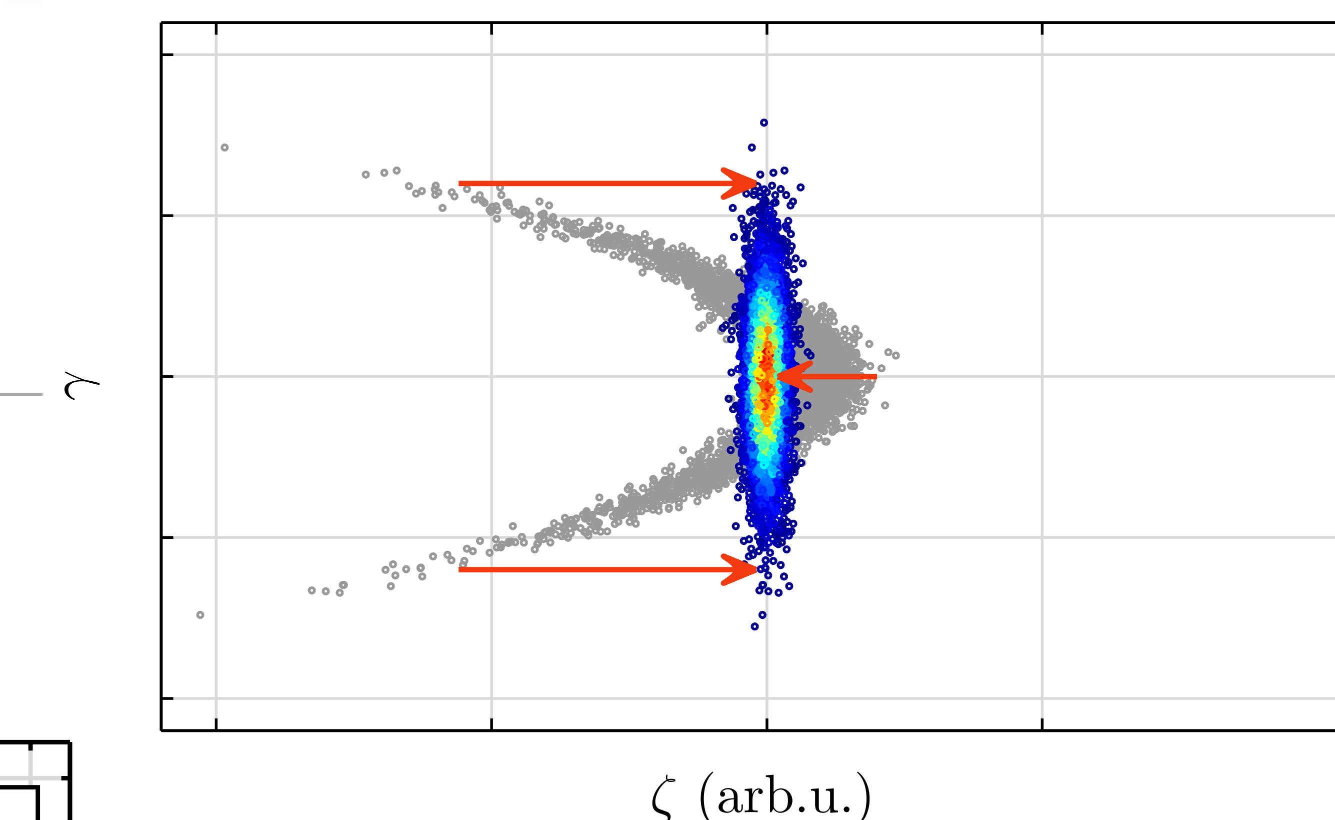


Phase Space Linearization

> solution 1: 3rd harmonic cavity
  > works: 700 as (K. Floettmann, Nucl. Instr. Meth. Phys. Res. A, Vol. 740, 2014)
  > no space & money: X-band structure…

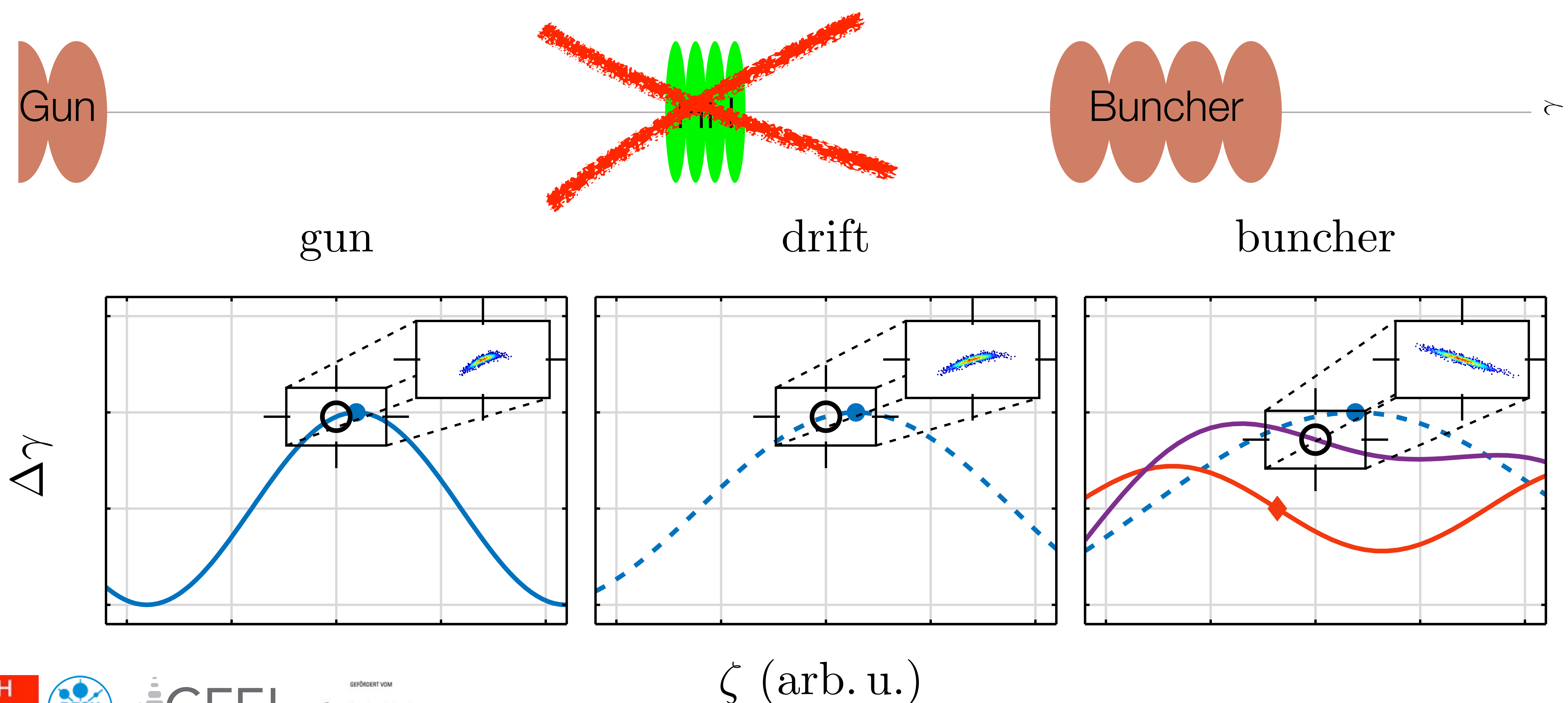


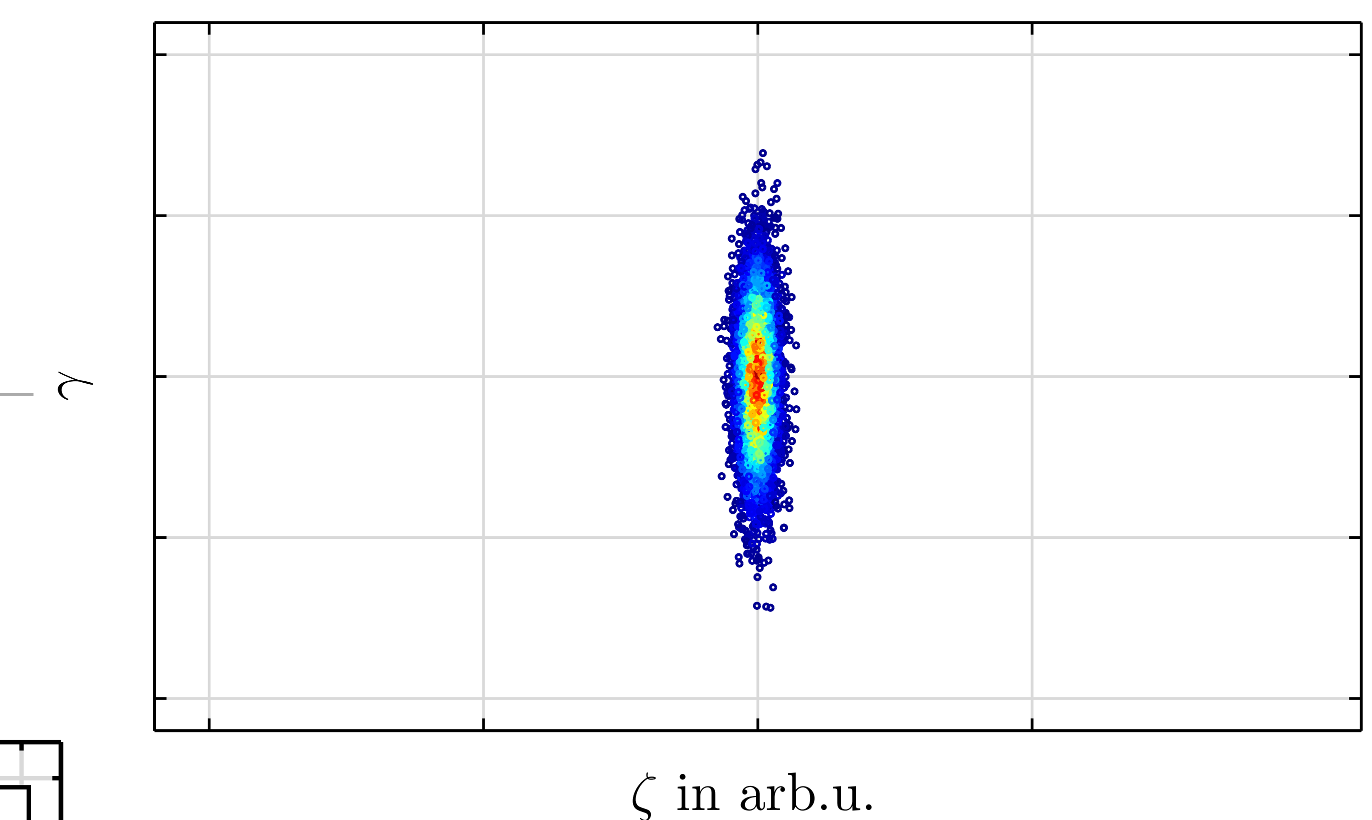




Phase Space Linearization

- solution 2: **stretcher mode**
  - gun settings: (far) off-crest
  - beam expansion between gun and buncher
    - pseudo HH with **same** RF system as gun

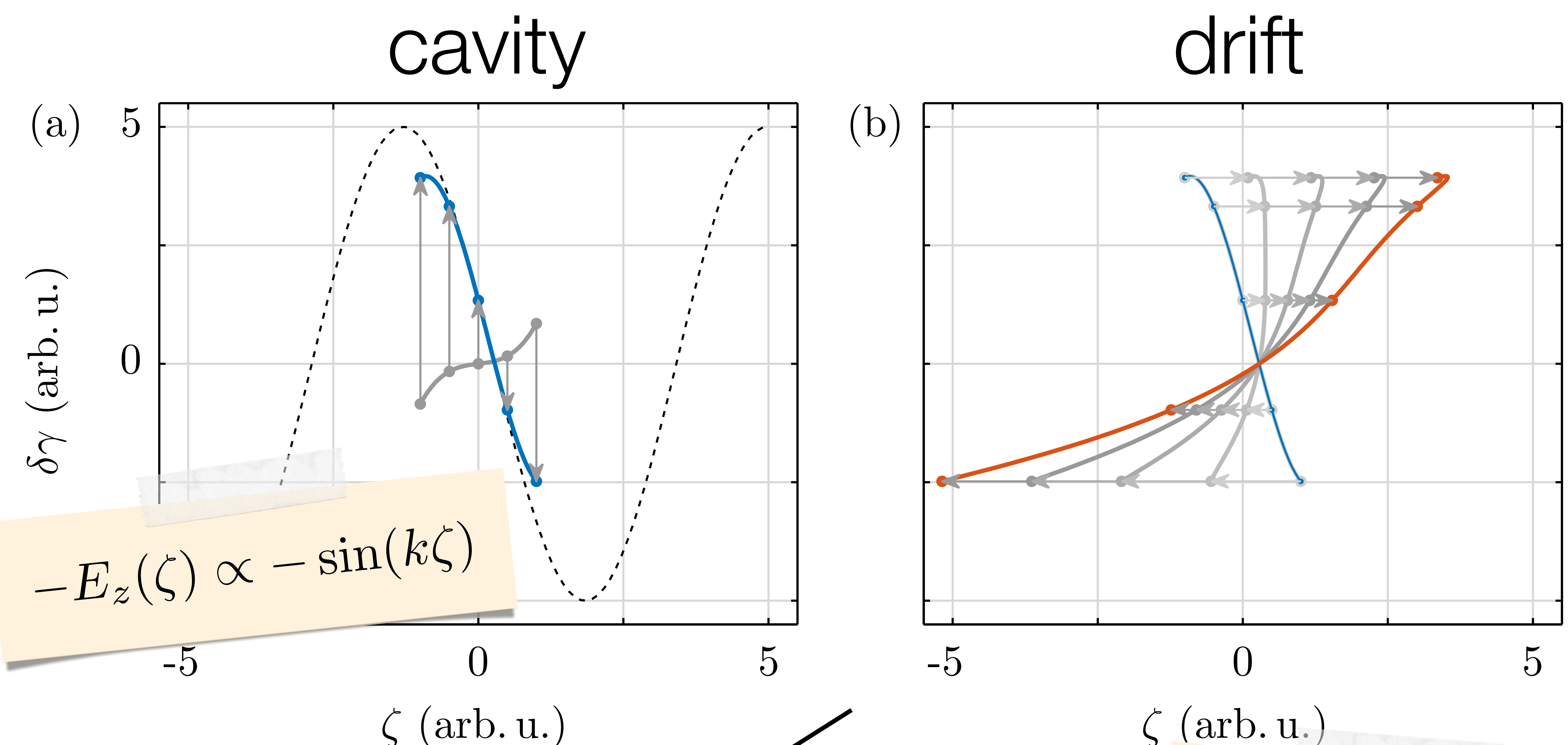




Phase Space Linearization

- nonlinear field structure
  - *finite* bunch length
  - field curvature passed on to bunch
  - no linearly correlated energy spread
  - changes γ

- nonlinear bunch evolution
  - no linear correlated *velocity* spread
  - curvature due to drift
  - changes ζ



$$-E_z(\zeta) \propto -\sin(k\zeta)$$

$$\beta(\gamma) = \sqrt{1 - \frac{1}{\gamma^2}}$$

Phase space coordinates: $\big(\zeta(z), \gamma(z)\big)$

Phase Space Linearization: Theory

> curvature due to drift…

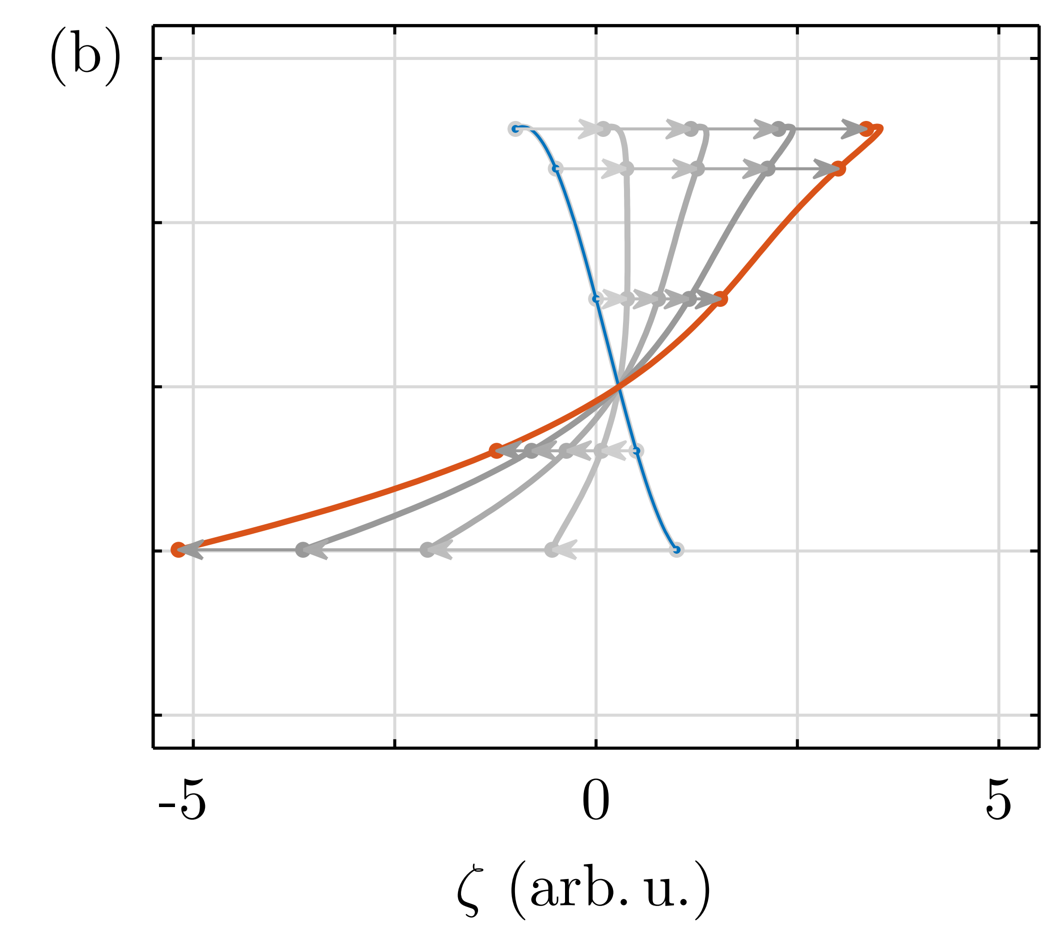
$$\Delta\beta \cdot c \qquad \frac{1}{\beta c}\cdot(z - z_0)$$

$$\Delta\zeta(z) = \Delta v \cdot \big(t(z) - t(z_0)\big) = \frac{1}{\bar\beta}\Delta\beta(\gamma)\cdot(z - z_0)$$

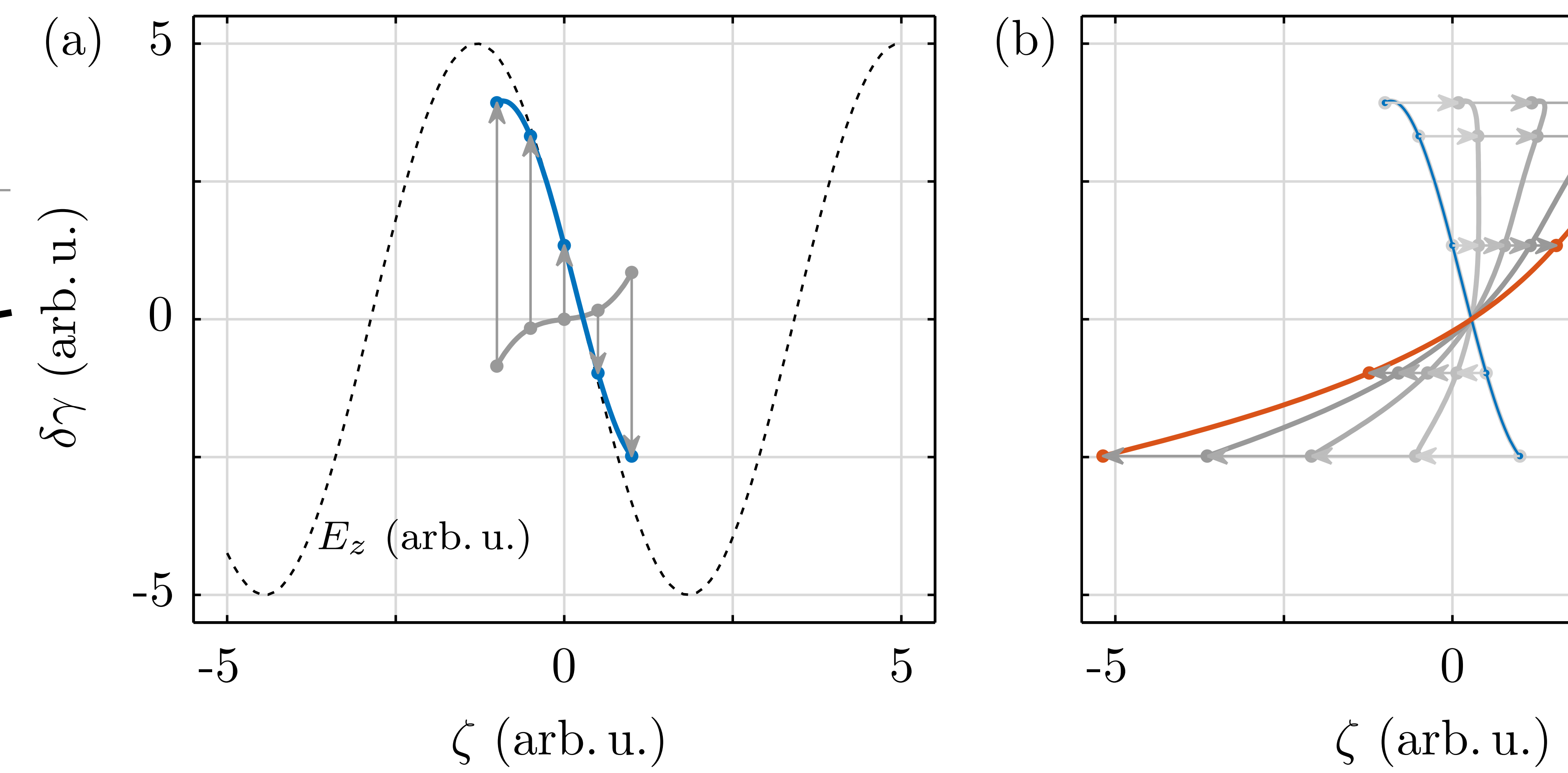
$$= \frac{1}{\bar\beta}\left[\frac{\mathrm{d}\beta}{\mathrm{d}\gamma}\bigg|_{\bar\gamma}\cdot\delta\gamma + \frac{1}{2}\frac{\mathrm{d}^2\beta}{\mathrm{d}\gamma^2}\bigg|_{\bar\gamma}\cdot(\delta\gamma)^2 + \frac{1}{6}\frac{\mathrm{d}^3\beta}{\mathrm{d}\gamma^3}\bigg|_{\bar\gamma}\cdot(\delta\gamma)^3 + \cdots\right]_{z_0}\cdot(z - z_0)$$

$$= \left[\underbrace{\frac{1}{\bar\gamma^3\bar\beta^2}}_{\eta_1(\bar\gamma)}\cdot\delta\gamma + \underbrace{\frac{2-3\bar\gamma^2}{2\bar\gamma^6\bar\beta^4}}_{\eta_2(\bar\gamma)}\cdot(\delta\gamma)^2 + \underbrace{\frac{2-5\bar\gamma^2+4\bar\gamma^4}{2\bar\gamma^9\bar\beta^6}}_{\eta_3(\bar\gamma)}\cdot(\delta\gamma)^3 + \cdots\right]_{z_0}\cdot(z - z_0)$$

$$\beta(\gamma) = \sqrt{1 - \frac{1}{\gamma^2}}$$



Phase Space Linearization: Theory



> start distribution at gun:

$$\gamma(\zeta_0) = \underbrace{A_0}_{\bar{\gamma}} + \underbrace{A_1\zeta_0 + A_2\zeta_0^2 + A_3\zeta_0^3}_{\delta\gamma(\zeta_0)}$$

> description of drift:

$$\zeta(z) = \zeta_0 + \Delta\zeta = \zeta_0 + \Big[\eta_1(\bar{\gamma})\cdot\delta\gamma + \eta_2(\bar{\gamma})\cdot(\delta\gamma)^2 + \eta_3(\bar{\gamma})\cdot(\delta\gamma)^3\Big]_{z_0}\cdot(z - z_0)$$

> multiplying out, rearranging, neglecting of higher orders:

$$\zeta(z) = \chi_1(z)\cdot\zeta_0 + \chi_2(z)\cdot\zeta_0^2 + \chi_3(z)\cdot\zeta_0^3$$

$$\chi_1(z) = 1 + (z - z_0)\cdot[\eta_1 A_1]$$

$$\chi_2(z) = (z - z_0)\cdot[\eta_1 A_2 + \eta_2 A_1^2]$$

$$\chi_3(z) = (z - z_0)\cdot[\eta_1 A_3 + 2\eta_2 A_1 A_2 + \eta_3 A_1^3]$$

$$\big(\zeta(z), \gamma(z)\big)$$

Phase Space Linearization: Theory

$$\zeta(z) = \chi_1(z) \cdot \zeta_0 + \chi_2(z) \cdot \zeta_0^2 + \chi_3(z) \cdot \zeta_0^3$$

> drift: constant γ for each particle!

$$\gamma(\zeta_0) = A_0 + A_1\zeta_0 + A_2\zeta_0^2 + A_3\zeta_0^3 \equiv a_0 + a_1\zeta + a_2\zeta^2 + a_3\zeta^3 = \gamma(\zeta)$$

$$= a_0 + [a_1\chi_1] \cdot \zeta_0 + \left[a_1\chi_2 + a_2\chi_1^2\right] \cdot \zeta_0^2 + \left[a_1\chi_3 + 2a_2\chi_1\chi_2 + a_3\chi_1^3\right] \cdot \zeta_0^3$$

> solving for a_i: general description of γ(ζ) during drift in phase space

$$\gamma\big(\zeta(z)\big) = a_0 + a_1 \cdot \zeta(z) + a_2 \cdot \big(\zeta(z)\big)^2 + a_3 \cdot \big(\zeta(z)\big)^3$$

$$\Big(\zeta(z), \gamma(z)\Big)$$

$$\downarrow$$

$$\Big(\zeta(z), \gamma\big(\zeta(z)\big)\Big)$$

$$a_0 = A_0 \quad (= \bar{\gamma}) \qquad a_1 = \frac{A_1}{\chi_1}$$

$$a_2 = \frac{A_2 - a_1\chi_2}{\chi_1^2} = \cdots = \frac{A_2\chi_1 - A_1\chi_2}{\chi_1^3}$$

$$a_3 = \frac{A_3 - a_1\chi_3 - 2a_2\chi_1\chi_2}{\chi_1^3}$$

Phase Space Linearization: Theory

> so…
>
> > curvature known at buncher
>
> > buncher described as polynomial
>
> > > simply add up coefficients

$$\gamma(\zeta_B) = a_0(z_B) + a_1(z_B)\cdot\zeta_B + a_2(z_B)\cdot(\zeta_B)^2 + a_3(z_B)\cdot(\zeta_B)^3$$

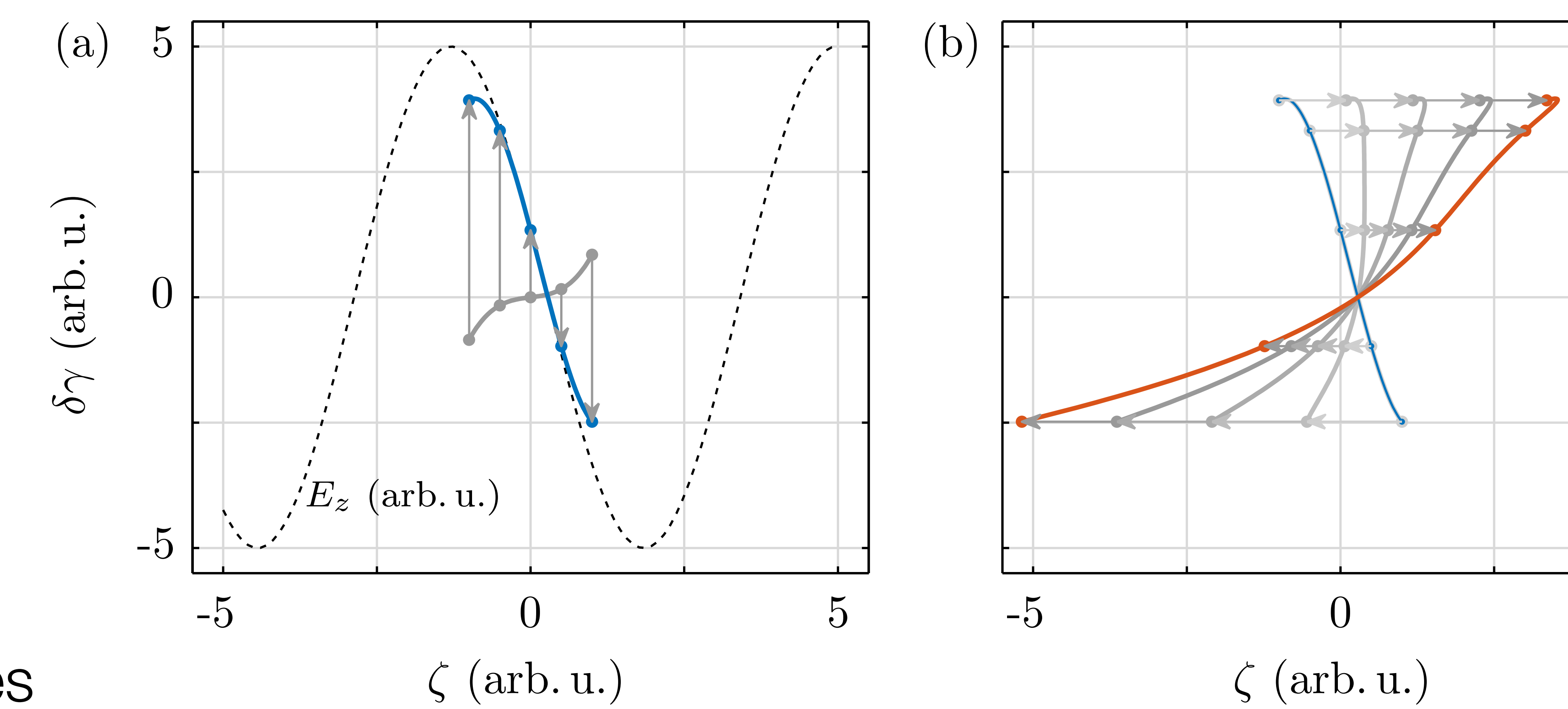
$$\Delta\gamma(\zeta_B) = B_0 + B_1\cdot\zeta_B + B_2\cdot(\zeta_B)^2 + B_3\cdot(\zeta_B)^3$$

> repeat whole cycle & end up with…

$$\zeta_z = \zeta_\mathrm{B} + \Delta\zeta(z) = X_1\zeta_\mathrm{B} + X_2\zeta_\mathrm{B}^2 + X_3\zeta_\mathrm{B}^3$$

$$X_1 := 1 + (z - z_\mathrm{B})\cdot[H_1 S_1 \cdot]$$
$$X_2 := (z - z_\mathrm{B})\cdot\left[H_1 S_2 + H_2 S_1^2\right]$$
$$X_3 := (z - z_\mathrm{B})\cdot\left[H_1 S_3 + 2H_2 S_1 S_2 + H_3 S_1^3\right]$$

$H_i=\eta_i(a_0+B_0)$: drift coefficients behind bunches
$S_i=a_i+B_i$: sum of γ-coefficients
ζ_B: bunch coordinates at buncher



buncher cavity

Stretcher Mode: Method

$$\zeta_F = \zeta_B + \Delta\zeta(z) = X_1\zeta_B + X_2\zeta_B^2 + X_3\zeta_B^3 \overset{!}{=} 0$$

$$X_1 := 1 + (z - z_B)\cdot[H_1 S_1] \overset{!}{=} 0$$
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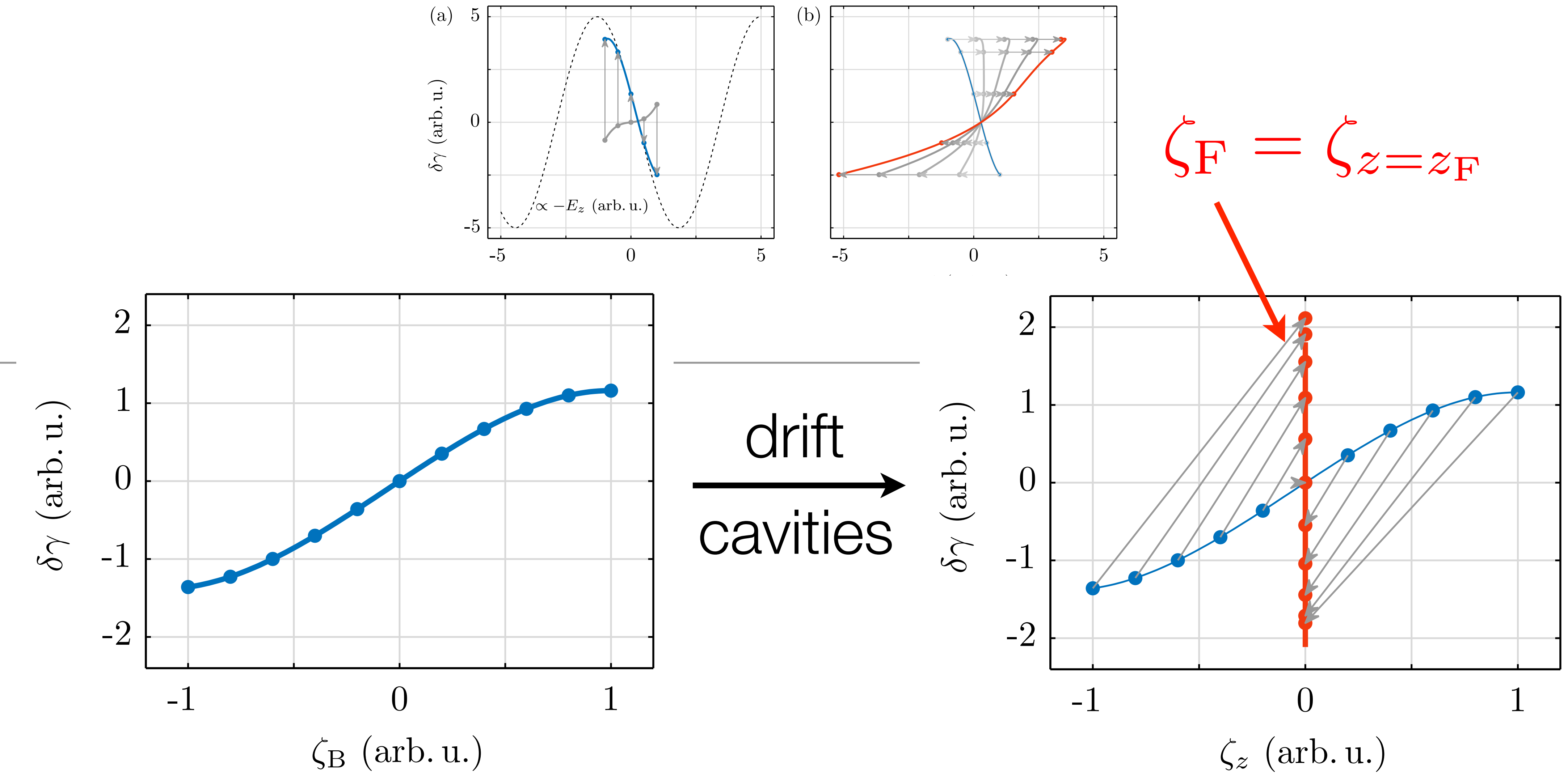
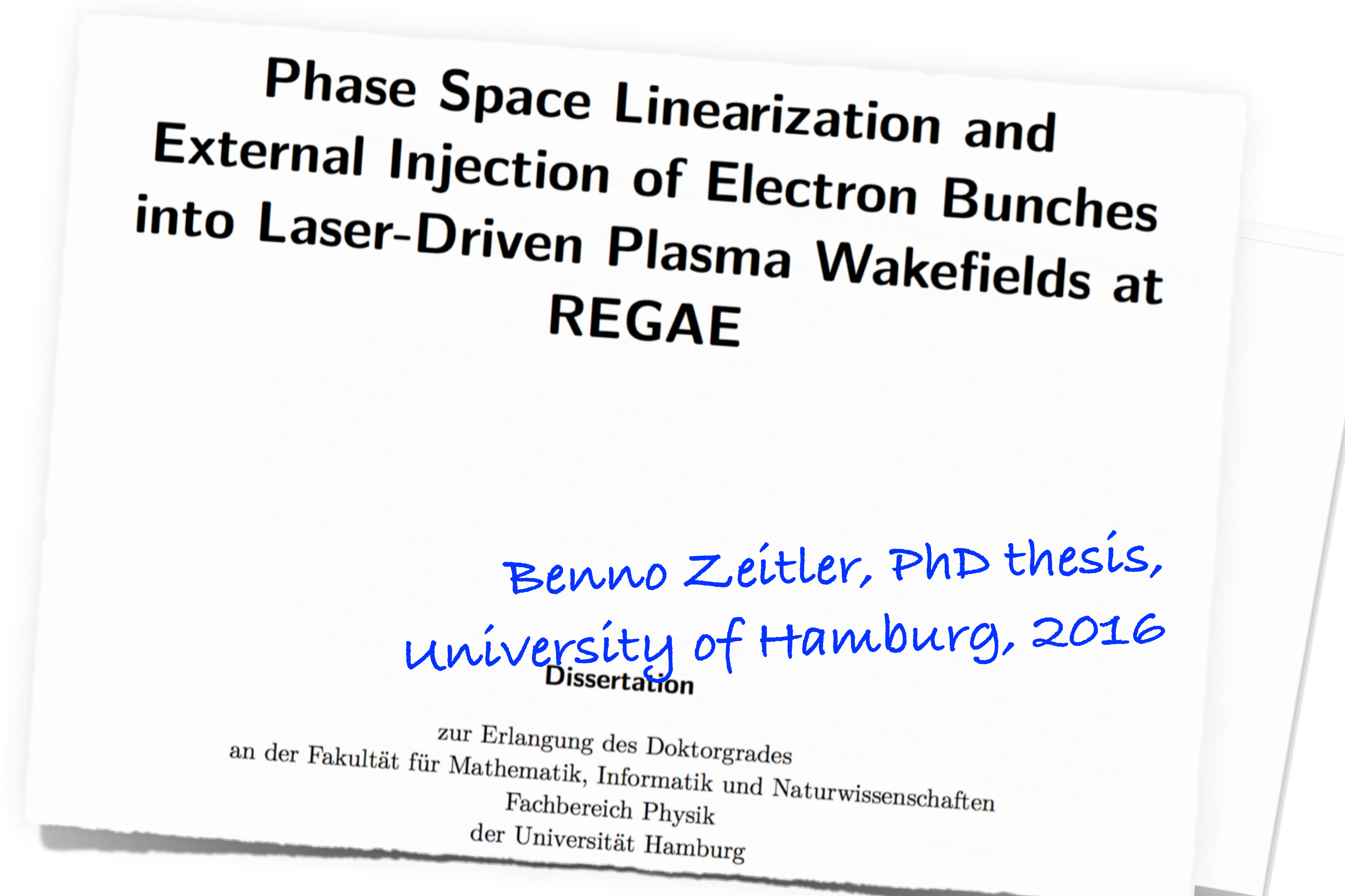
$H_i = \eta_i(a_0 + B_0)$: drift coefficients behind buncher
$S_i = a_i + B_i$: sum of γ-coefficients
ζ_B: bunch coordinates at buncher

> third order polynomial in ζ ⟶ three coefficient-equations X_i

> four free parameters (cavity fields & phases)

> seed parameters for numerical optimization (ASTRA)




Phase Space Linearization and External Injection of Electron Bunches into Laser-Driven Plasma Wakefields at REGAE

Benno Zeitler, PhD thesis, University of Hamburg, 2016

Dissertation

zur Erlangung des Doktorgrades
an der Fakultät für Mathematik, Informatik und Naturwissenschaften
Fachbereich Physik
der Universität Hamburg

Benno Zeitler, Klaus Floettmann, and Florian Grüner
Phys. Rev. ST Accel. Beams 18, 120102, 2015

$$\zeta_{\mathrm{F}} = \zeta_{\mathrm{B}} + \Delta\zeta(z) = \boxed{X_1\zeta_{\mathrm{B}} + X_2\zeta_{\mathrm{B}}^2} + X_3\zeta_{\mathrm{B}}^3 \overset{!}{=} 0$$

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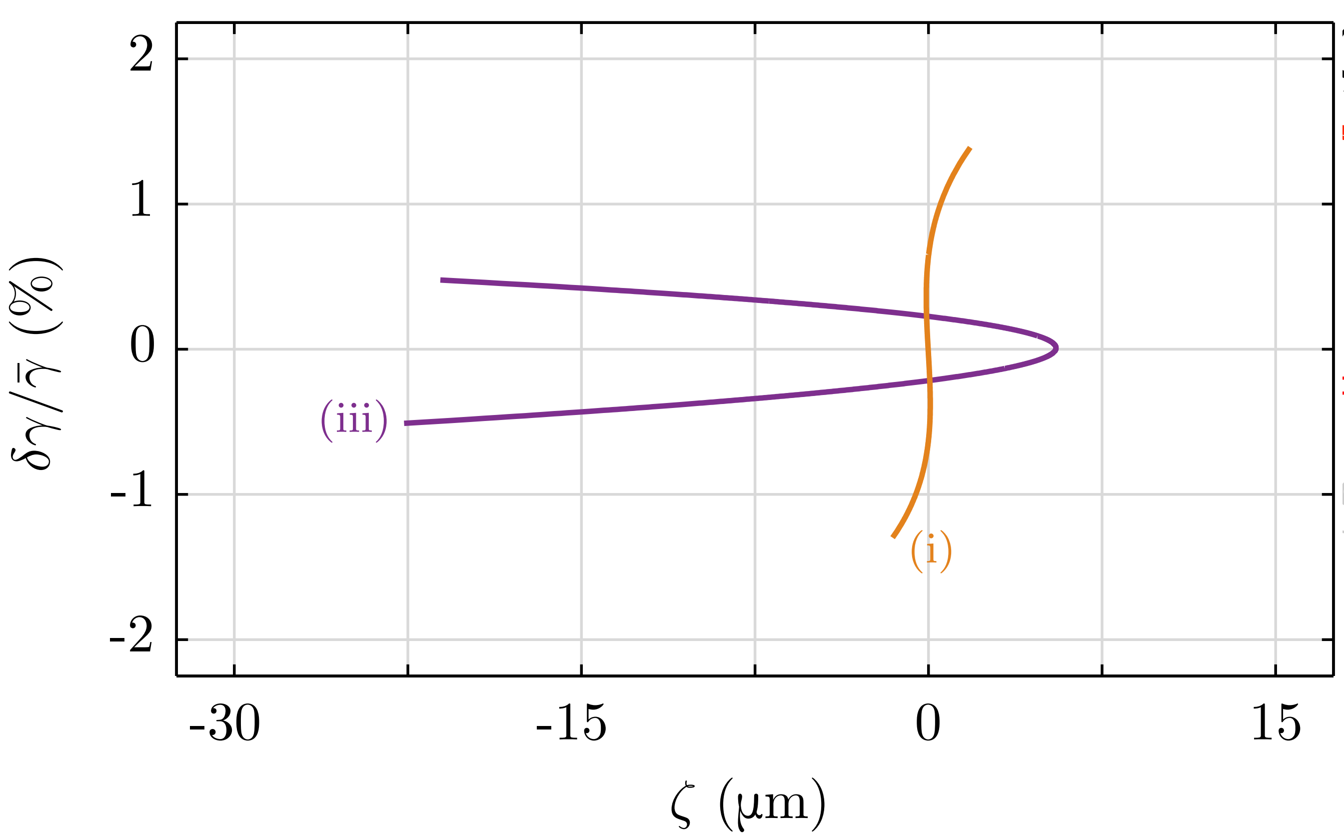
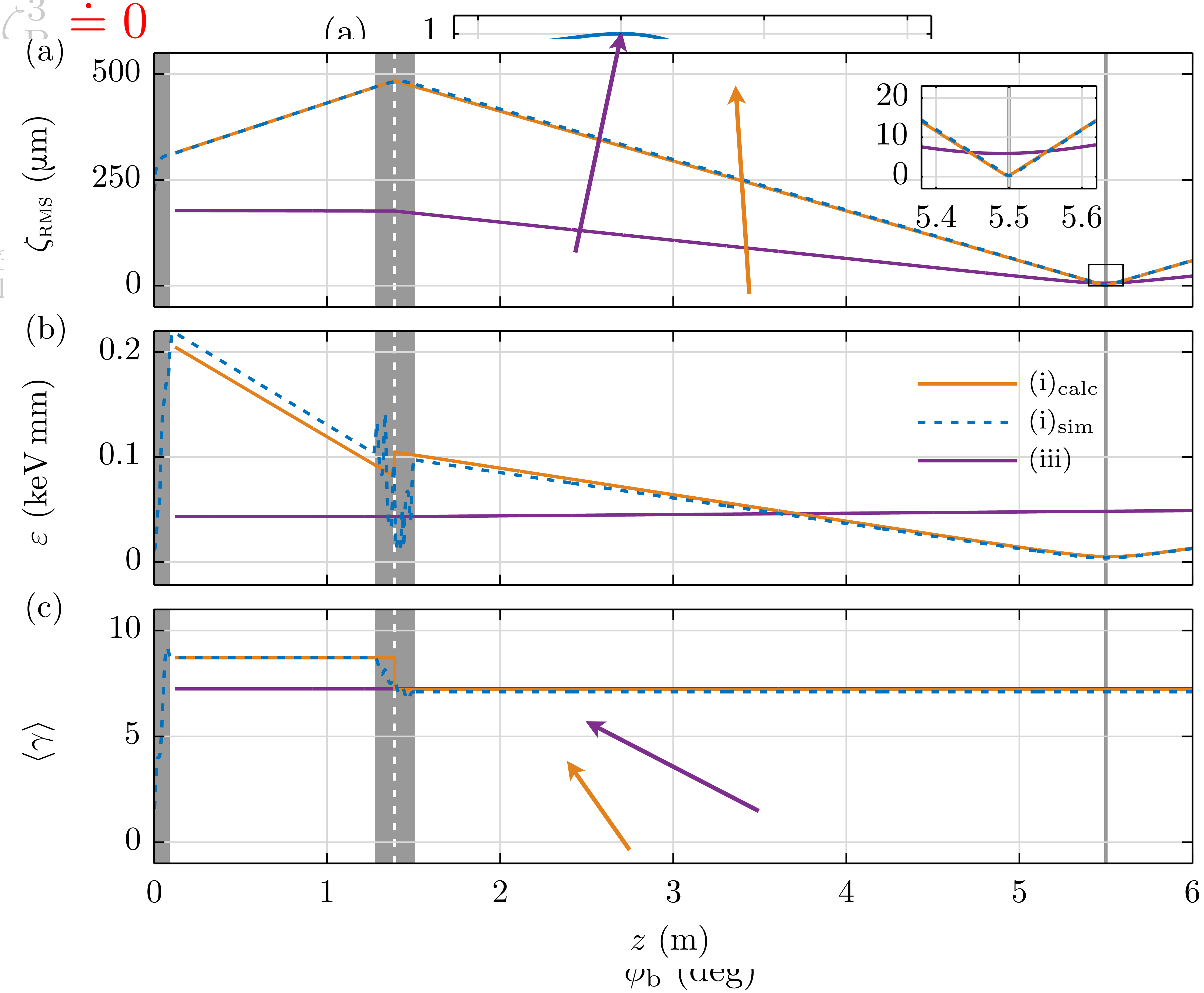
Applications: Second Order





- stretcher mode: case (i)
  - calc & sim: E_g = 100.0 MV/m, ϕ_g = 34.1 deg
    calc: E_b = 21.2 MV/m, ϕ_b = -109.3 deg
    sim: E_b = 21.7 MV/m, ϕ_b = -112.0 deg
- standard bunching scheme: case (iii)
  - calc: E_g = 70.0 MV/m, ϕ_g = 0.0 deg
    calc: E_b = 6.6 MV/m, ϕ_b = -90.0 deg

$$\zeta_F = \zeta_B + \Delta\zeta(z) = \boxed{X_1\zeta_B + X_2\zeta_B^2} + X_3\zeta_B^3 \overset{!}{=} 0$$

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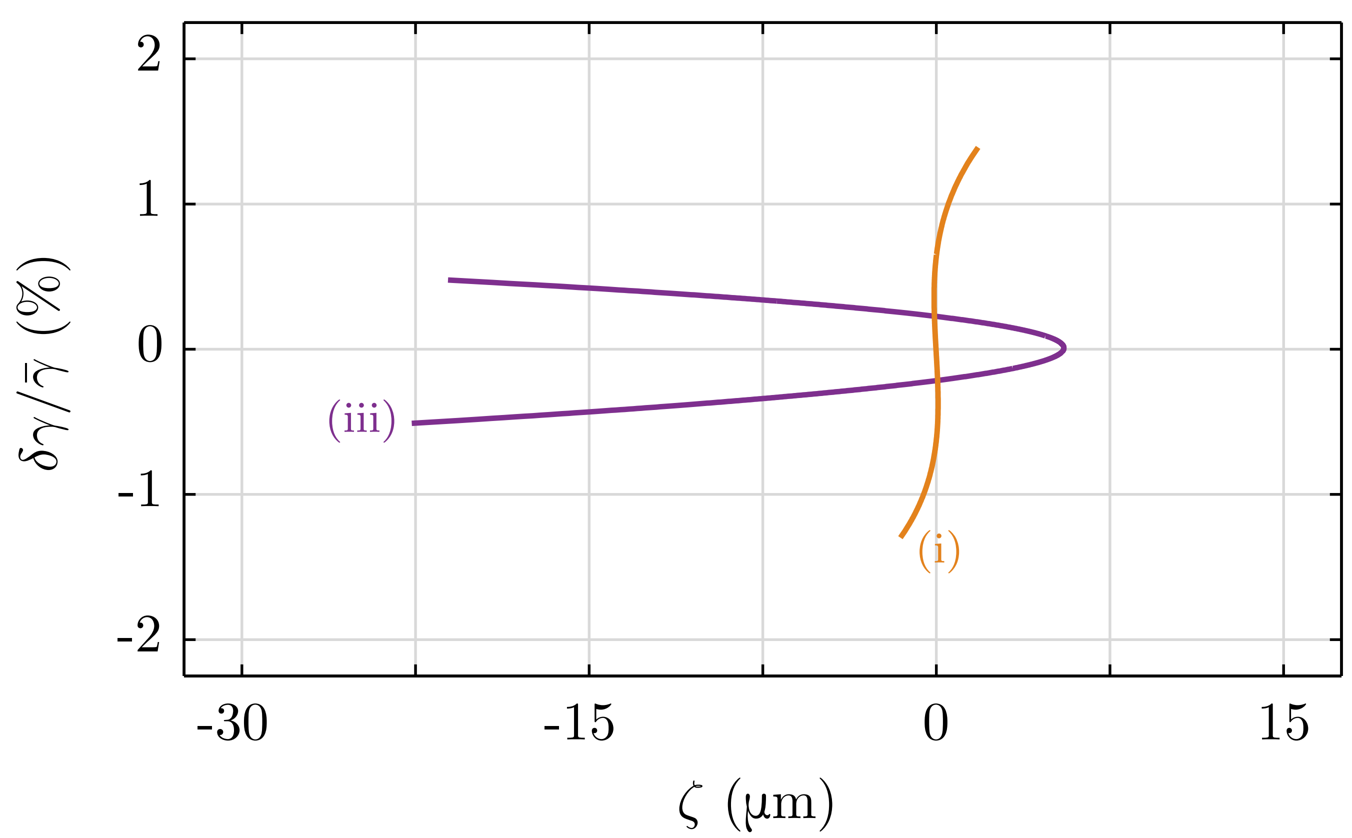
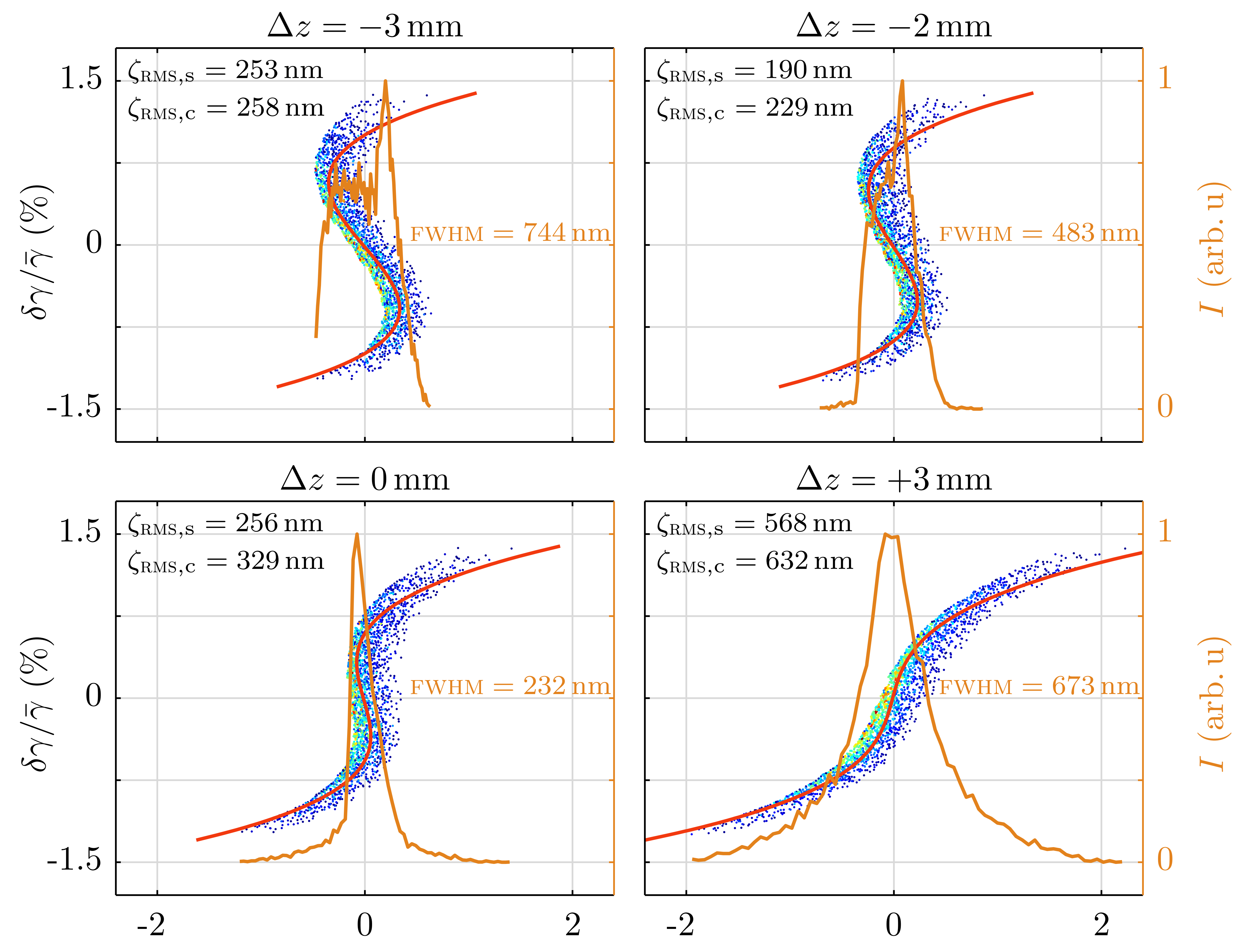
Applications: Second Order





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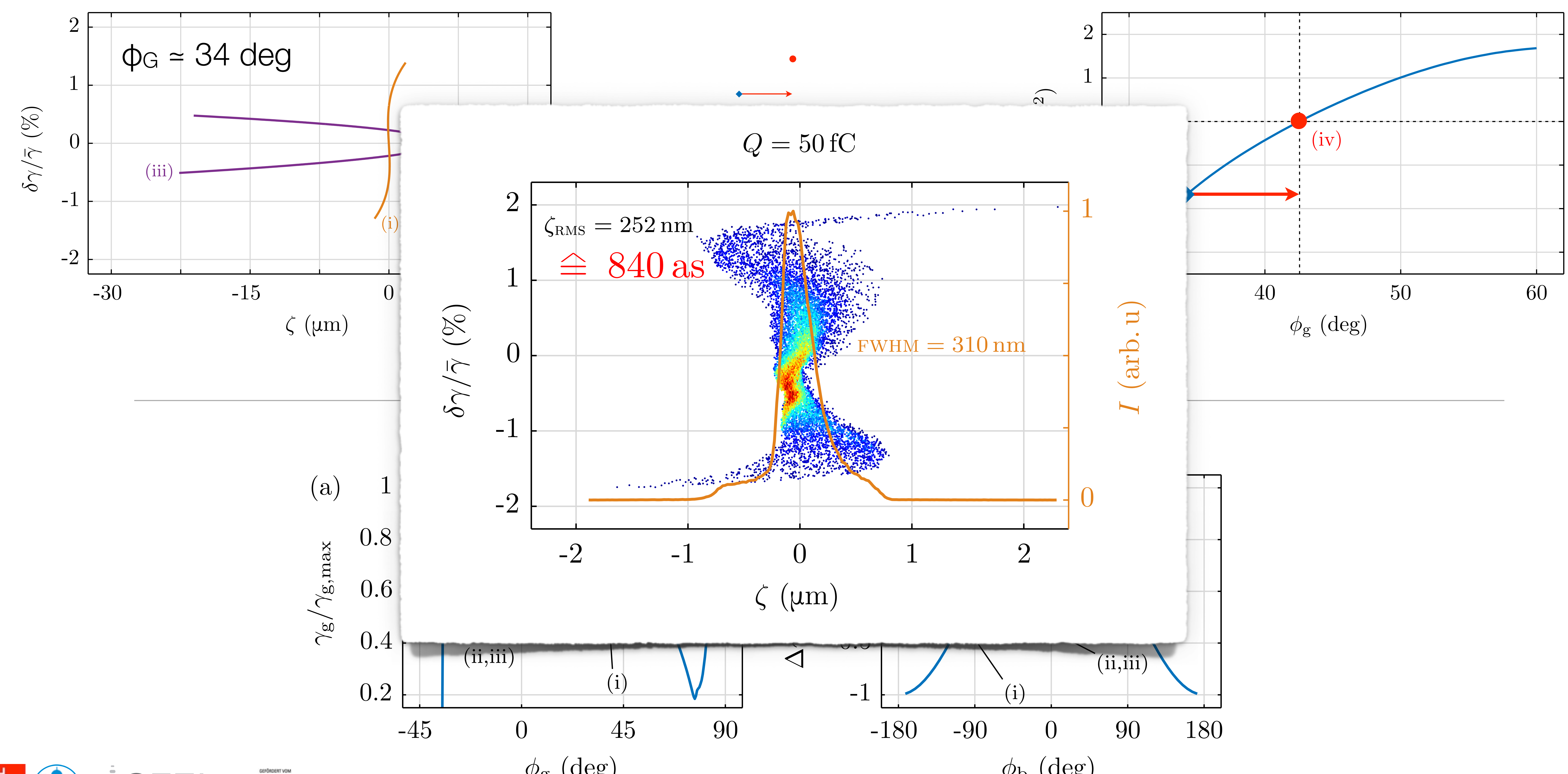
Applications: Third Order



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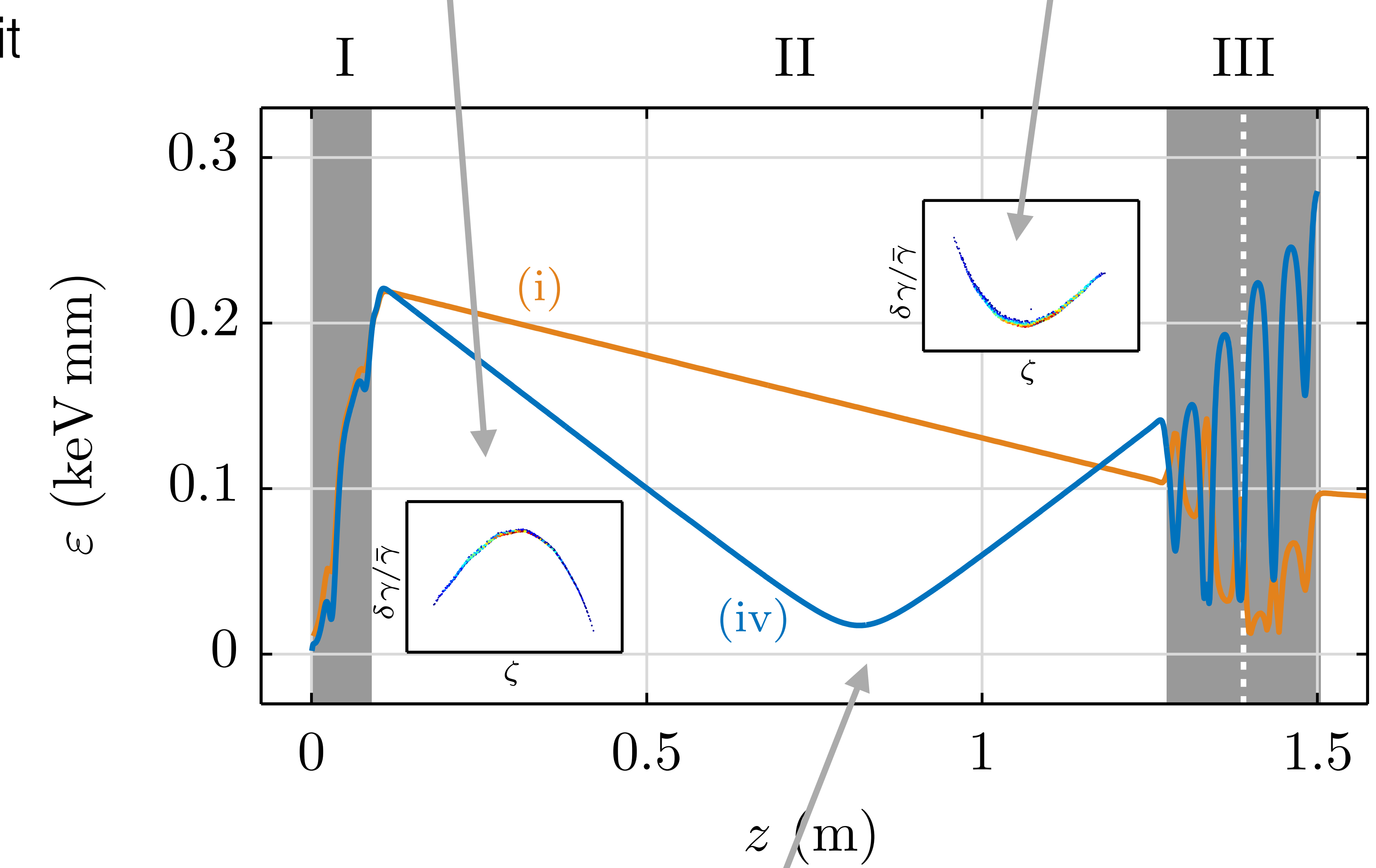
Applications: Third Order

- observation:
  - high off-crest phase ⟶ (first) emittance minimum between cavit
- requirement(s) for $X_3 = 0$: (one can show…)
  - change of sign in a_3
    - only possible if $a_2 > 0$, but $A_2 = a_2(z=0) < 0$
      - change of sign in a_2
        - minimum in emittance
        - **overcompensation mode**

$a_2 < 0$ $a_2 > 0$



$$X_3 = 0 = H_1 S_3 + 2H_2 S_1 S_2 + H_3 S_1^3$$
$$= \cdots = H_1 S_3 + \underbrace{\left(H_3 - 2\frac{H_2^2}{H_1}\right)}_{<0} \cdot S_1^3.$$

$$\epsilon_{1,\mathrm{RMS}}(z) = D(z)\cdot|a_2(z)|\cdot\sigma_\zeta(z)^3$$
$$\approx D(z)\cdot|a_2(z)\cdot\chi_1(z)^3|\cdot\zeta_{C,\mathrm{RMS}}^3$$
$$= D(z)\cdot|A_2\chi_1(z) - A_1\chi_2(z)|\cdot\zeta_{C,\mathrm{RMS}}^3$$

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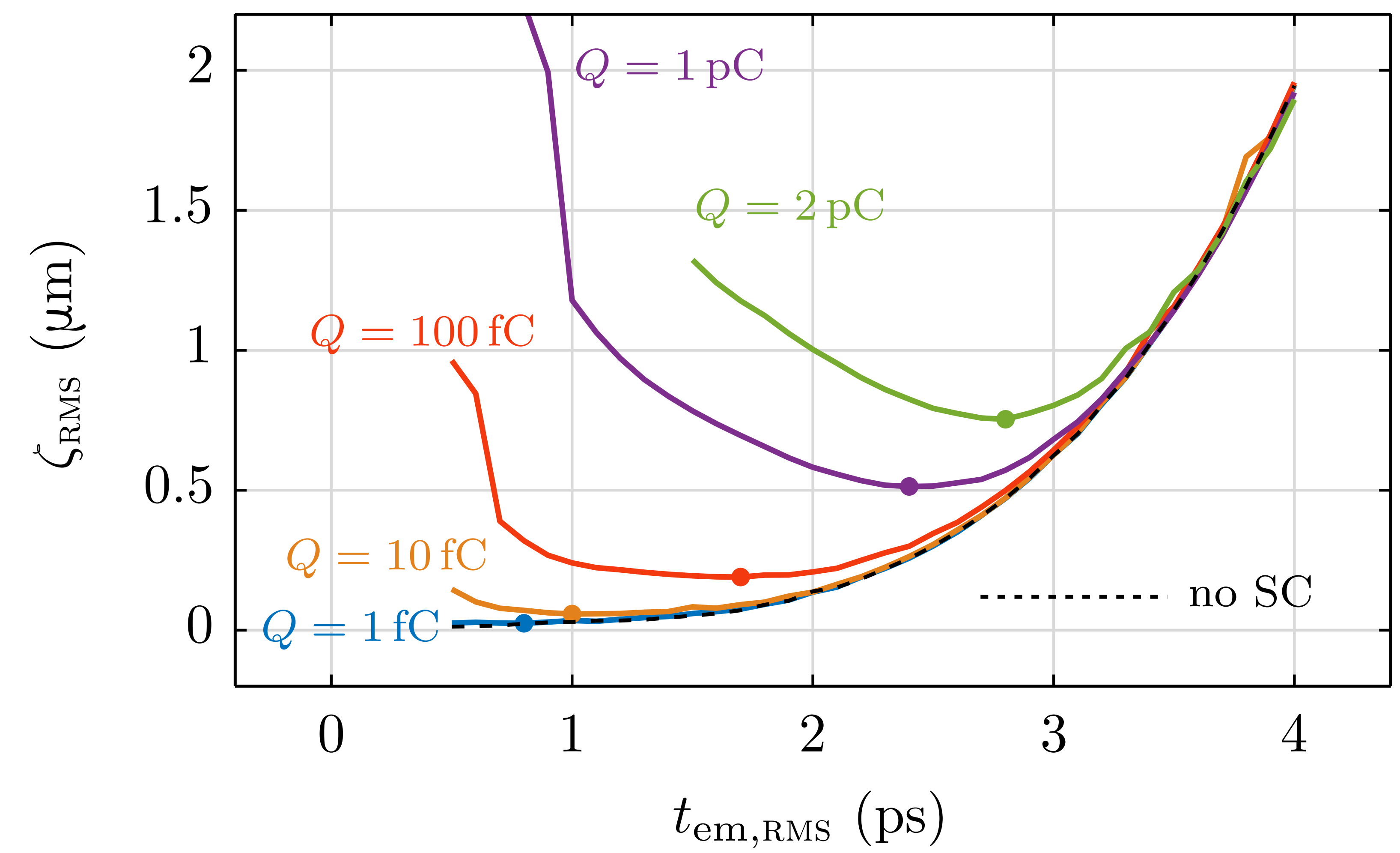
Applications: Third Order

- additional parameters
  - shorter focal length: „beyond REGAE“
  - scan of electron emission time
  - variation of bunch charges

- **high peak currents (500 A)**
- **extremely short bunches (< 100 as)**



Q (fC)	$t_{em,RMS}$ (ps)	ζ_{RMS} (nm)	t_{RMS} (fs)	CR
1	0.8	24	0.08	10000
10	1.0	58	0.2	5000
100	1.7	190	0.6	2500
1000	2.4	513	1.7	1400
2000	2.8	753	2.5	1000

$$\zeta_{\mathrm{F}} = \zeta_{\mathrm{B}} + \Delta\zeta(z) = X_1\zeta_{\mathrm{B}} + X_2\zeta_{\mathrm{B}}^2 + X_3\zeta_{\mathrm{B}}^3 \overset{!}{=} 0$$

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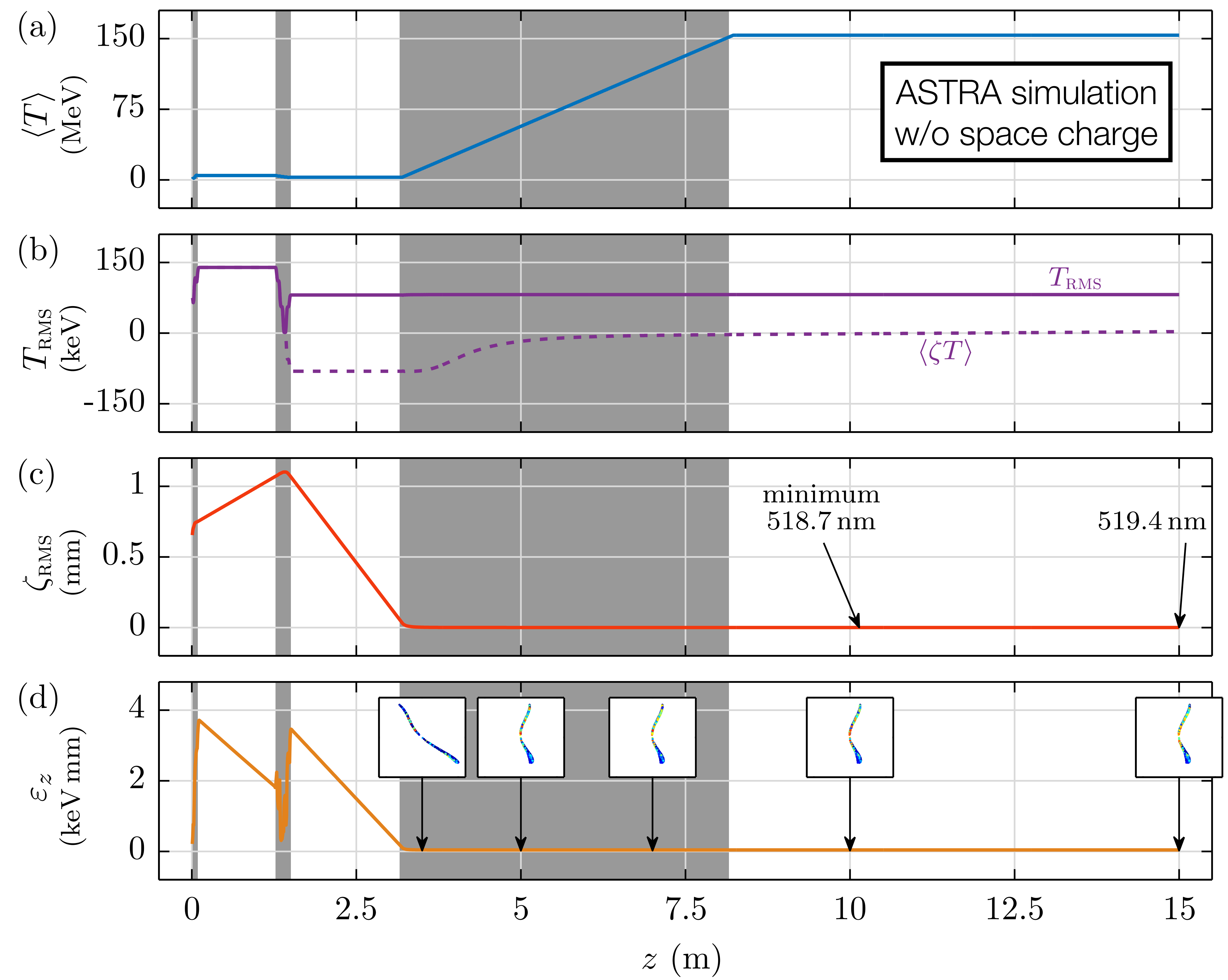
Applications: Freeze Out

- injection of linearized & compressed bunch into booster cavity
- drift factors scale as

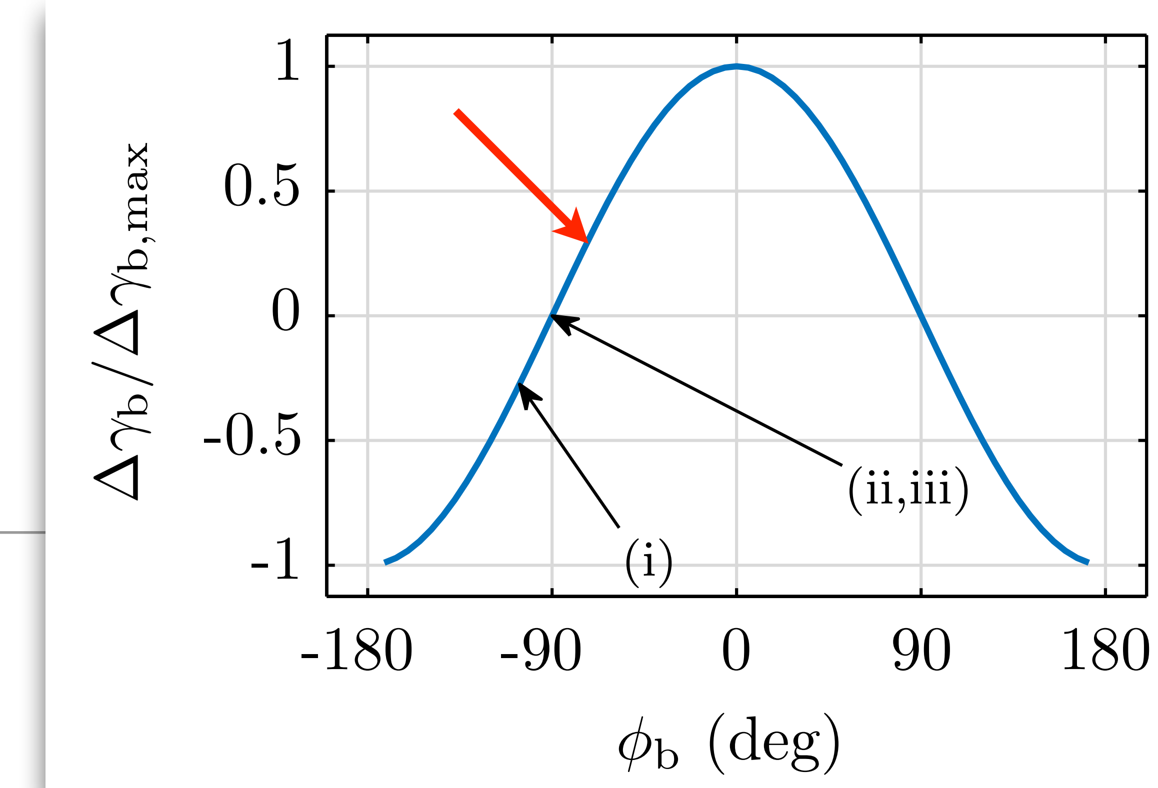
$$H_1 \propto \gamma^{-3},\ H_2 \propto \gamma^{-4},\ H_3 \propto \gamma^{-5}$$

  - evolution of bunch length (and especially nonlinearities) suppressed at higher energies
- bunch short compared to wavelength of booster cavity
  - e.g. 500 nm vs. 10 cm
  - (almost) no additional cavity effects

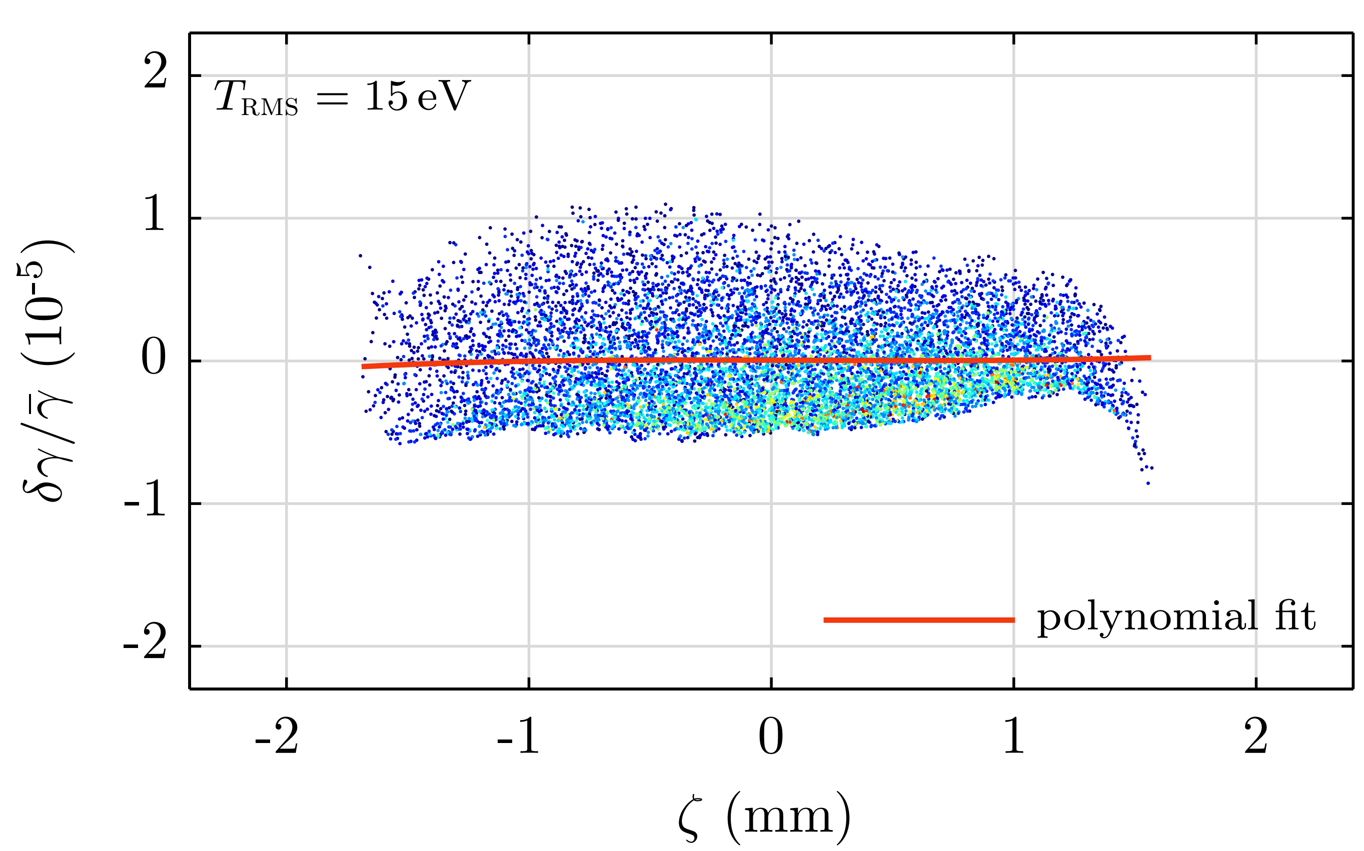
- **bunch shape frozen by rapid increase in energy**

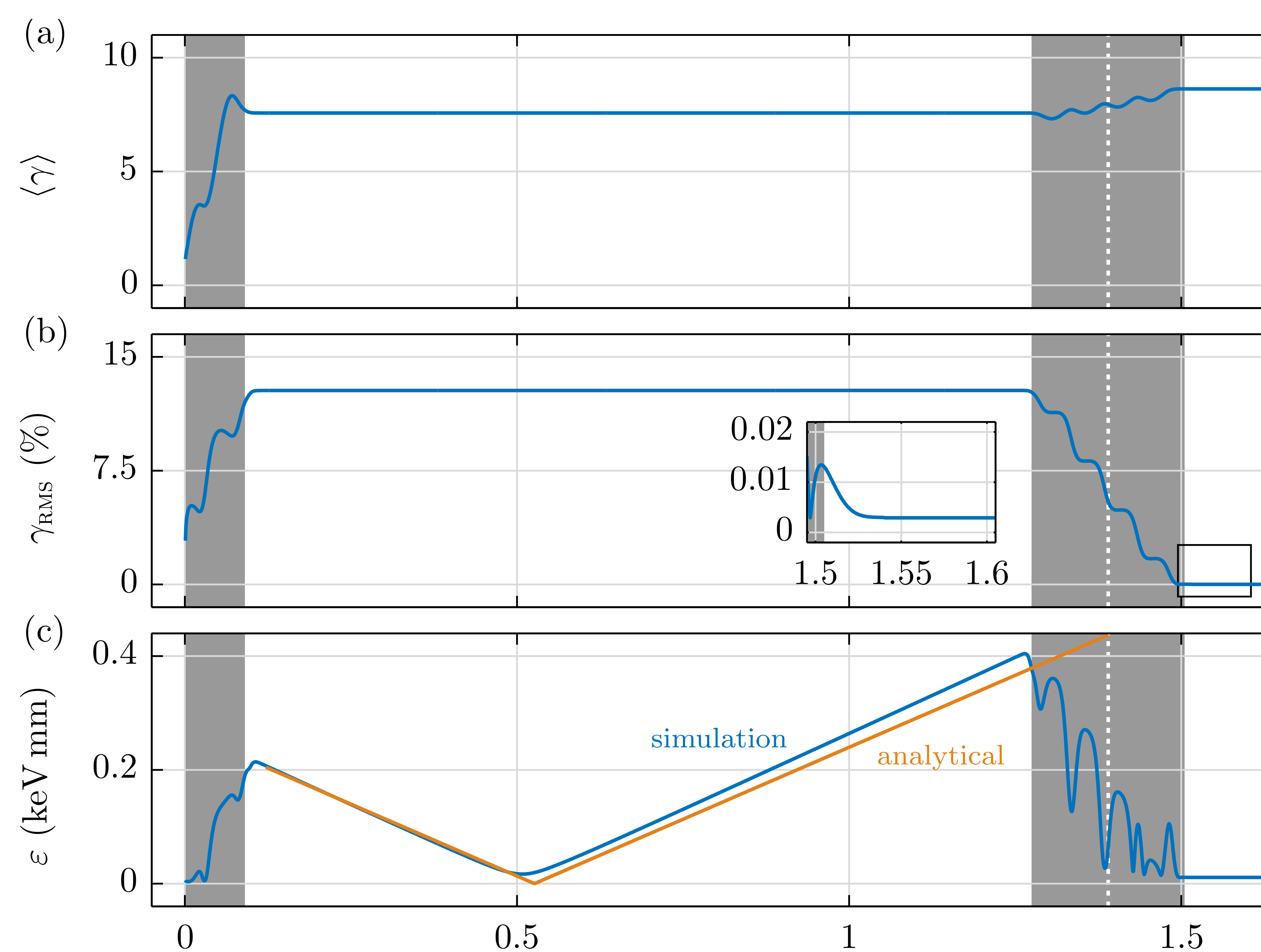


Applications: Energy Spread Compensation



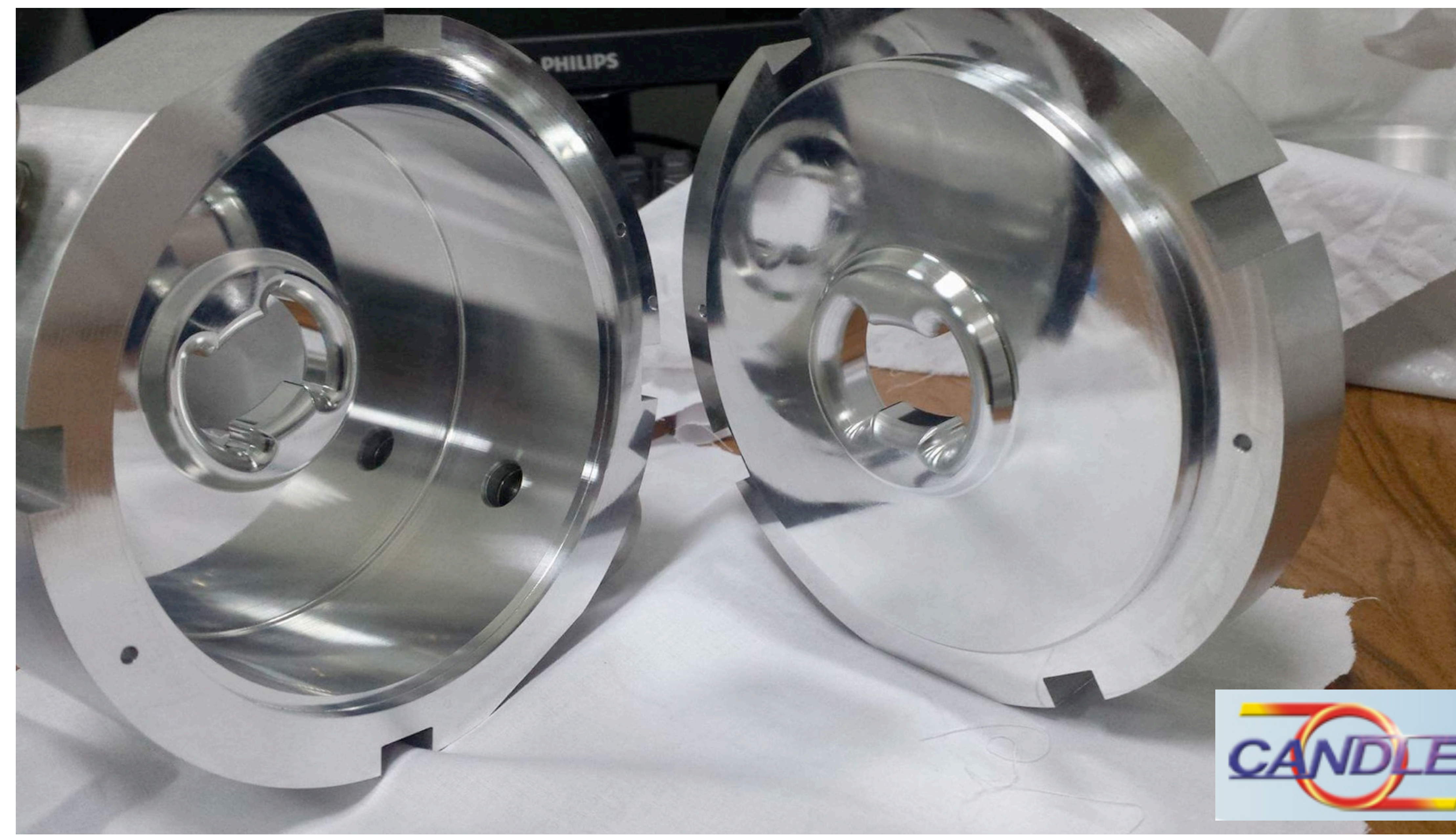
- parameters:
  - gun: E_g = 100.0 MV/m, ϕ_g = 45.7 deg
    buncher: E_b = 14.1 MV/m, ϕ_b = -69.9 deg
  - Q = 50 fC
- bunch properties:
  - T = 3.9 MeV, T_{RMS} = 15 eV





How to Measure?

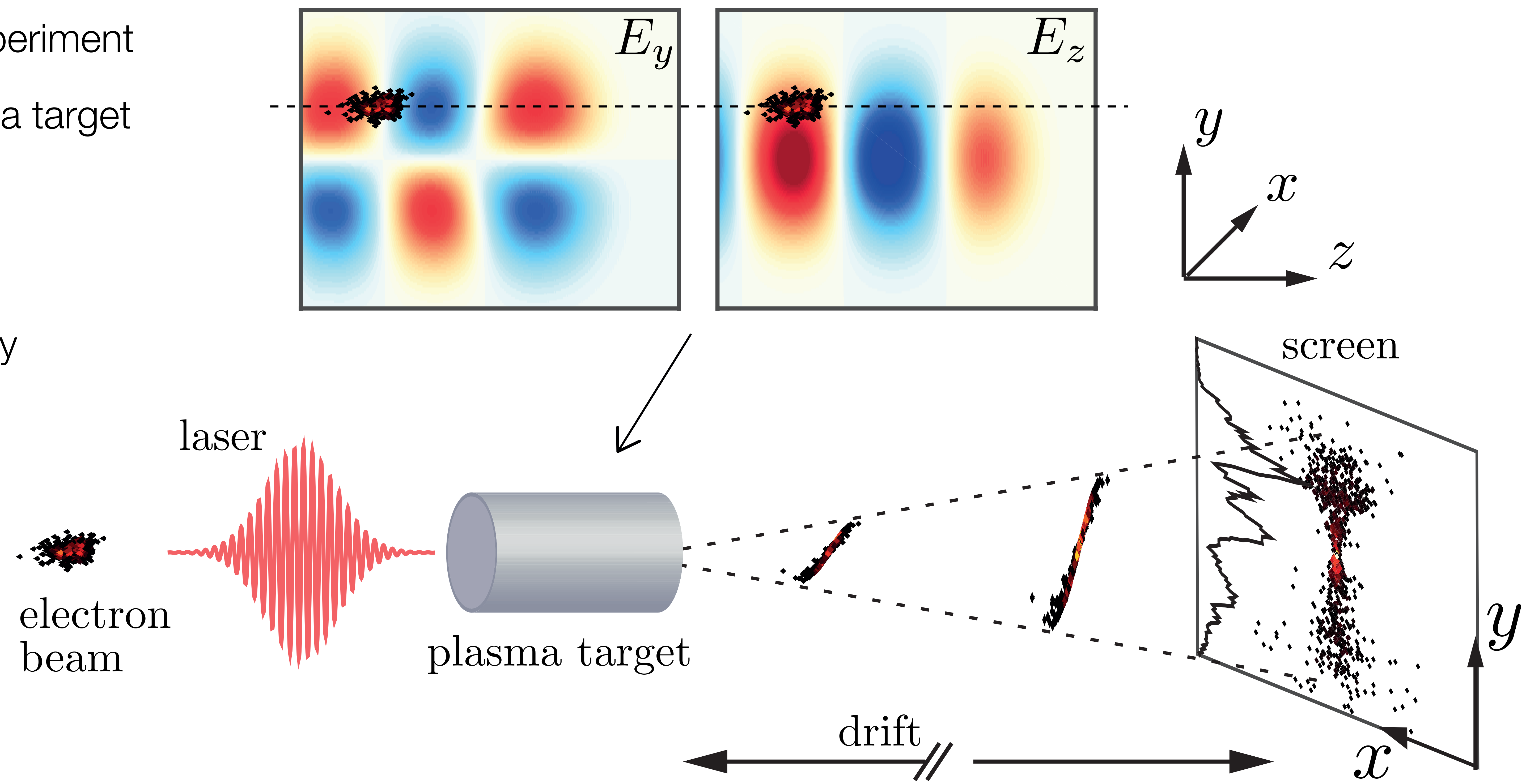
- planned beamline upgrade at REGAE
  - (external injection project)
  - shift of electron spectrometer
  - transverse deflecting structure (TDS)
    - resolution: 10 fs
    - cold test model manufactured at CANDLE
    - cavity: manufacturing in progress at CANDLE
  - mapping of longitudinal phase space
    - proof of concept



see also talks on Friday by H. Delsim-Hashemi and F. Lemery

How to Measure? pTDS

- external injection project at REGAE
  - beamline upgrade for LWFA experiment
  - ANGUS laser (200 TW) & plasma target
- laser drives linear wakefield
  - inject electron bunch off-axis in y
  - experiences streaking field
- advantages:
  - strong fields
  - short (plasma) wavelength
  - short target



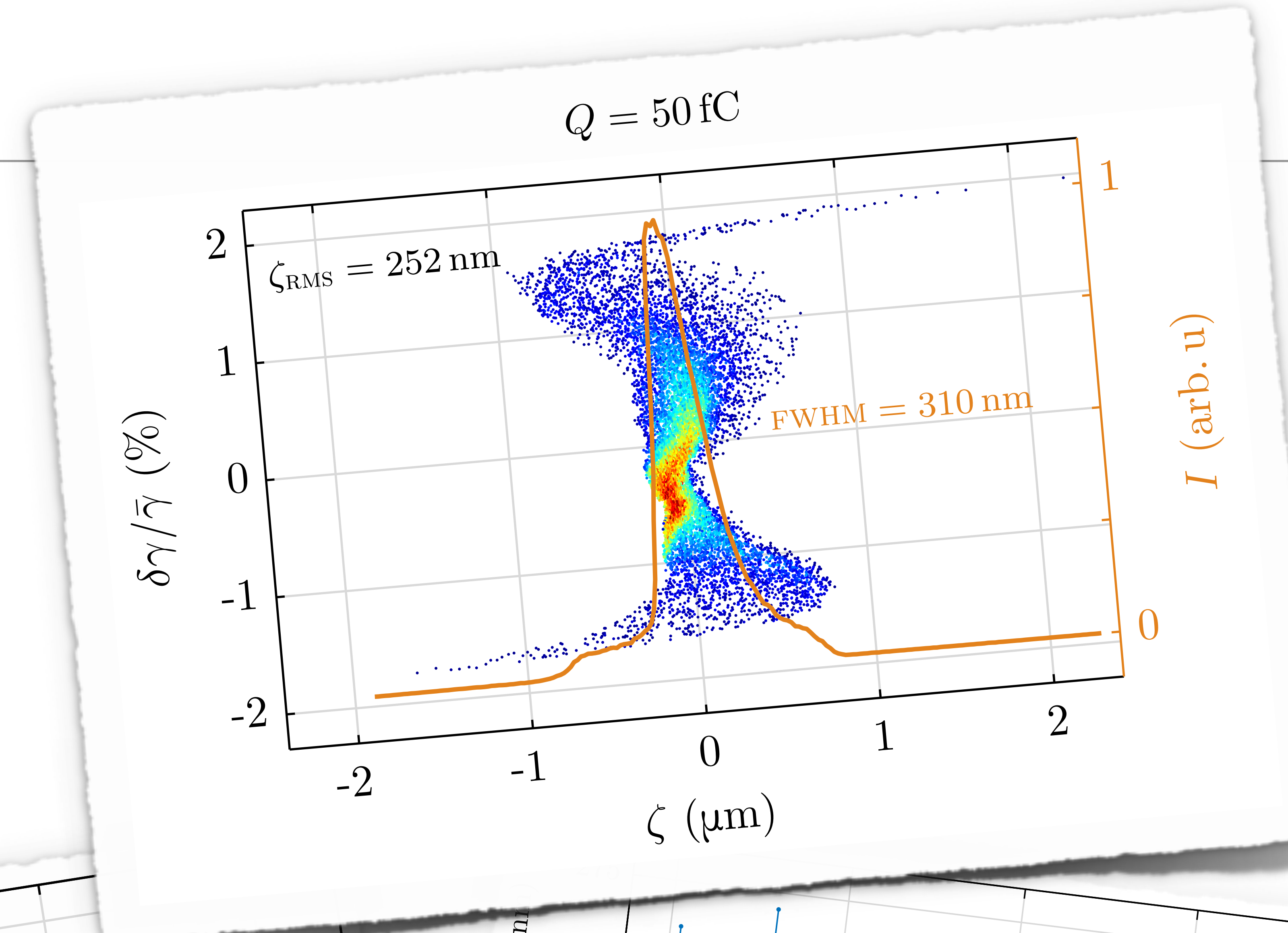
Courtesy: I. Dornmair

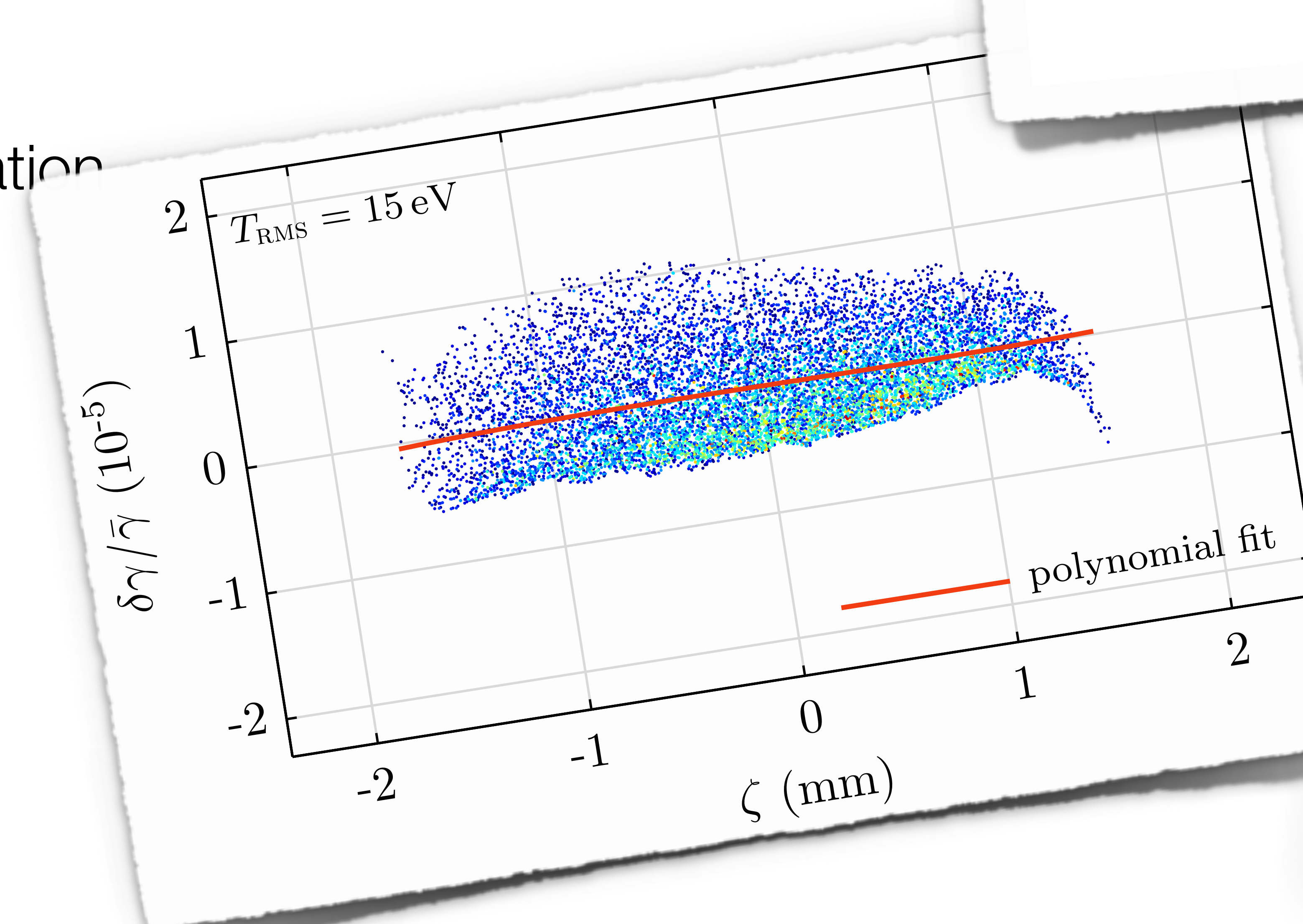
I. Dornmair et al., PRAB 19, 062801 (2016)

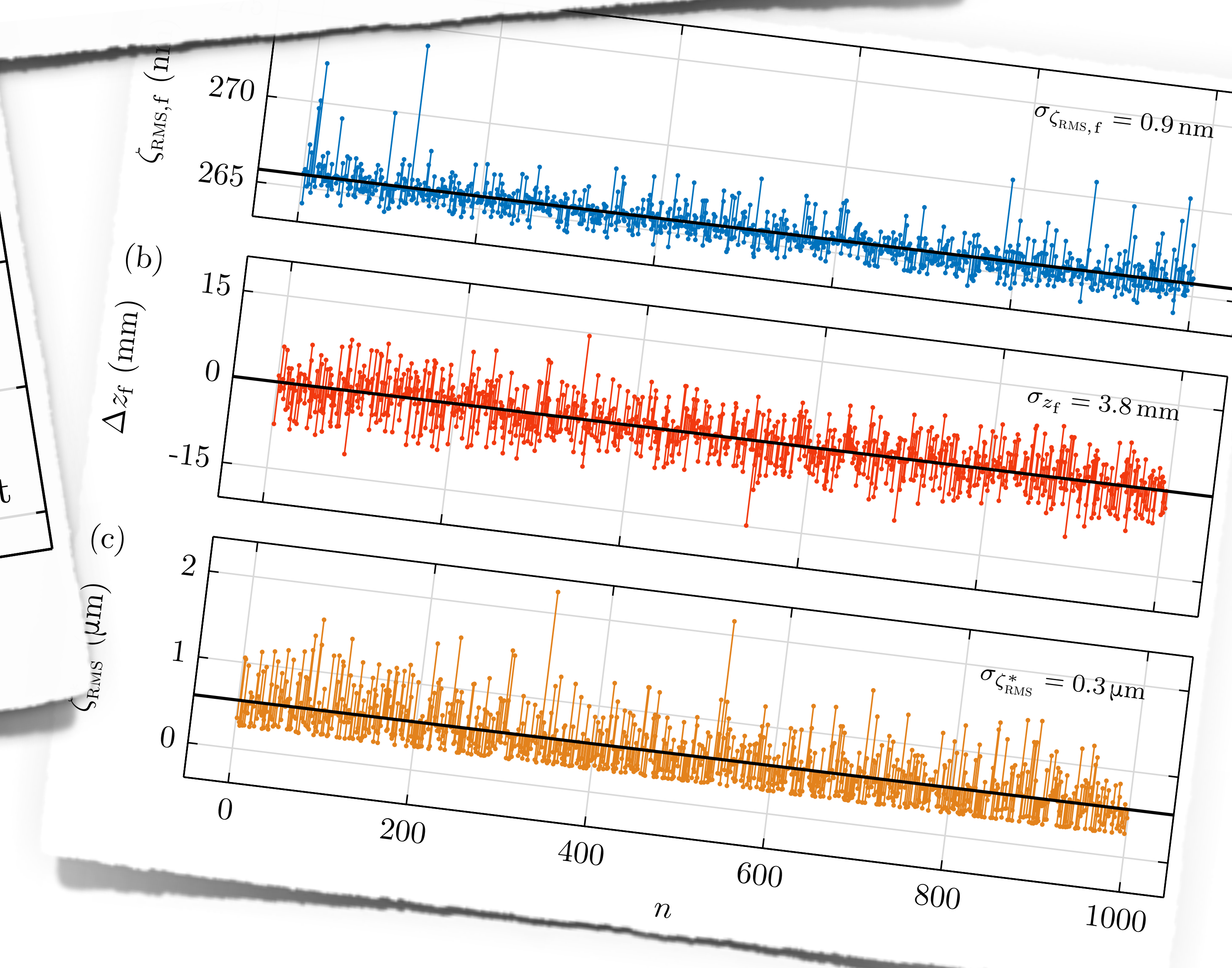
Summary

> phase space linearization
  > origin of curvature
  > novel method: stretcher mode
    > overcompensation
  > sub-fs bunches at REGAE
  > freeze out
  > energy spread compensation

> how to measure?
  > TDS at REGAE
  > plasma based TDS
  > ideas and suggestions welcome!







Thank you for your attention

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GEFÖRDERT VOM

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DESY

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