Synchrotron Radiation Reflection from Outer Wall of Vacuum Chamber

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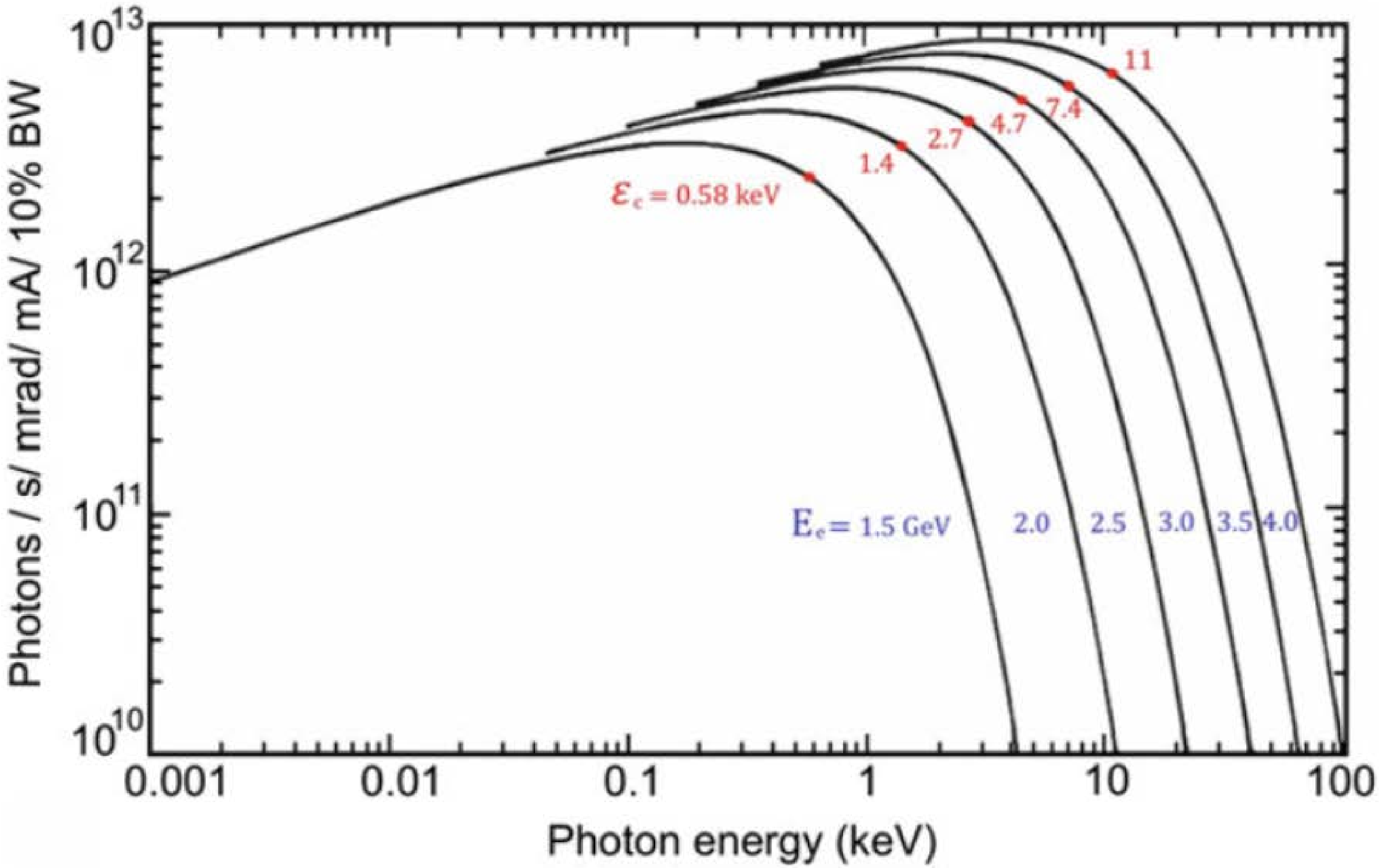
The presentation is devoted to the eightieth anniversary of the birth of Garry Nagorsky and memory of Andrey Amatuni

Synchrotron radiation is produced by relativistic electrons rotating in a magnetic field . The spectral synchrotron radiation energy emitted by one electron [H. Wiedemann, Particle Accelerator Physics, Third Ed., Springer-Verlag Berlin Heidelberg, 2007] [http://xdb.lbl.gov/Section2/Sec_2-1.html]:

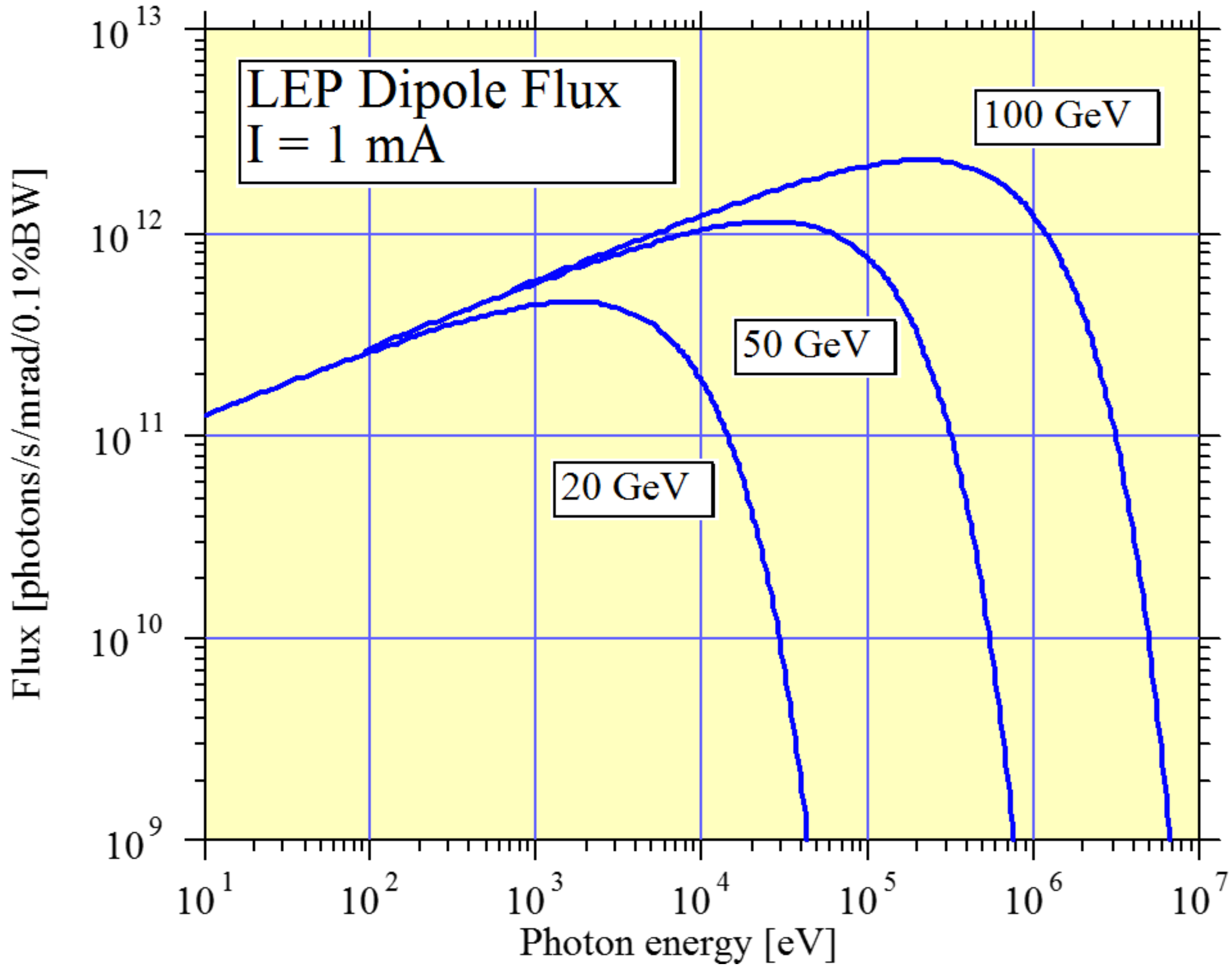
$$\frac{d^2W}{d\Omega d\omega} = \frac{3r_0 mc}{4\pi^2}\gamma^2\frac{\omega^2}{\omega_c^2}(1+\gamma^2\psi^2)^2\left[K_{2/3}^2(x)+\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(x)\right]$$

where $d\Omega = d\theta d\psi$ (ψ is angle between orbit plane and radiation detection direction, θ is observation angle in the horizontal plane), ω angular frequency of photon, γ is electron energy divided by $m_e c^2$ (m_e is electron mass, c velocity of light), $r_0 = e^2/m_e c^2$ is electron classical radius, $x = \frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}$, $\omega_c = 3\gamma^3 c/2\rho_0$ critical frequency, defined as the frequency that divides the emitted power into equal halves. $K_{1/3}, K_{2/3}$ are modified Bessel functions of the second kind. For ultrarelativistic electrons $\gamma \gg 1$ the distribution is very wide and involves harmonics up to γ^3 order of charge circulating frequency $\omega_0 = c/\rho_0$ (ρ_0 is radius of charge trajectory).

A. Balerna and S. Mobilio

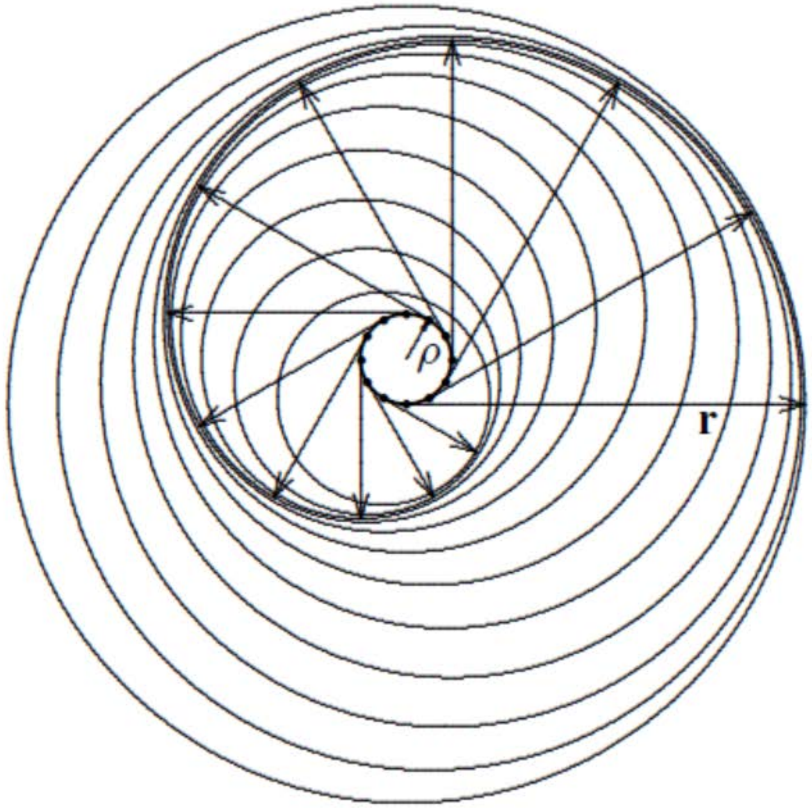


Spectral distribution of SR as a function of the value of critical energy $\varepsilon_c = 3\hbar c\gamma^3/(2\rho_0)$ of the storage ring (ρ_0 radius of storage ring) [Synchrotron Radiation Basics, Methods and Applications, S. Mobilio, F. Boscherini, C. Meneghini Ed., Springer, Springer-Verlag Berlin Heidelberg 2015]

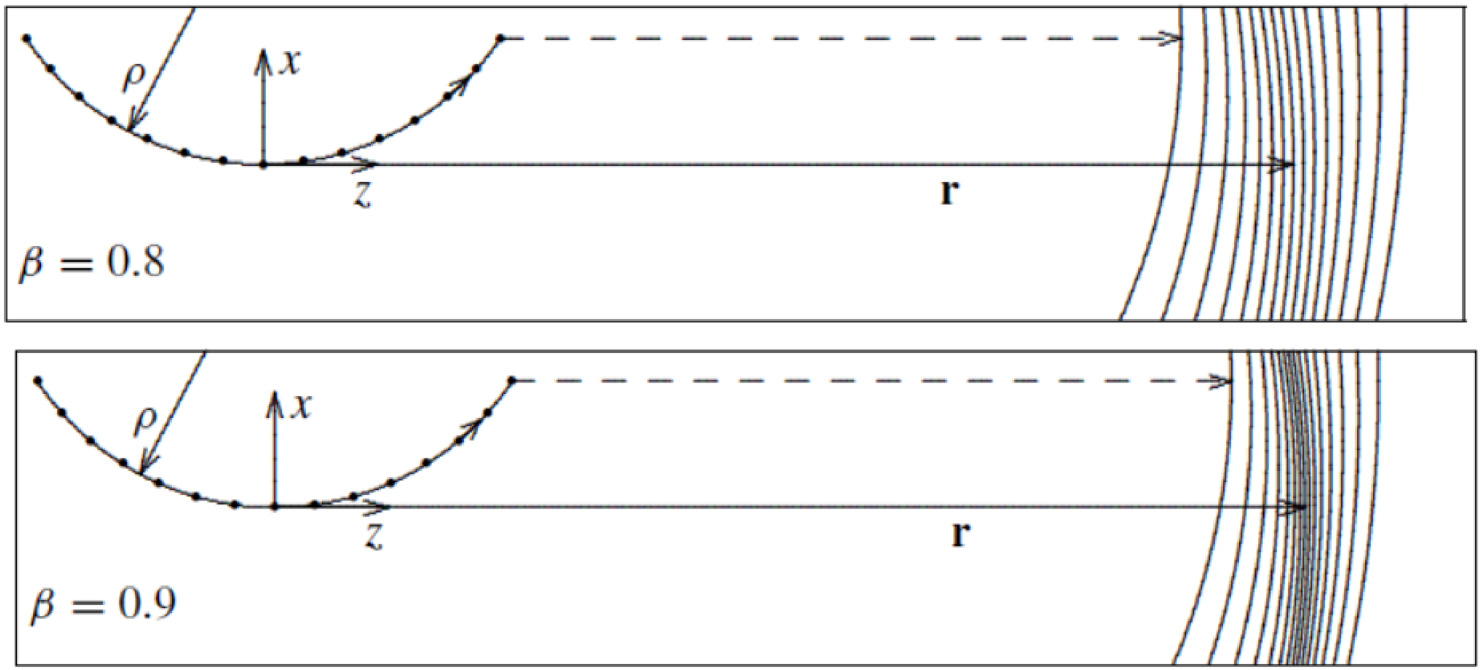


LEP synchrotron radiation [L. Rivkin, Electron dynamics with Synchrotron Radiation, SRBasicsCAS16_LRivkin.pptx].

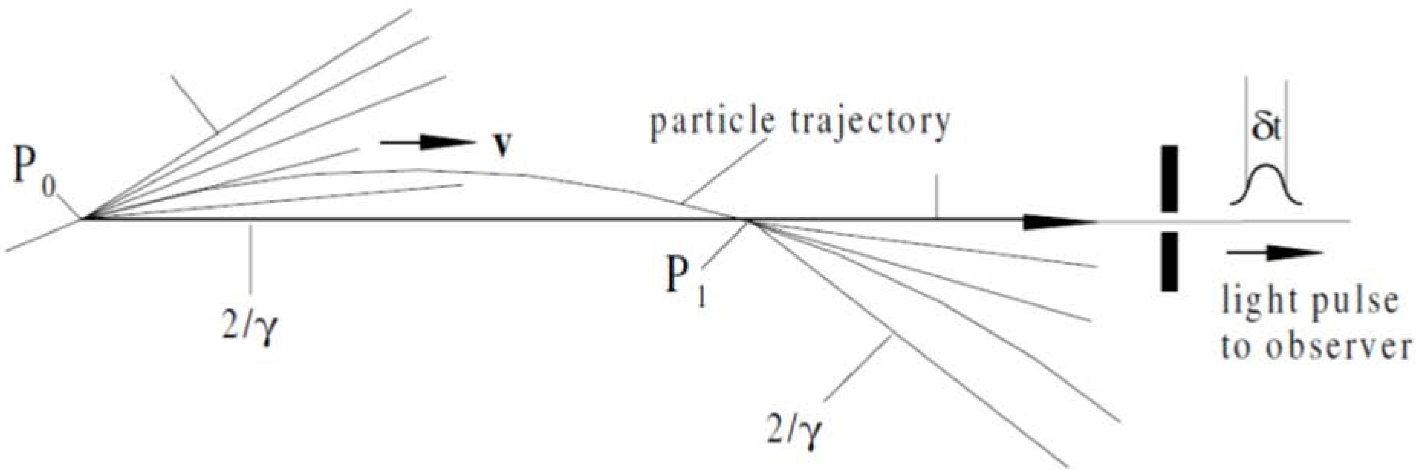
Wide Fourier expansions correspond to narrow spatial structures. The pulse width detected by a detector has a duration $\Delta t \approx 1/\gamma^3\omega_0$, where $\omega_0 = eB/\gamma m_e$ is orbiting frequency. Therefore, synchrotron radiation is dominated by frequency components in the range $\omega = \gamma^3\omega_0$.



The wave fronts of synchrotron radiation for β = 0.9 [Hofmann].

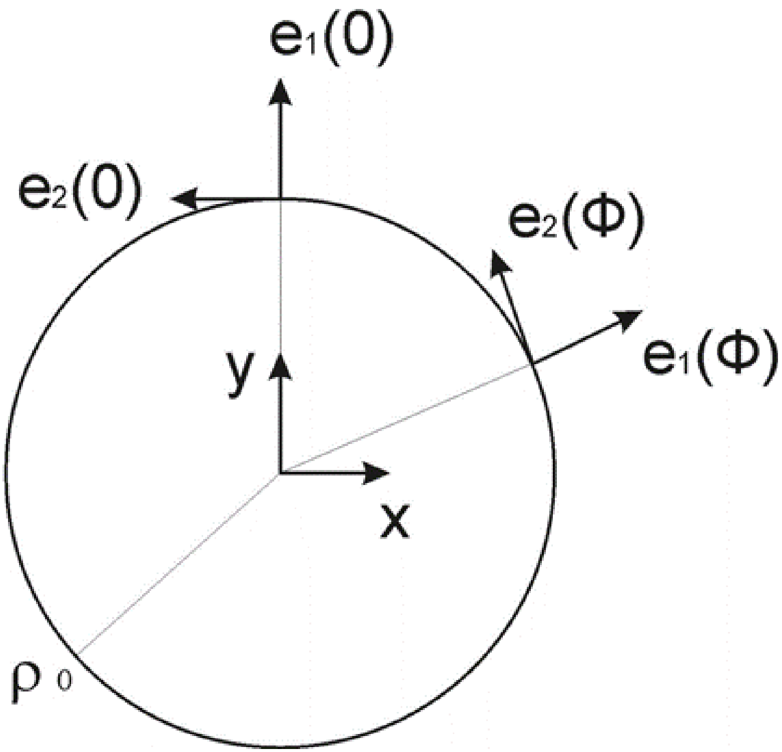


Forward propagation of synchrotron radiation for ß = 0.8 and 0.9 [Hofmann].



Pulse of synchrotron radiation: $\delta t = 4\rho_0 / 3c\gamma^3$ [Wiedemann, Particle Accelerator Physics, Springer, 2007].

circulating electron with constant velocity βc in following coordinate system



$$\vec{r}_0(t) = \rho_0 \vec{e}_1(\Phi)$$

$$\Phi = \frac{\beta c}{\rho_0} t$$

The electric and magnetic fields of a circulating charge are

$$\vec{E}(t) = e\left\{ \frac{\gamma^{-2}(\vec{n}-\vec{\beta})}{\rho_0^2(1-n\beta)^3} + \frac{[\vec{n}\times[(\vec{n}-\vec{\beta})\times d\vec{\beta}/dt]]}{c\rho_0(1-n\beta)^3} \right\}$$

$$\vec{H}(t) = [\vec{n}\times\vec{E}]$$

all values in right side are taken at retardation time t' coupled with t by retardation equation

$$c(t-t') = \left|\vec{r} - \vec{r}_0(t')\right|$$

Our concept since 1979 [С.Г. Арутюнян, Одновременная картина поля вблизи движущейся по кругу ультрарелятивистской заряженной частицы, препринт ЕФИ-387(45)-79, Ереван, 1979] was to investigate direct Lienard–Wiechert fields by solving retardation equation in vicinity of charge but far from Coulomb area.

$$\vec{r} = \rho_0(1+\delta)\vec{e_1}(\Phi) + \rho_0\zeta\vec{e_3}$$

$$\chi = (\Phi + \beta c(t-t')/\rho_0)/2$$

$$\chi^4 + 3\chi^2(\gamma^{-2} - \delta) - 3\chi\Phi + 3(\Phi^2 - \delta^2 - \zeta^2)/4 = 0$$

Synchrotron radiation is concentrated in area near the line

$$\vec{r}_\gamma(\Phi) = \rho_0(1+(3\Phi)^{2/3}/2)\vec{e}_1(\Phi)$$

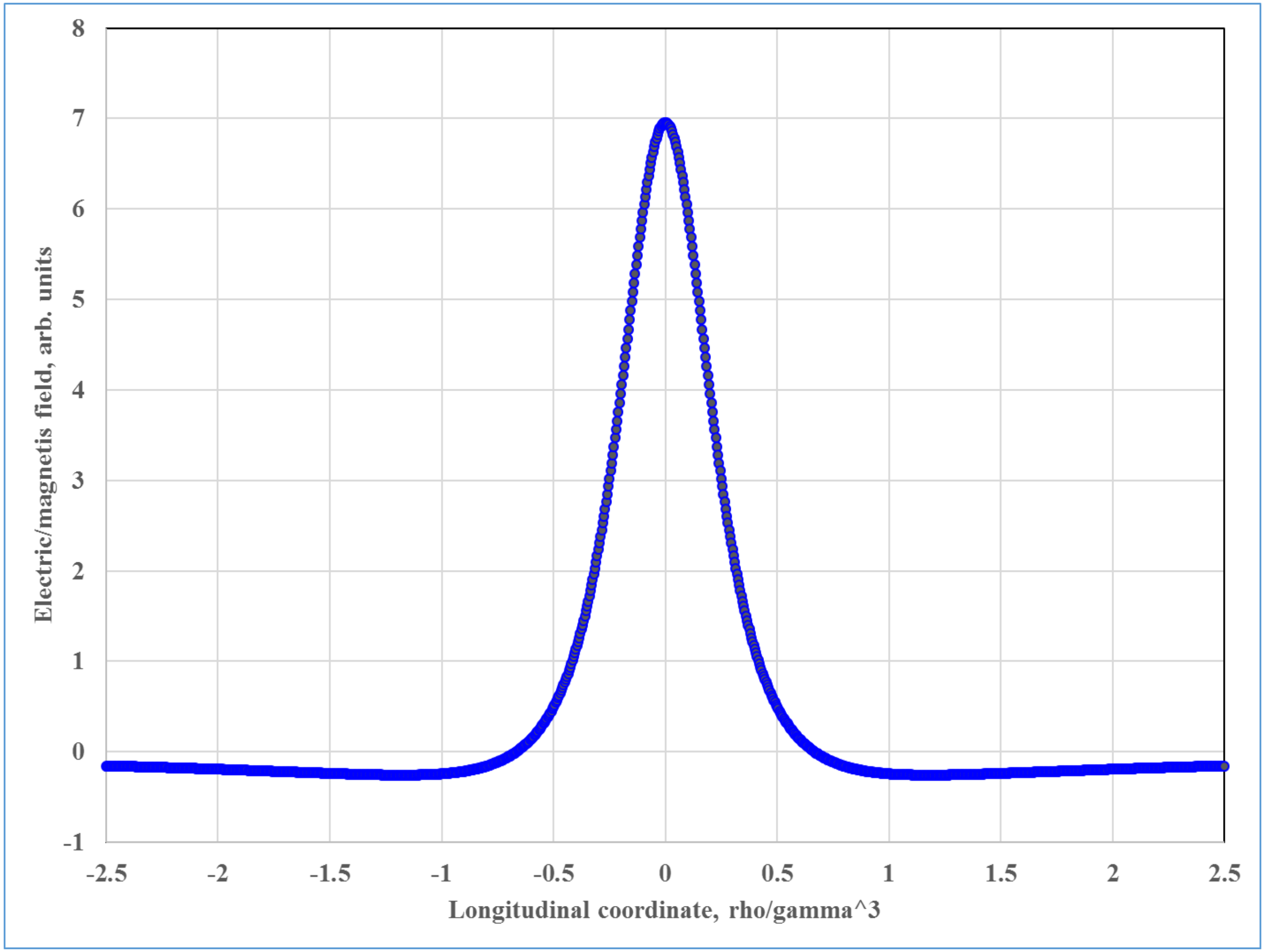
$$\varphi = (\Phi - (2\delta)^{3/2}/3)\gamma^3$$

$$\chi = \sqrt{\delta / 2} + \eta \qquad \eta = \left(\sqrt{9\varphi^2 + 1} + 3\varphi\right)^{1/3} - \left(\sqrt{9\varphi^2 + 1} - 3\varphi\right)^{1/3}$$

electric and magnetic field in synchrotron radiation area (in orbit plane):

$$\frac{\rho_0^2}{e}\vec{E} = \frac{2\sqrt{2}\gamma^4(1-\eta^2)}{(1+\eta^2)^3}\left\{\frac{\vec{e}_1(\phi)}{\delta^{1/2}} - \sqrt{2}\vec{e}_2(\Phi)\right\}$$

$$\frac{\rho_0^2}{e}\vec{H} = -\frac{2\sqrt{2}\gamma^4(1-\eta^2)\vec{e}_3}{(1+\eta^2)^3}$$



$$\frac{2\sqrt{2}\gamma^4(1-\eta^2)}{(1+\eta^2)^3}$$

The main factor as function of angular (longitudinal) shift

To illustrate the field of synchrotron radiation we use electric field lines. First complete formula for lines of field of arbitrary moving charge was obtained in [M.A. Aginian, S.G. Arutunian, Force lines of electric and magnetic fields of arbitrary moving charge, Preprint YerPhI 684(74)-83, Yerevan, 1983].

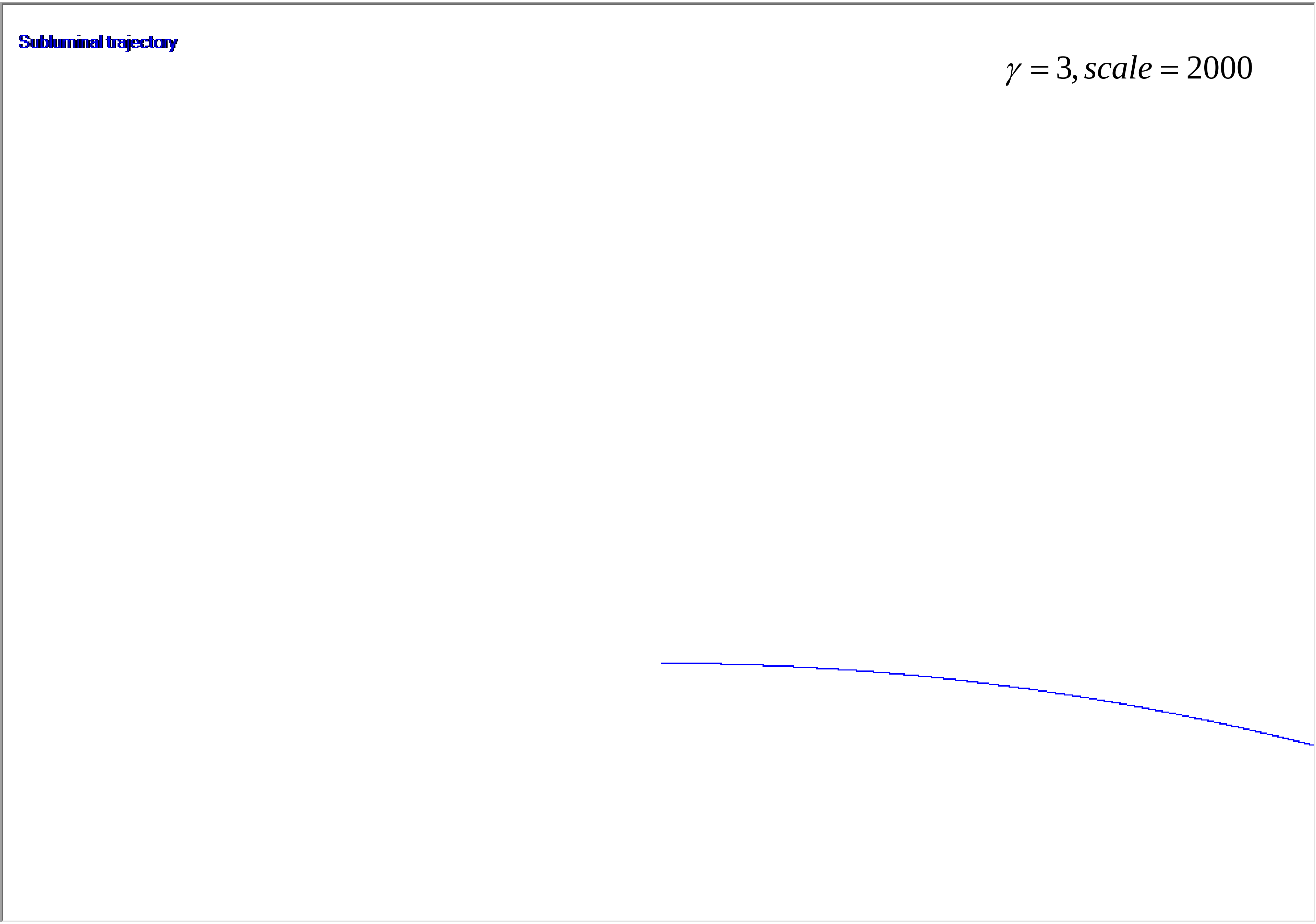
$$\vec{r}_0(\tau)/\rho_0 = \vec{e}_1(\beta\tau)$$

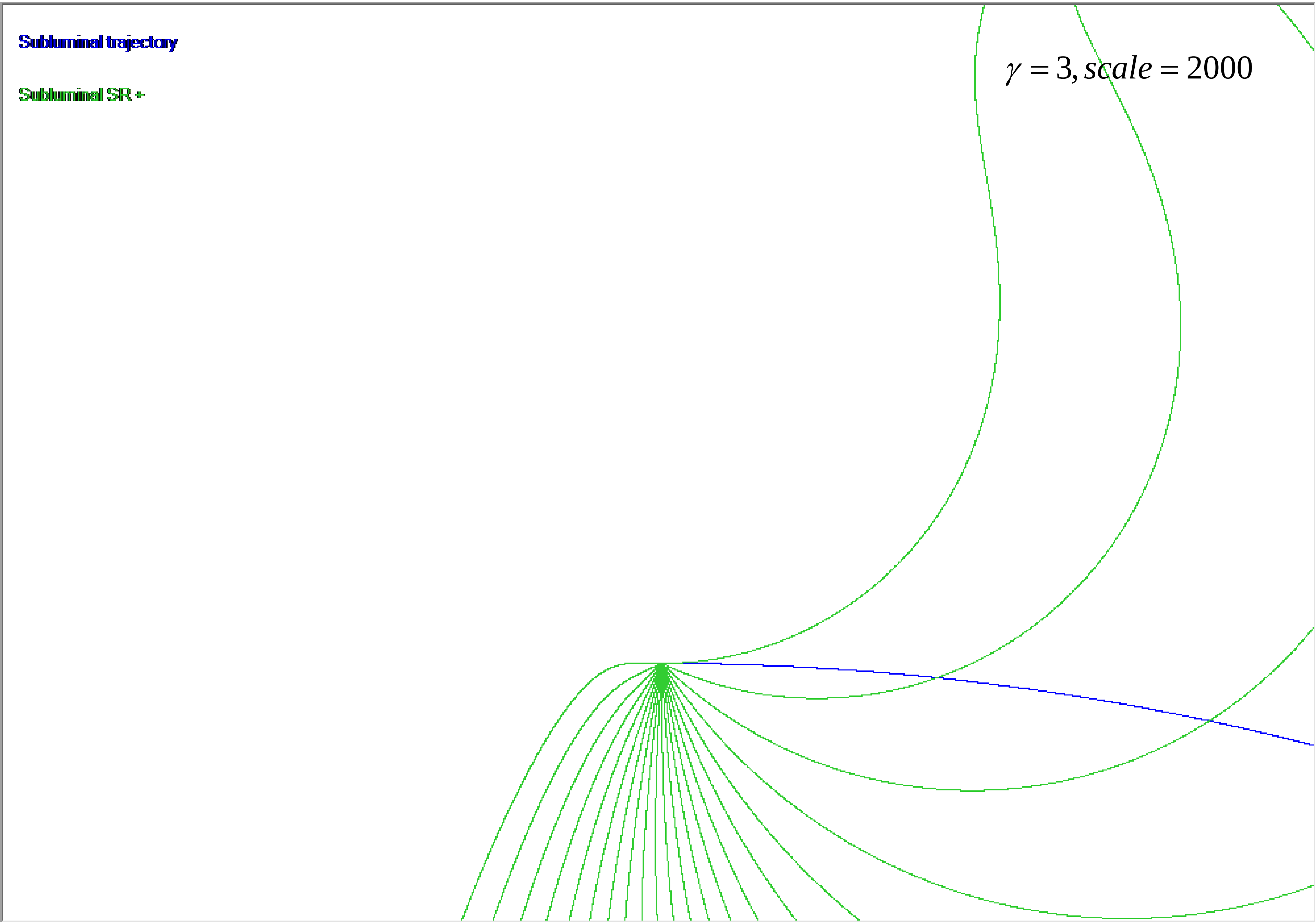
$$\vec{L}(\tau)/\rho_0 = \vec{e}_1(\beta\sigma) + (\tau-\sigma)(n_1\vec{e}_1(\beta\sigma) + n_2\vec{e}_2(\beta\sigma) + n_3\vec{e}_3)$$

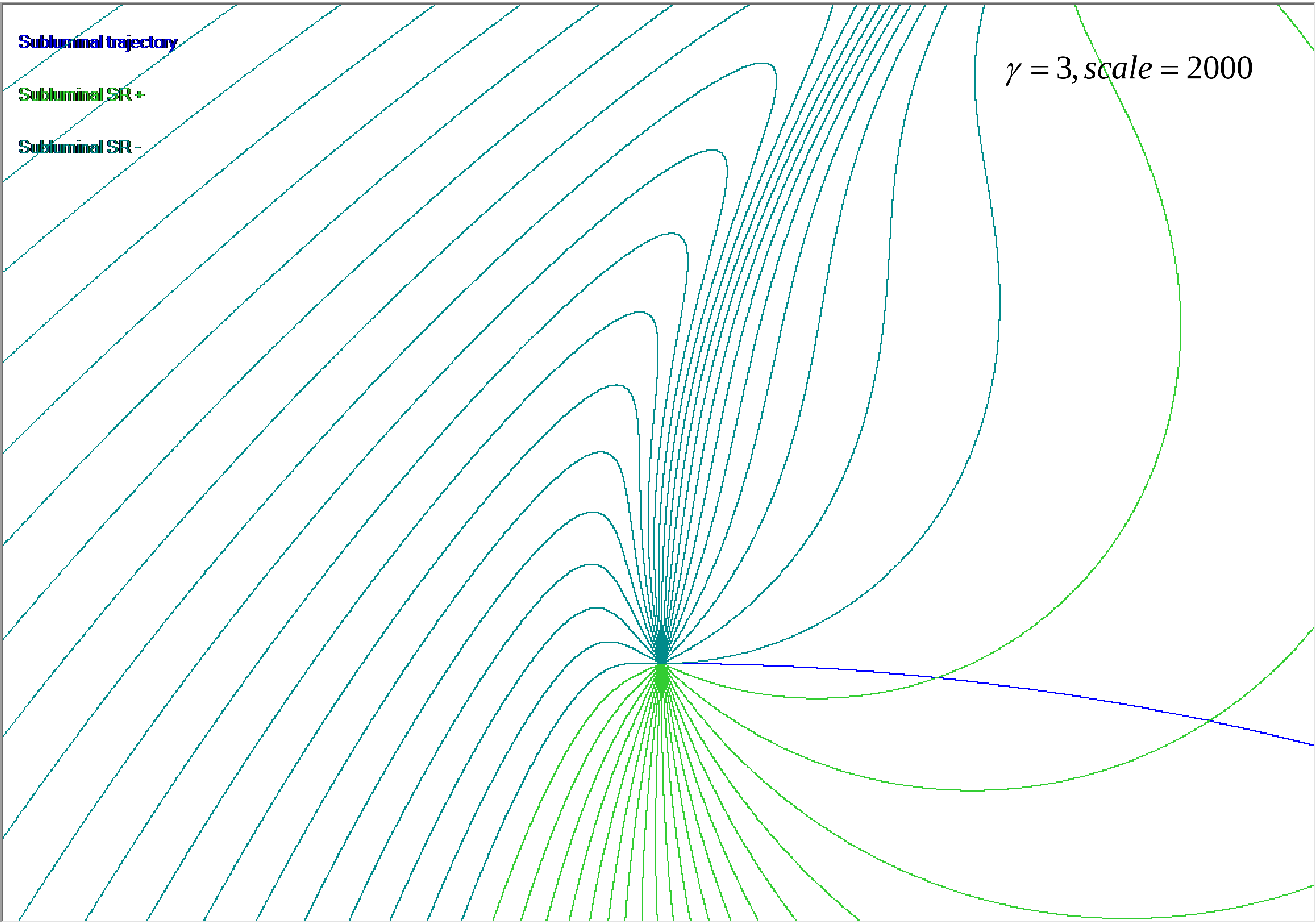
$$n_1 = -\frac{\sqrt{1-c_2^2}\,\sin(\beta\gamma(\tau-\sigma)+c_1)}{\gamma(1+\beta\sqrt{1-c_2^2}\,\cos(\beta\gamma(\tau-\sigma)+c_1))}$$

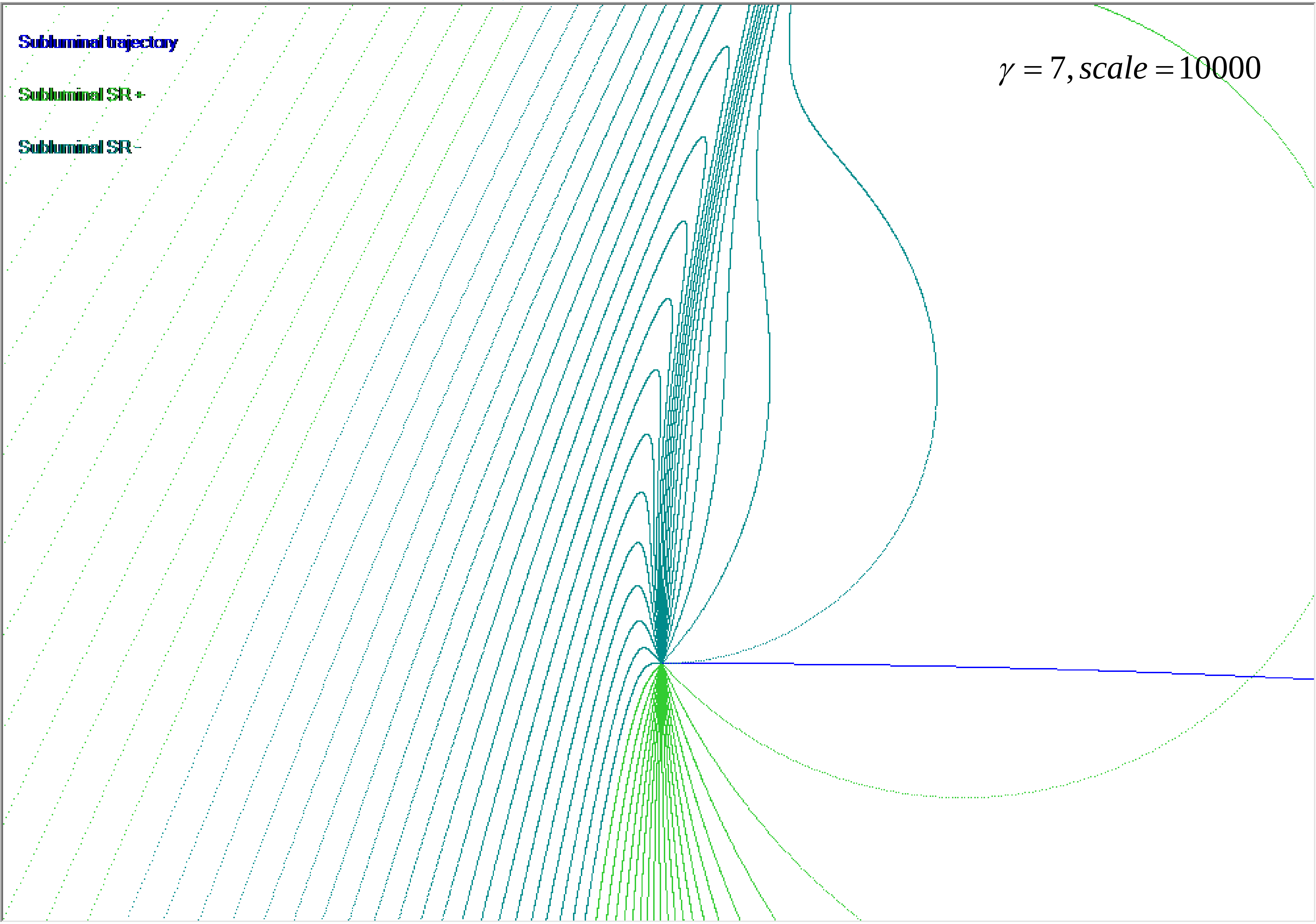
$$n_2 = \frac{\beta+\sqrt{1-c_2^2}\,\sin(\beta\gamma(\tau-\sigma)+c_1)}{1+\beta\sqrt{1-c_2^2}\,\cos(\beta\gamma(\tau-\sigma)+c_1)}$$

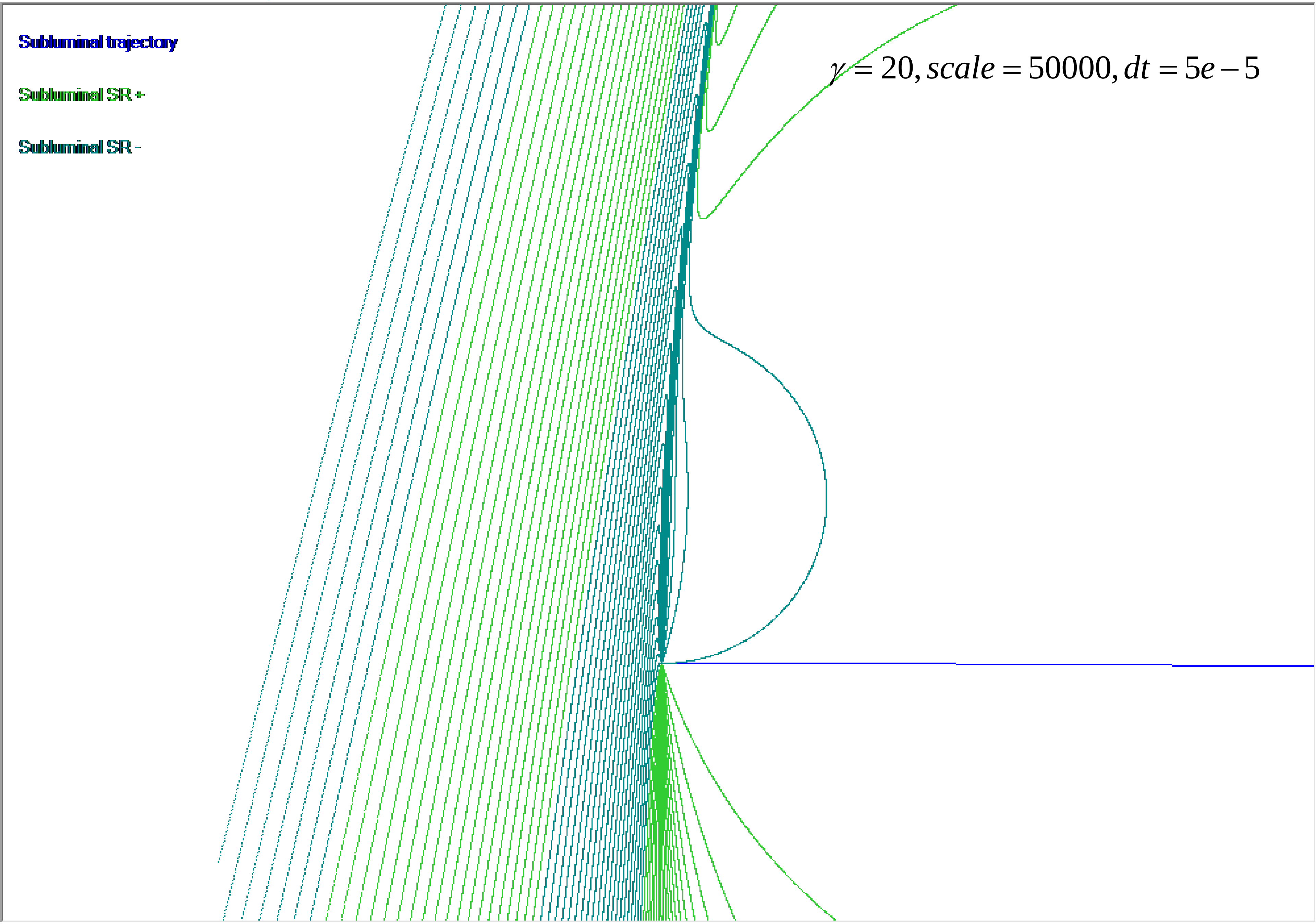
$$n_3 = \frac{c_2}{\gamma(1+\beta\sqrt{1-c_2^2}\,\cos(\beta\gamma(\tau-\sigma)+c_1))}$$

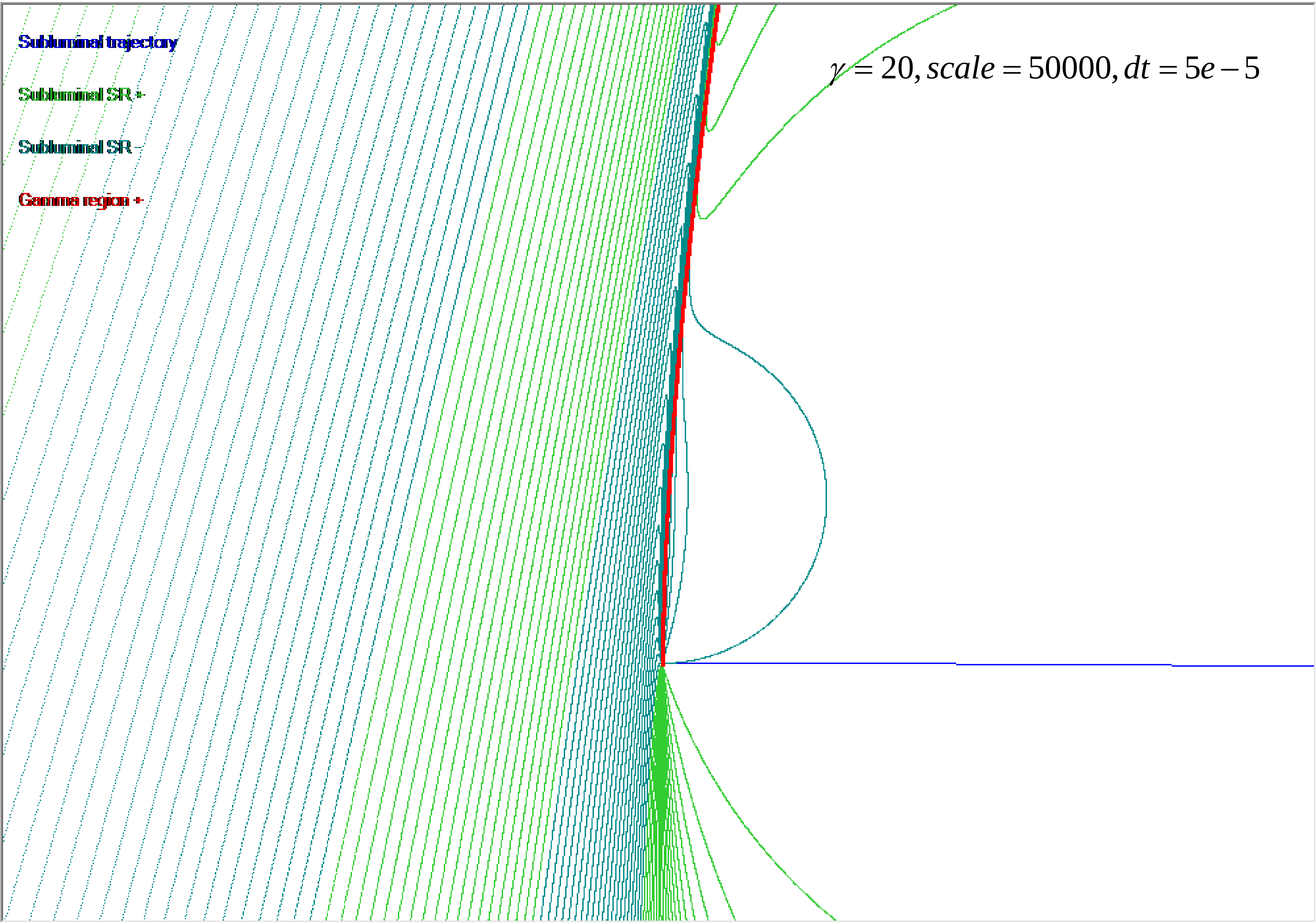

Subluminal trajectory
$\gamma = 3, scale = 2000$

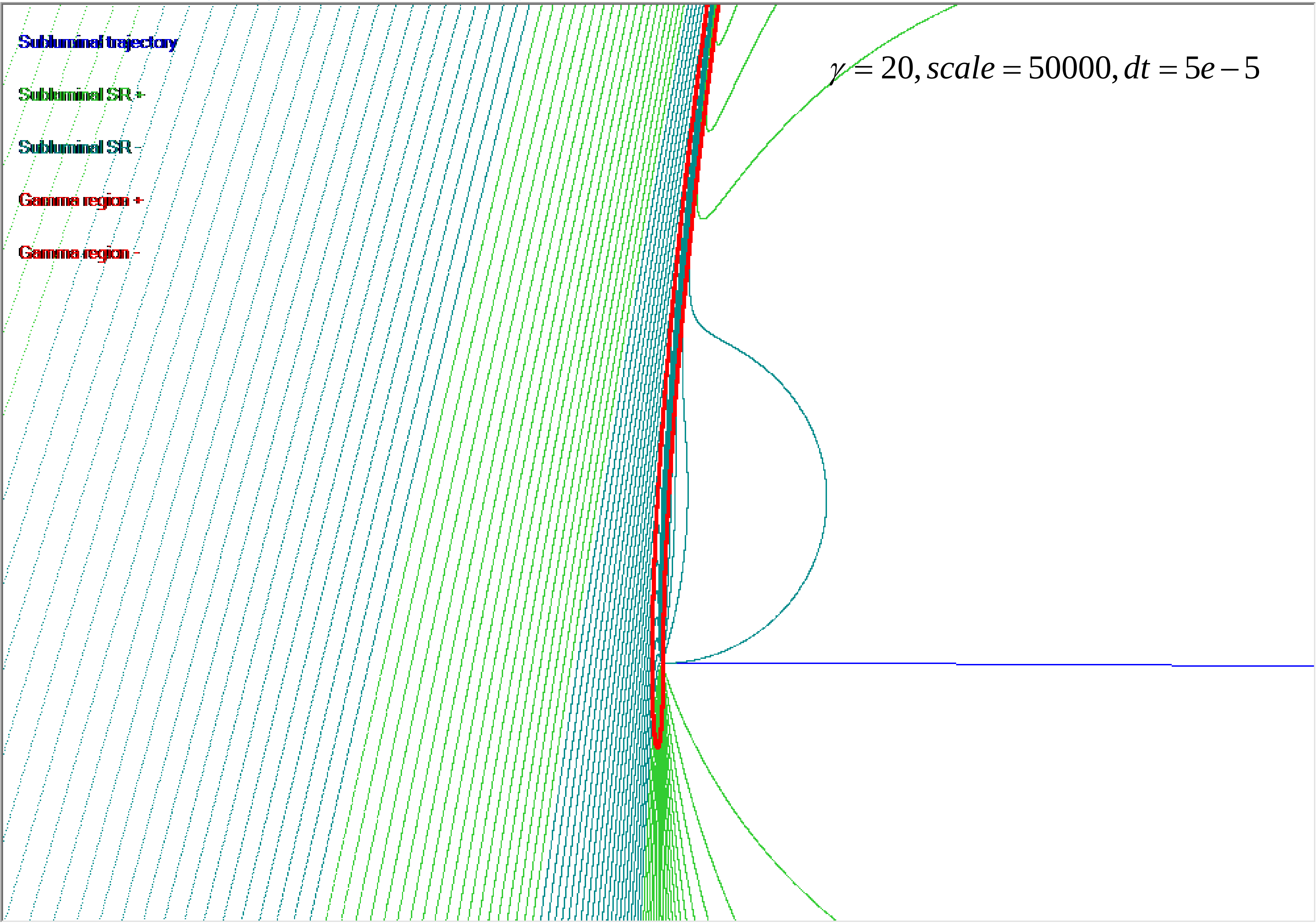

Subluminal trajectory
Subluminal SR +
$\gamma = 3, scale = 2000$

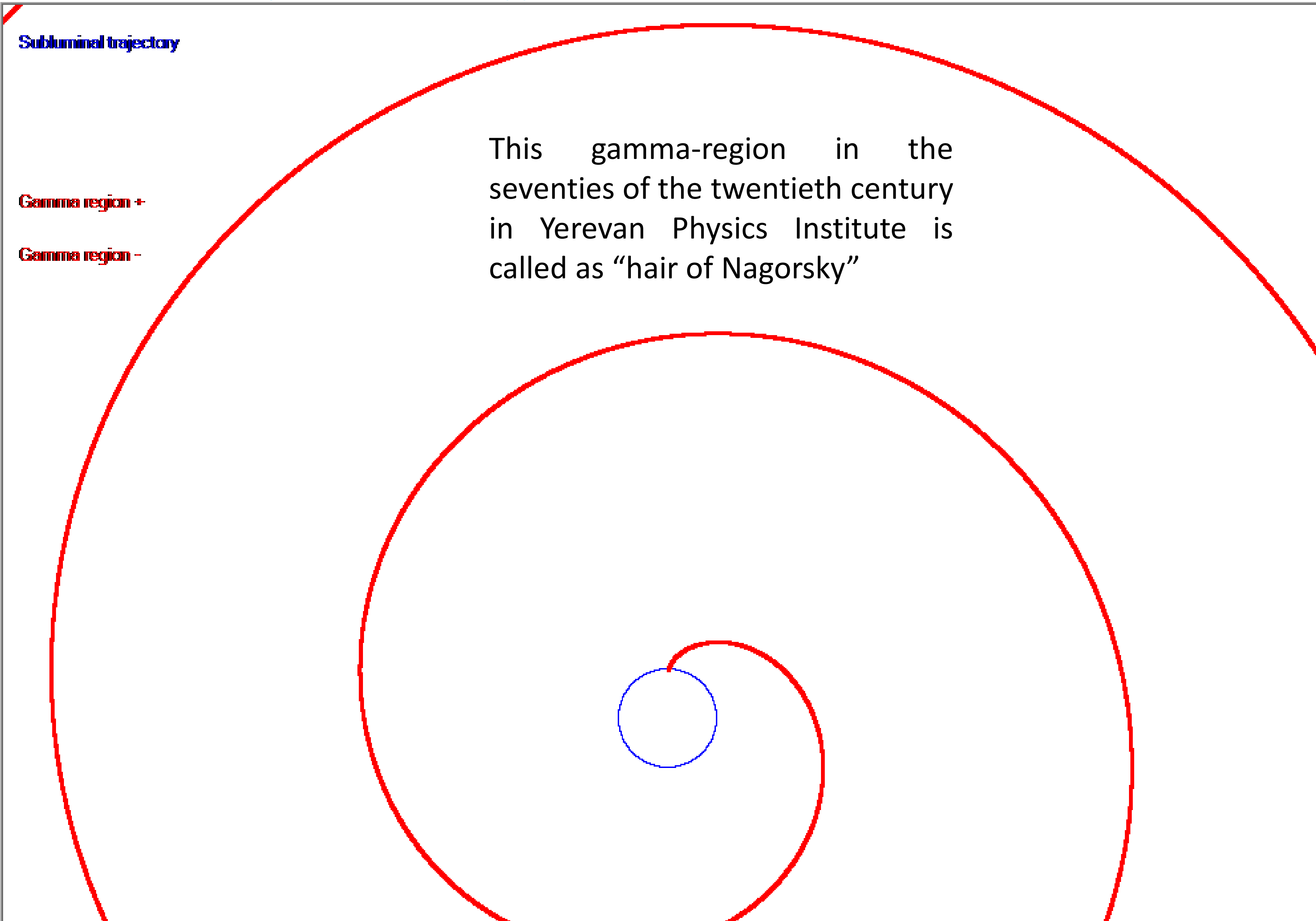

Subluminal trajectory
Subluminal SR +
Subluminal SR -
$\gamma = 3, scale = 2000$


Subluminal trajectory
Subluminal SR +
Subluminal SR -
$\gamma = 7, scale = 10000$

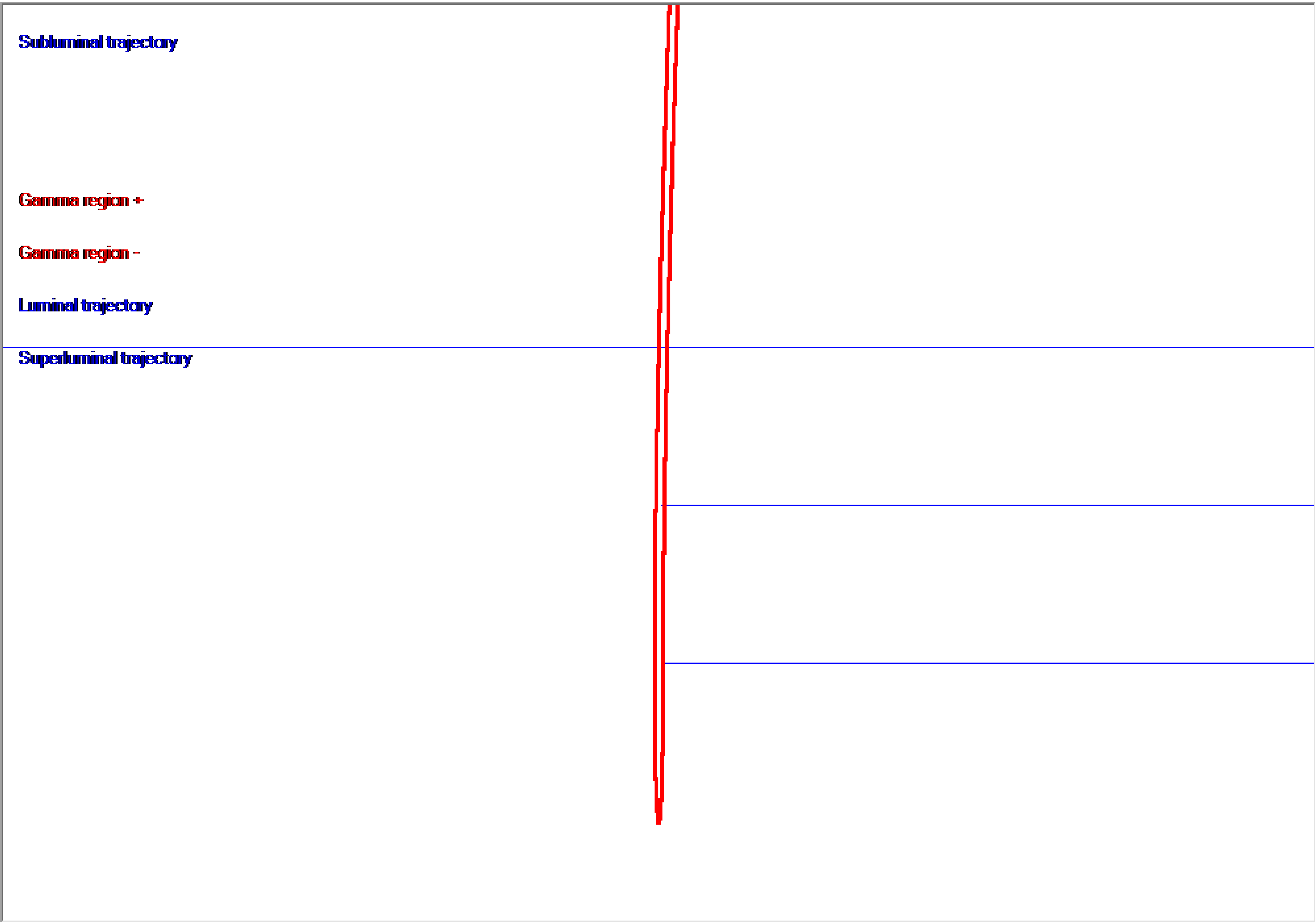

Subluminal trajectory
Subluminal SR +
Subluminal SR -
$\gamma = 20, scale = 50000, dt = 5e-5$


Subluminal trajectory
Subluminal SR +
Subluminal SR -
Gamma region +
$\gamma = 20, scale = 50000, dt = 5e-5$


Subluminal trajectory
Subluminal SR +
Subluminal SR -
Gamma region +
Gamma region -
$\gamma = 20, scale = 50000, dt = 5e-5$



This gamma-region in the seventies of the twentieth century in Yerevan Physics Institute is called as “hair of Nagorsky”


Subluminal trajectory
Gamma region +
Gamma region -
Luminal trajectory
Superluminal trajectory

On the outer wall of vacuum chamber, the "spot" of synchrotron radiation is running with angular velocity $\omega_0 = \beta c / \rho_0$. If distance from charge trajectory to outer wall of vacuum chamber is $\delta\rho_0$. the linear velocity of the "spot" is $\beta c(1+\delta)$. For typical values of synchrotron light sources $\beta c(1+\delta)$ is greater than c at very small values of δ. For CANDLE project E=3 GeV (γ=5.87E+03) and radius of trajectory in dipole magnet ρ_0=7.4 m (corresponds to magnetic field B=1.35 T) [CANDLE Design Report, July 2002] the distance $\delta\rho_0$ at which velocity of "spot" exceed the speed of light is only 0.5 μm. So, it is evidently that the "spots" of synchrotron radiation running on the outer wall of vacuum chamber with velocities greater than the light velocity.

The sizes of the "spot" are $(\rho_0 / \gamma^3) \times (\rho_0 \sqrt{\delta} / \gamma)$.

$$q = -\sigma_e S \qquad\qquad q \sim -\frac{e}{\rho_0^2} \frac{\gamma^4}{\sqrt{\delta}} \times \frac{\rho_0}{\gamma^3} \times \frac{\rho_0 \sqrt{\delta}}{\gamma} = -e$$

[J.D. Jackson, Classical Electrodynamics, Third Ed., Wiley, 1999 /Models for the Molecular Polarizability].

$$\Delta x \sim (eE / m)\delta t^2 \qquad \Delta x \sim r_0 \frac{\gamma^{-2}}{\sqrt{\delta}}$$

$$E_m \sim en_e \times \Delta x = \frac{en_e r_0}{\sqrt{\delta}} \gamma^{-2} \qquad n_e r_0 \rho_0^2 \gamma^{-6}$$

For values of CANDLE project this value is about 3.2×10^{-7} (for n_e $=8.5\times10^{28}$ m^{-3} copper value). It means that charge induced in synchrotron radiation spot is about 3.2×10^{-7}e.

Superluminal synchrotron radiation

$$\vec{r}^{\,*}(\tau) = \rho \vec{e}_1\left(\frac{B\tau}{\rho}\right)$$

$$\vec{L}^{\,*}(\tau) = \rho \vec{e}_1\left(\frac{B\sigma}{\rho} + \Phi^*\right) + (\tau - \sigma)\left(n_1 \vec{e}_1\left(\frac{B\sigma}{\rho} + \Phi^*\right) + n_2 \vec{e}_2\left(\frac{B\sigma}{\rho} + \Phi^*\right)\right)$$

$$B = \rho\beta = \rho\sqrt{1 - \gamma^{-2}} > 1$$

$$G = 1/\sqrt{B^2 - 1} = 1/\sqrt{\rho^2(1 - \gamma^{-2}) - 1}$$

$$G = \gamma \qquad\qquad \rho = \sqrt{(\gamma^2 + 1)/(\gamma^2 - 1)}$$

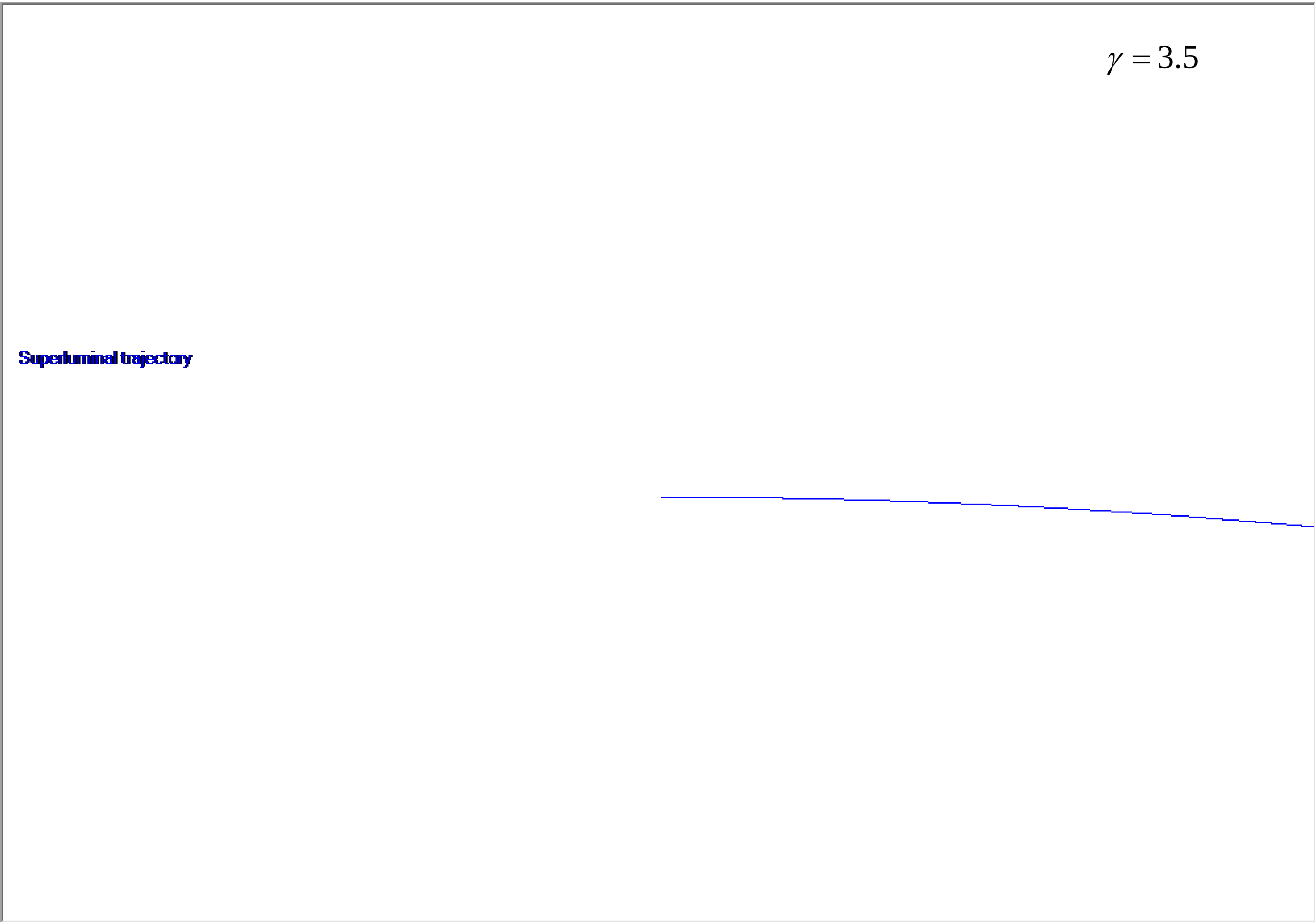
$$n_1 = \mp \frac{\sinh(BG(\tau - \sigma) + c_1)}{G(1 \pm B\cosh(BG(\tau - \sigma) + c_1))}$$

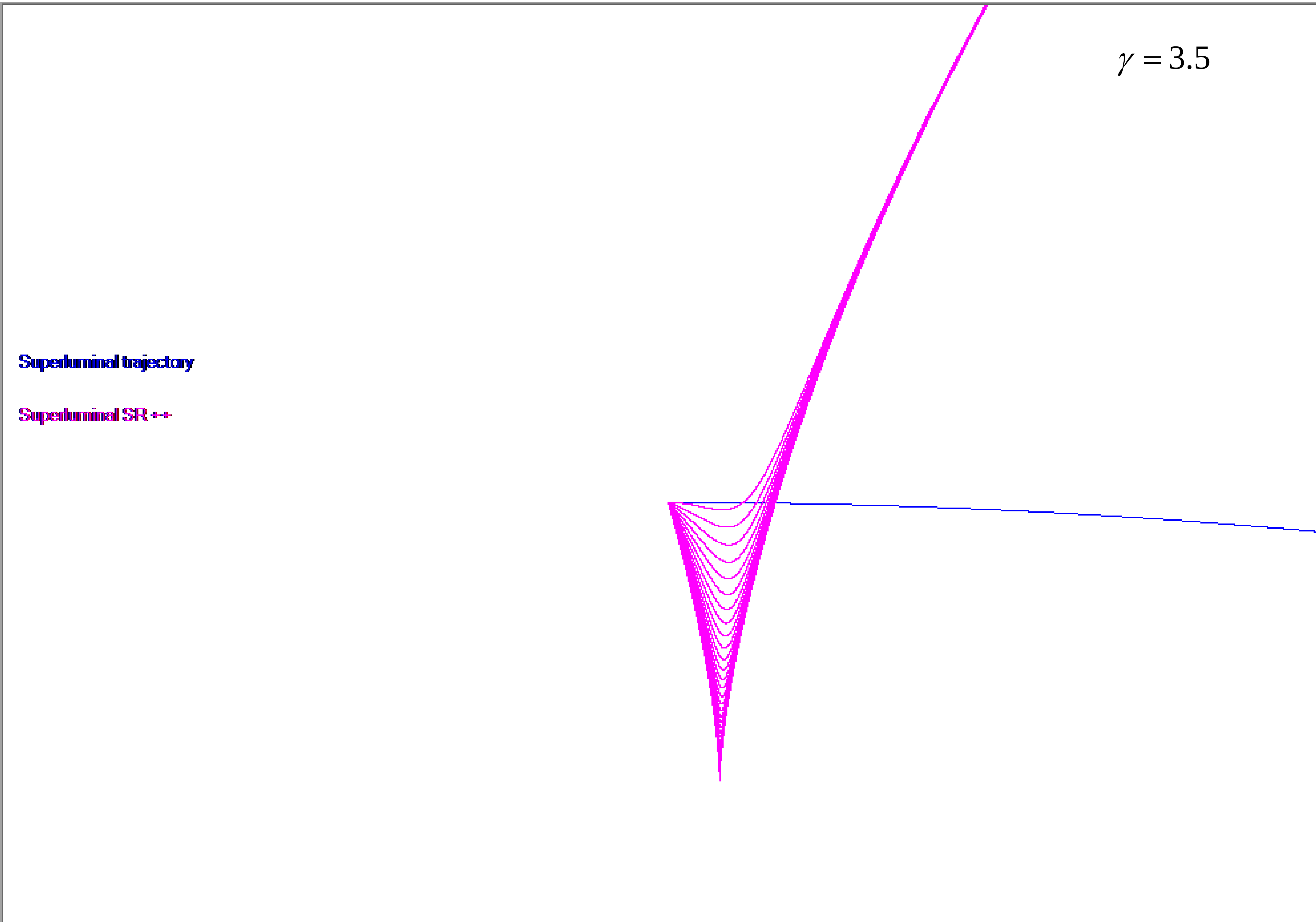
$$n_2 = \frac{B \pm \sinh(BG(\tau - \sigma) + c_1)}{1 \pm B\cosh(BG(\tau - \sigma) + c_1)}$$

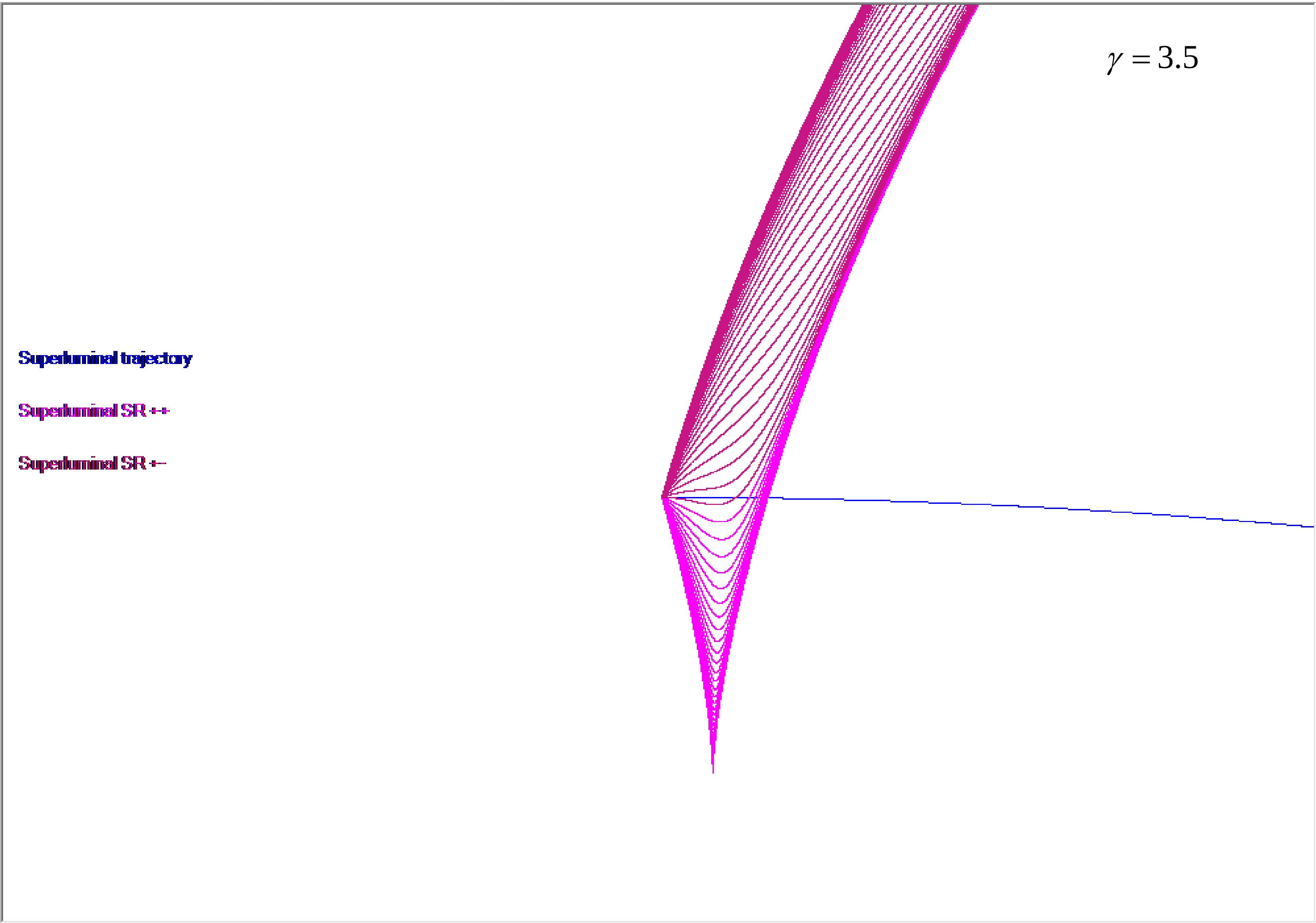
Mach surface

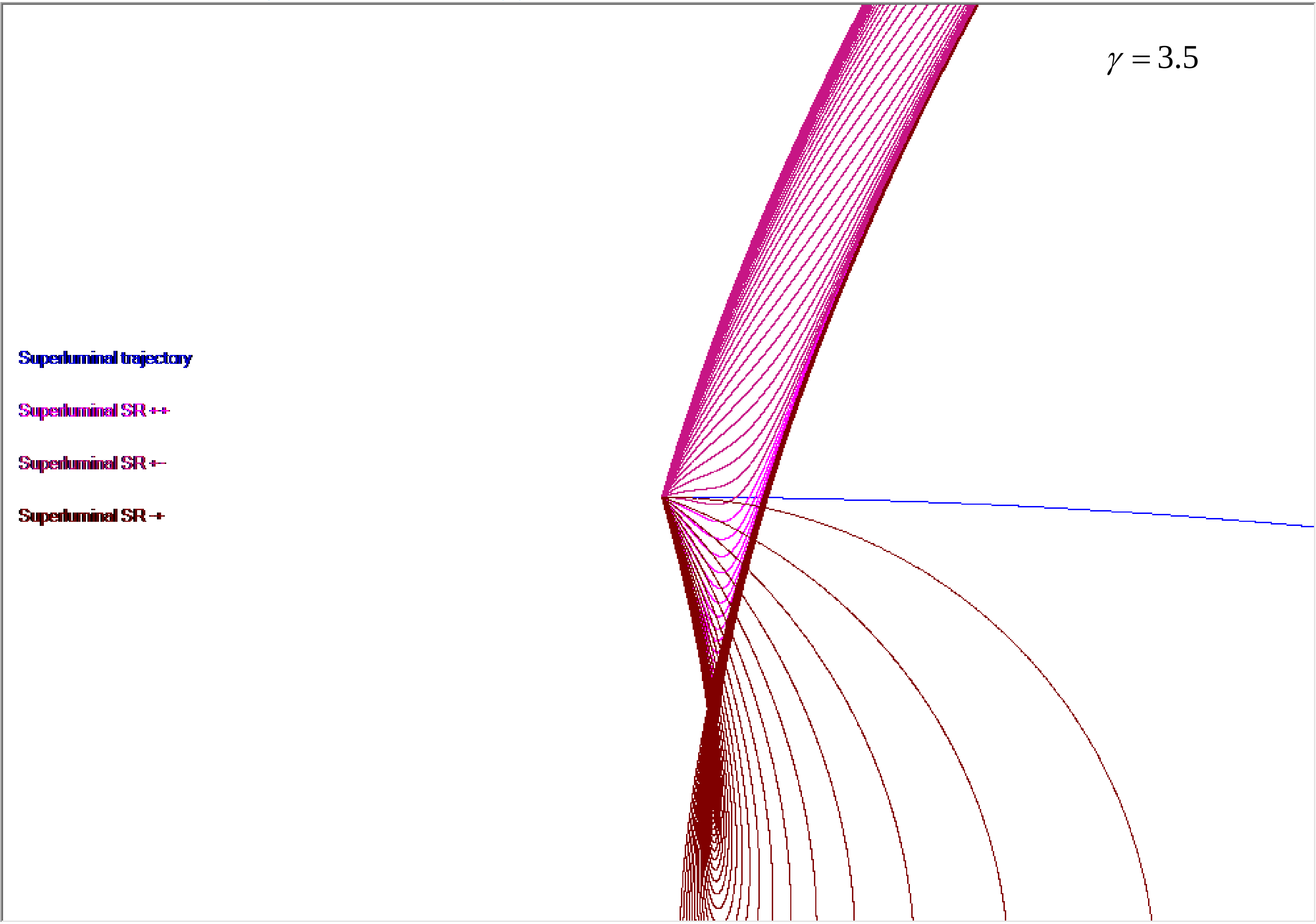
$$\vec{n}\vec{B} = 1$$

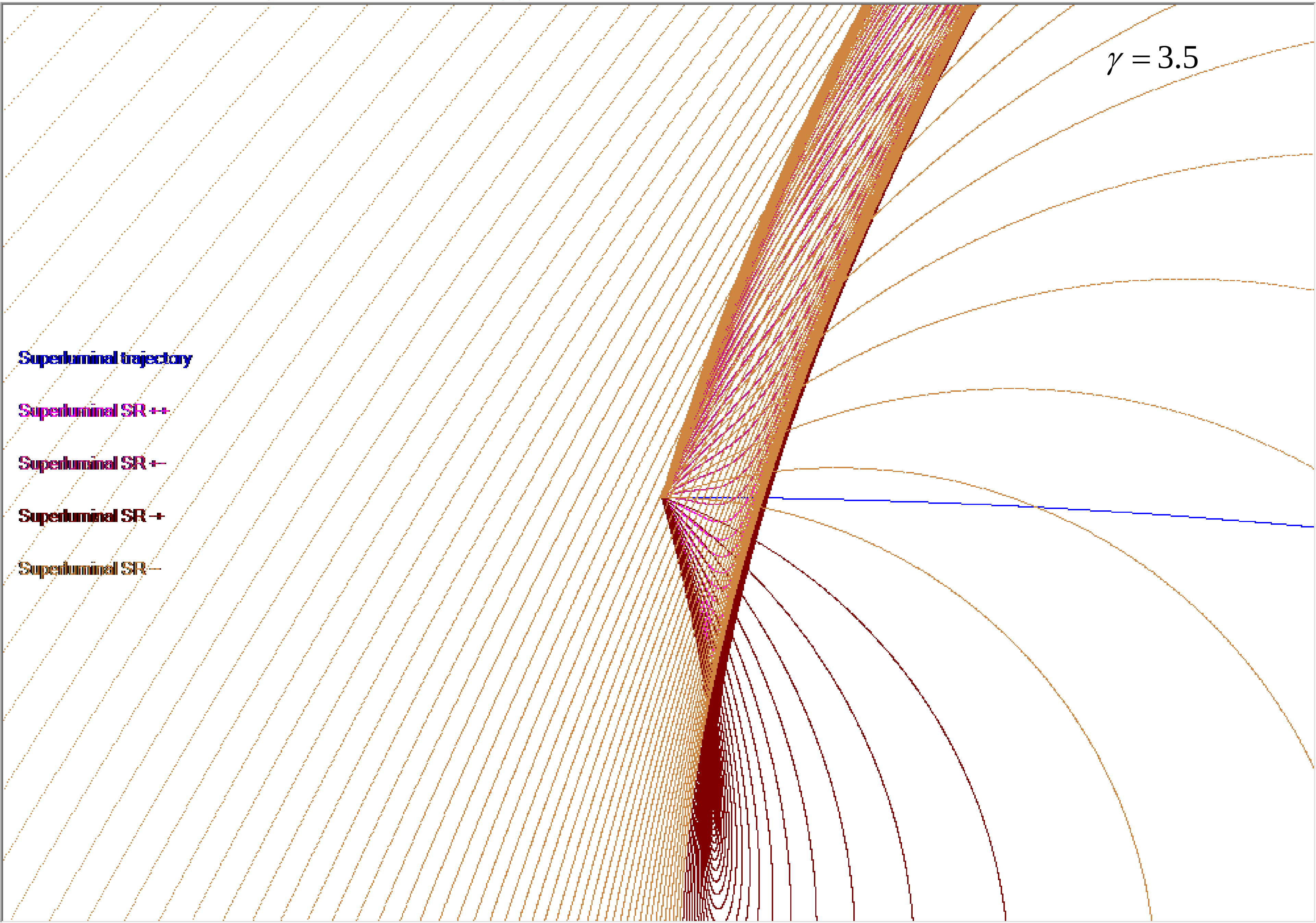
Here $\vec{B}c$ is velocity of superluminal charge

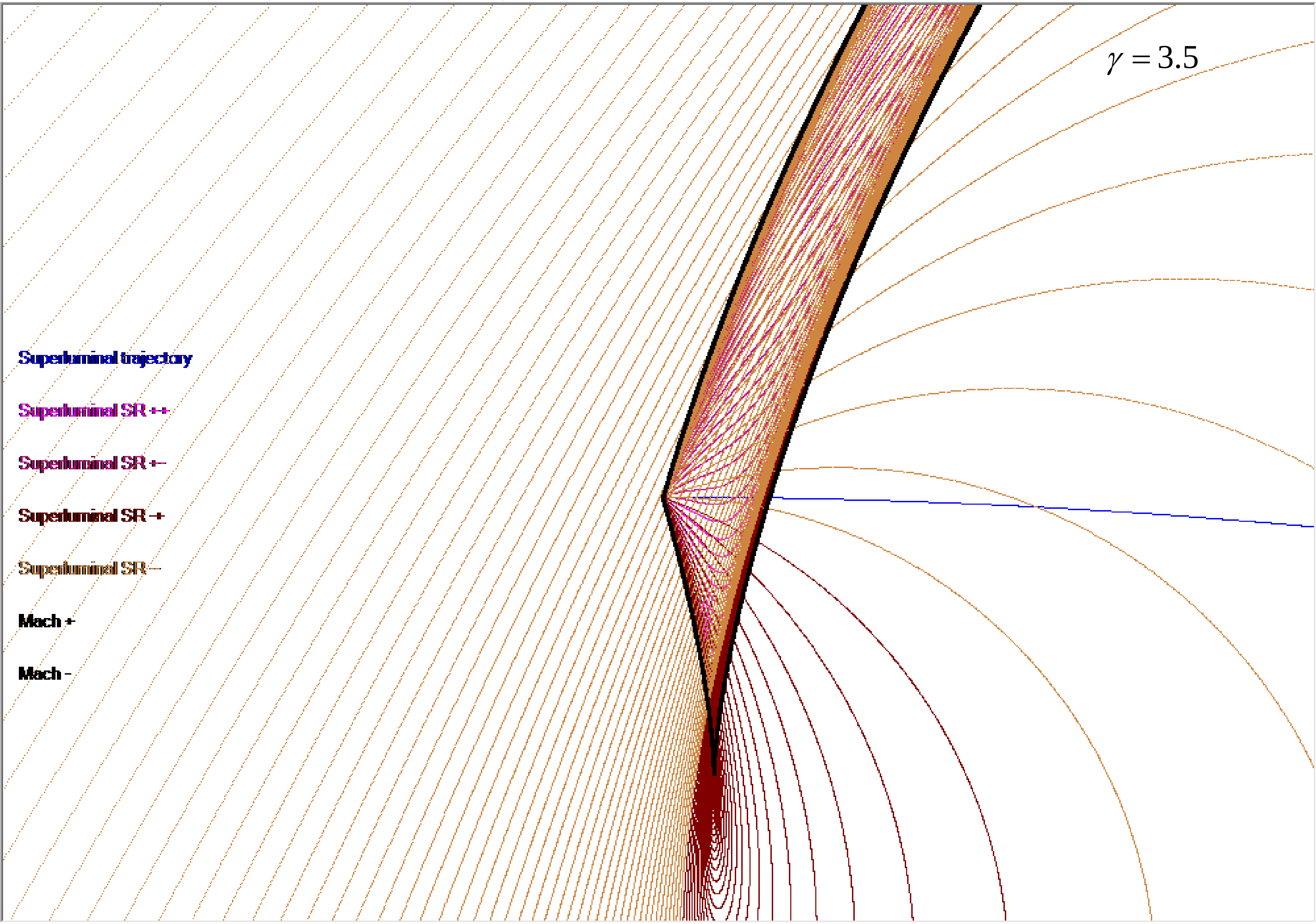

γ = 3.5
Superluminal trajectory

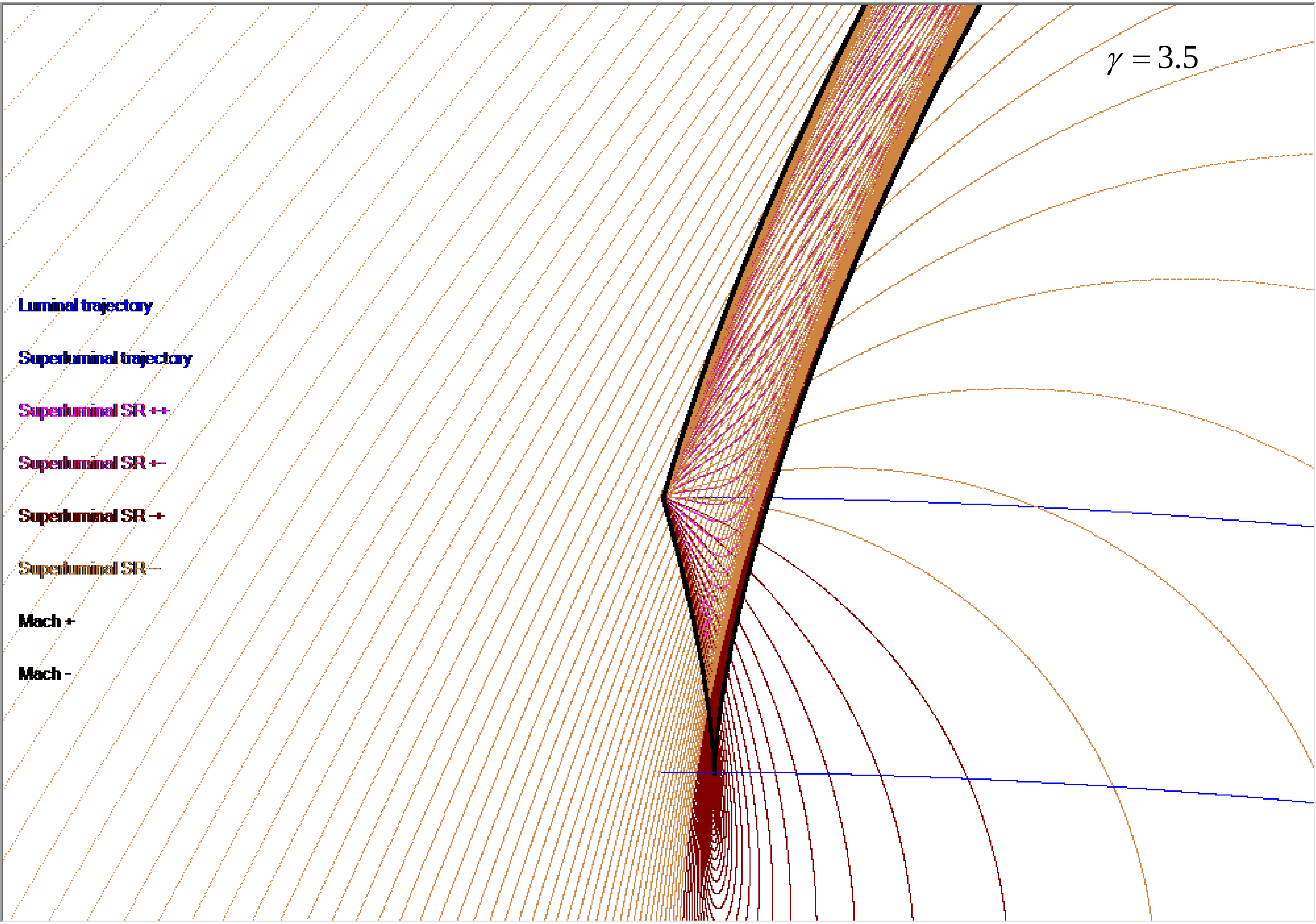

$\gamma = 3.5$
Superluminal trajectory
Superluminal SR ++


$\gamma = 3.5$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-

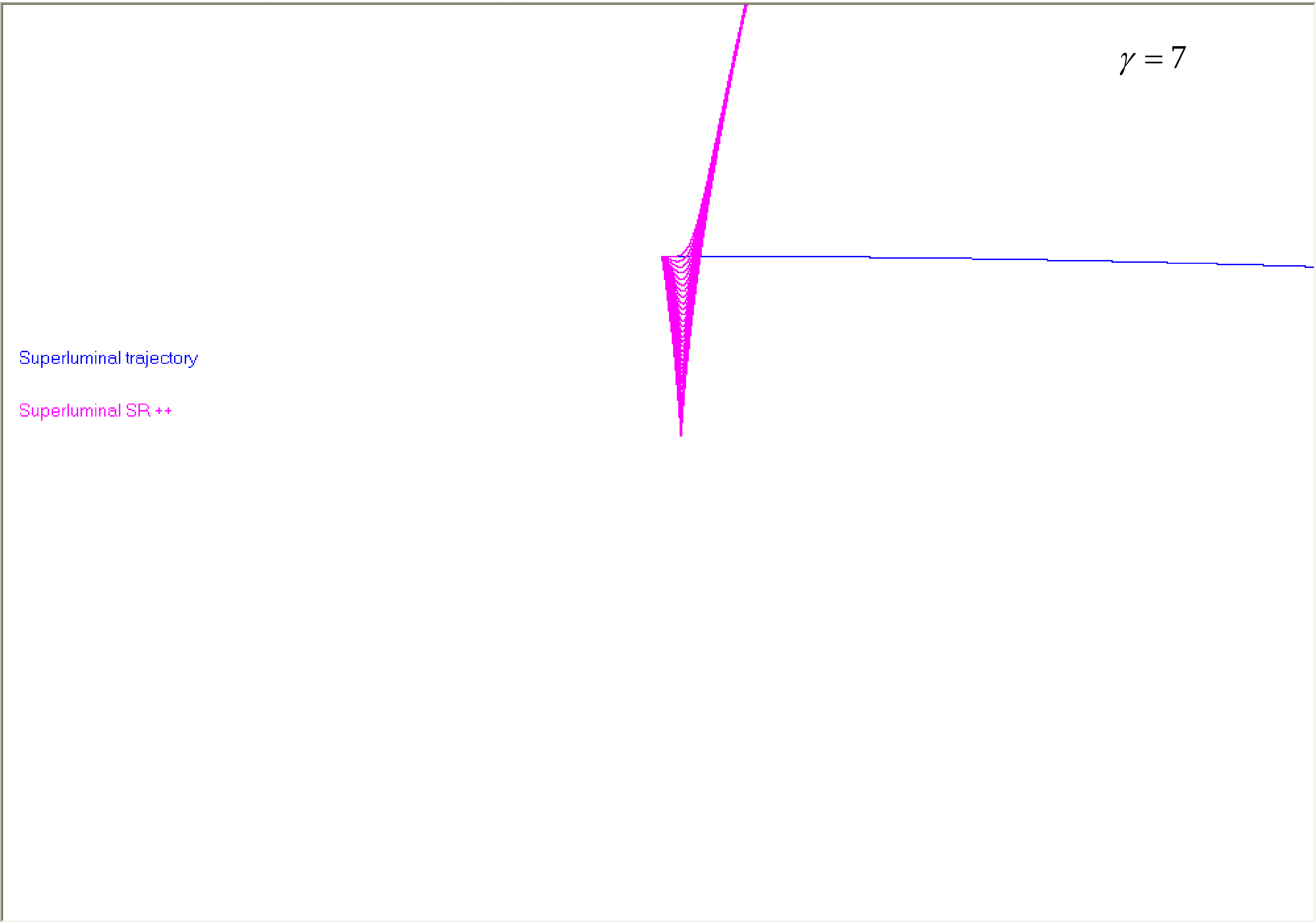

$\gamma = 3.5$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-
Superluminal SR -+


$\gamma = 3.5$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−
Superluminal SR −+
Superluminal SR −−

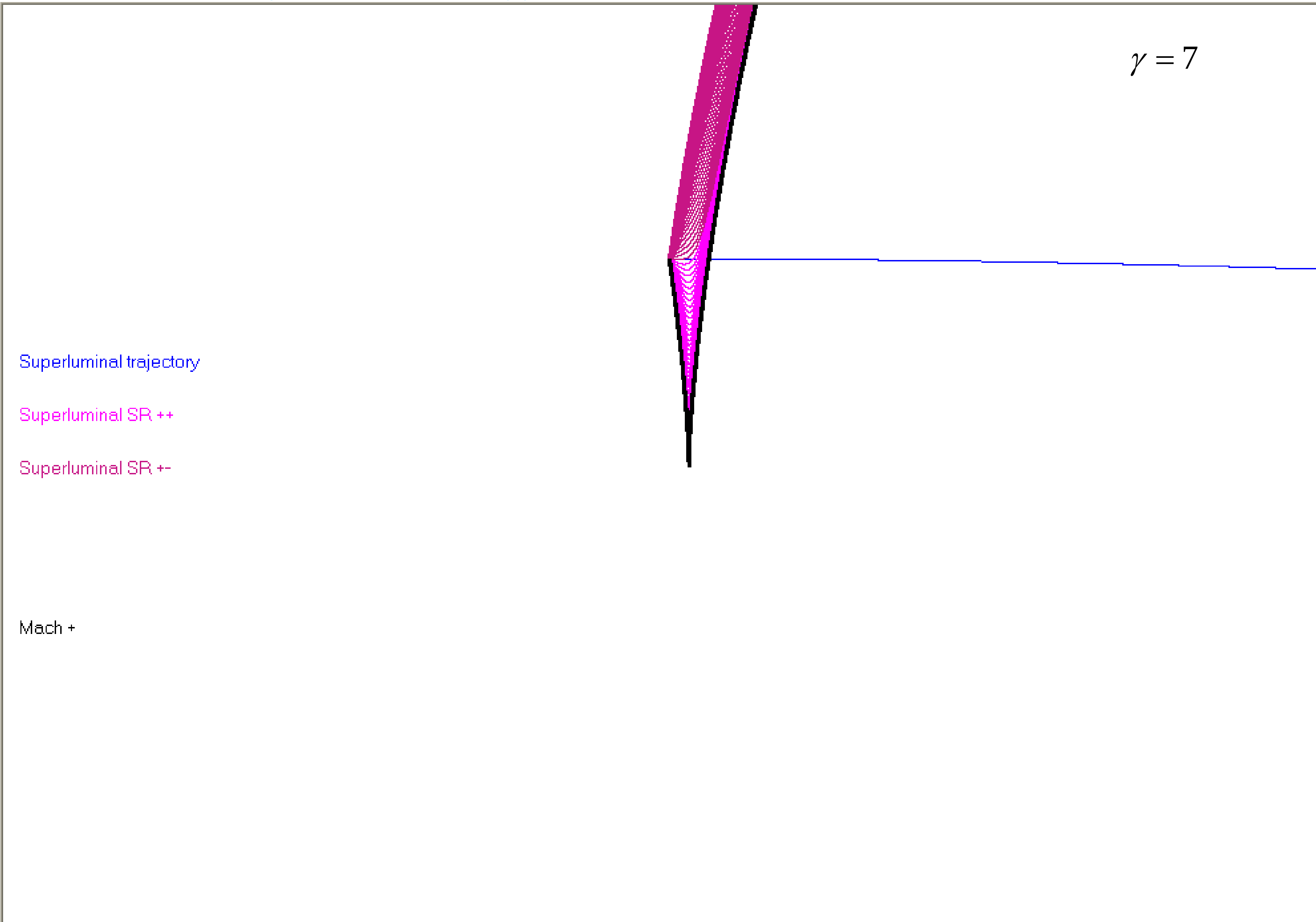

$\gamma = 3.5$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-
Superluminal SR -+
Superluminal SR --
Mach +
Mach -

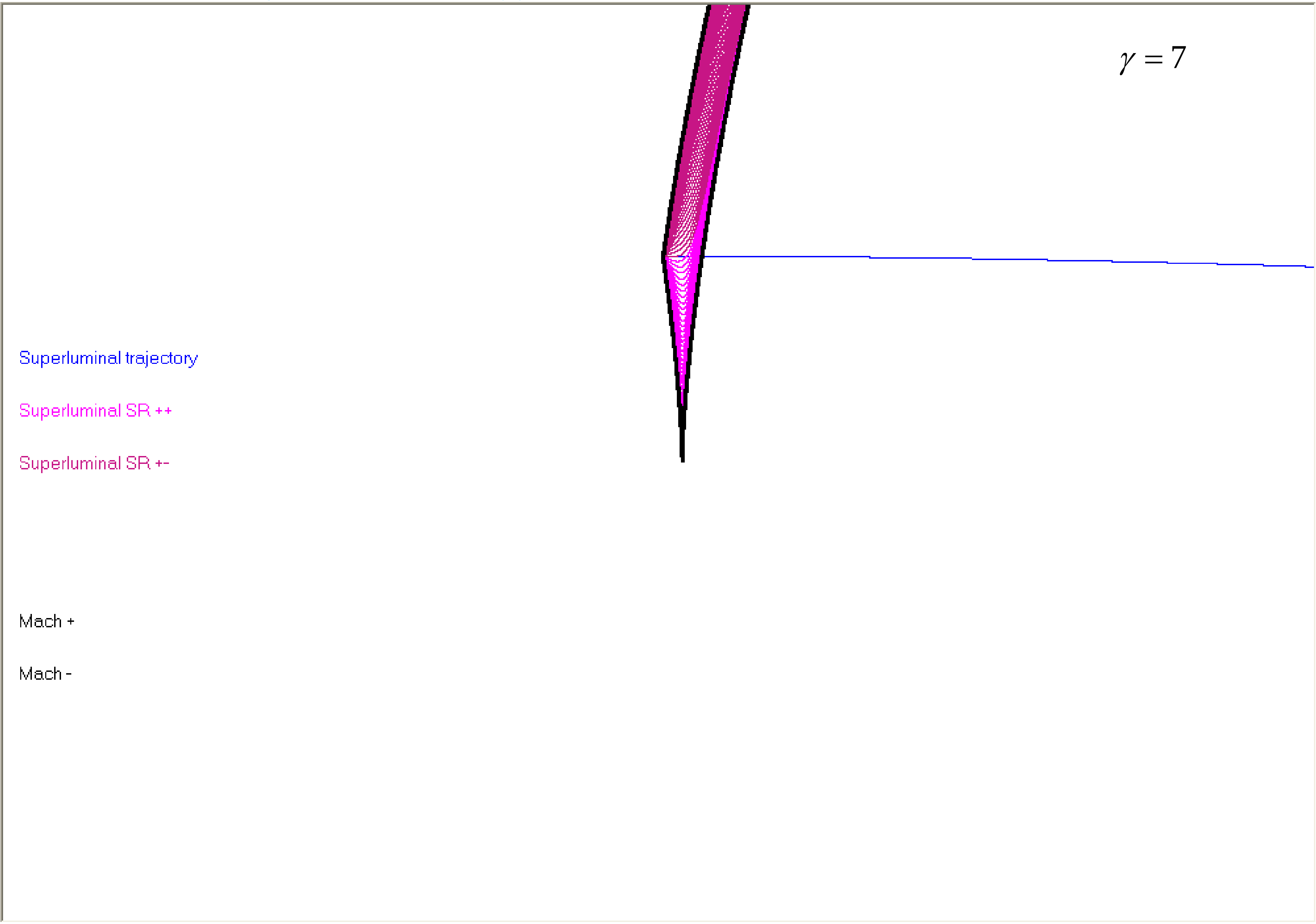

$\gamma = 3.5$
Luminal trajectory
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−
Superluminal SR −+
Superluminal SR −−
Mach +
Mach −

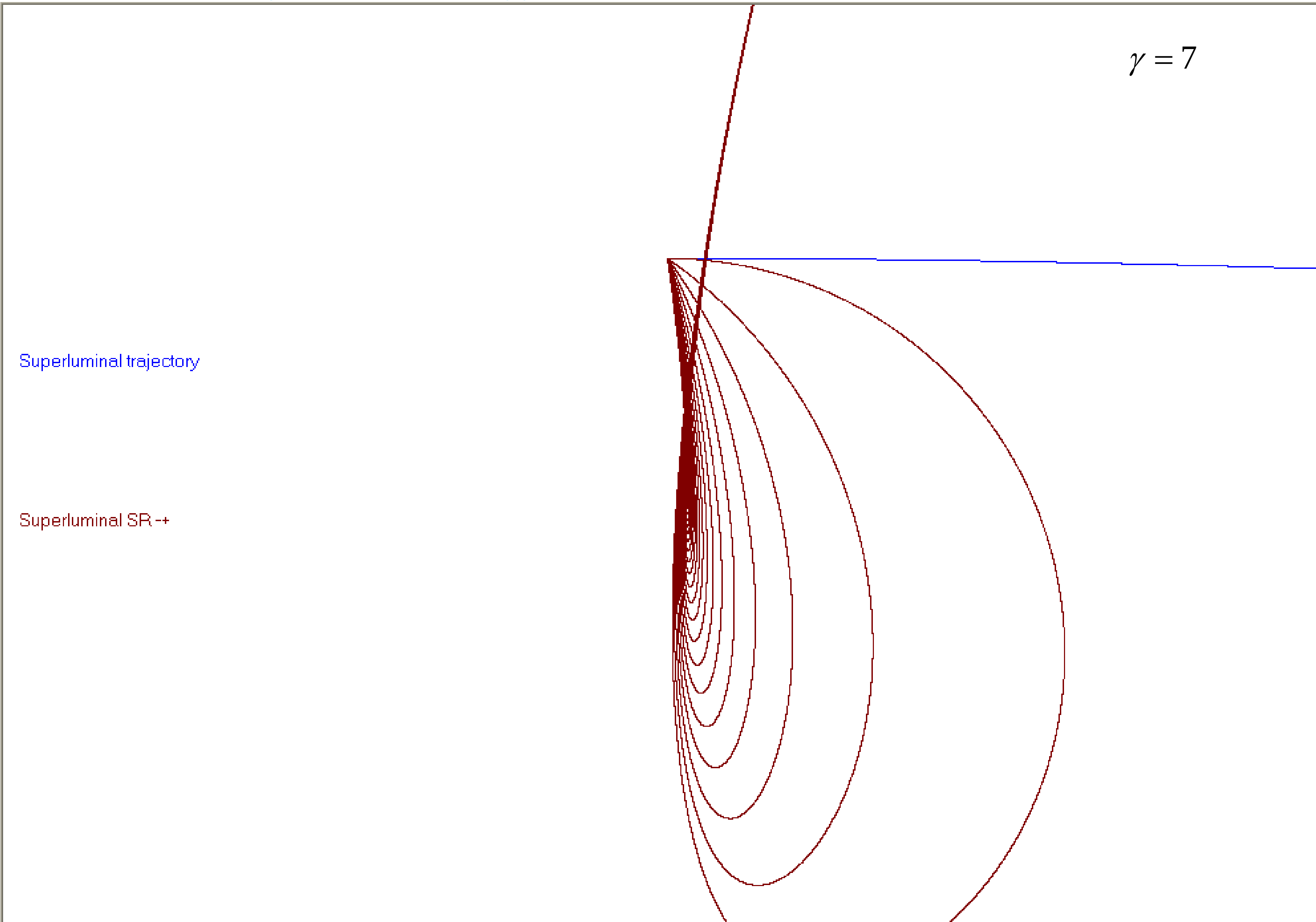

$\gamma = 7$
Superluminal trajectory

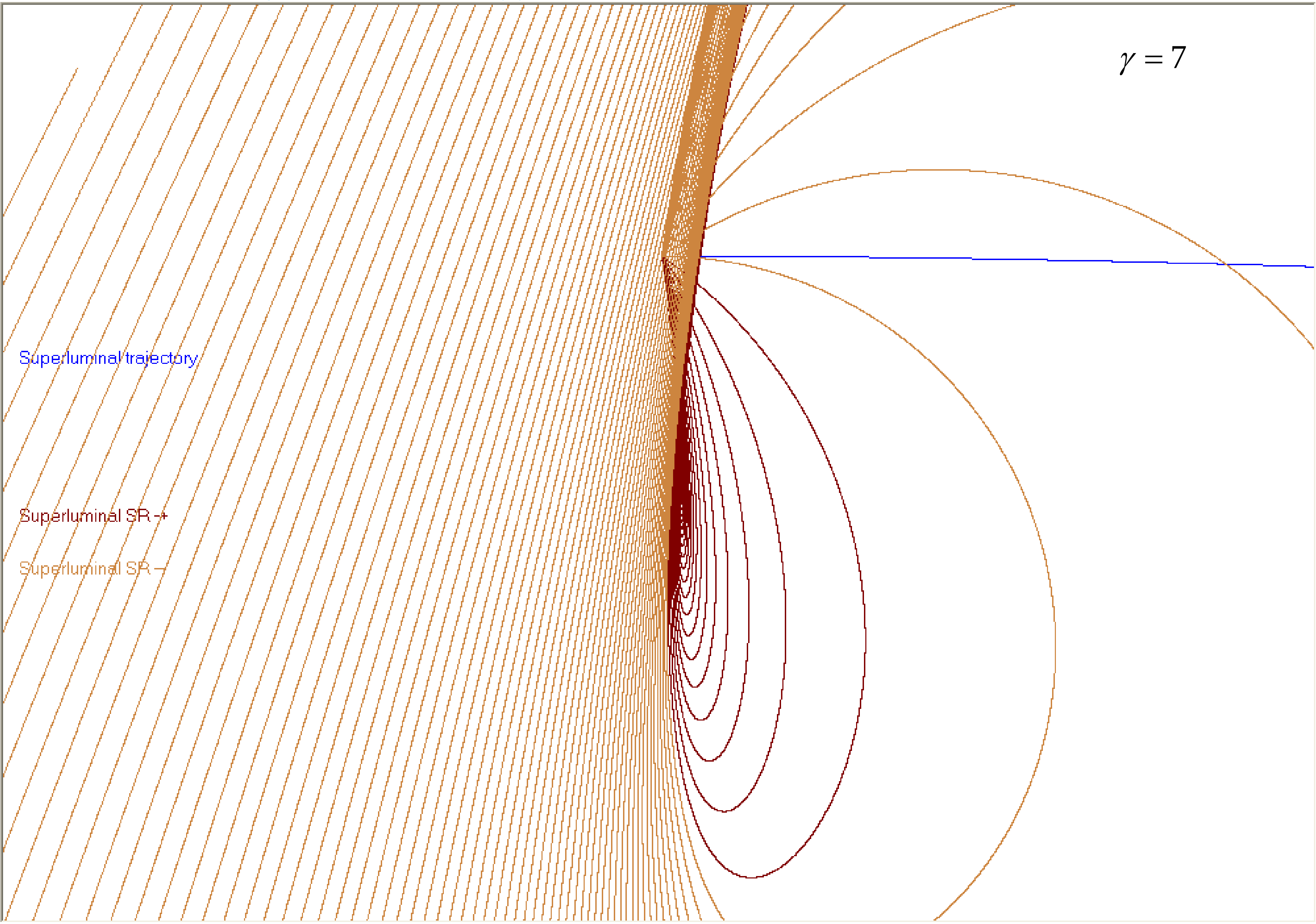

$\gamma = 7$
Superluminal trajectory
Superluminal SR ++

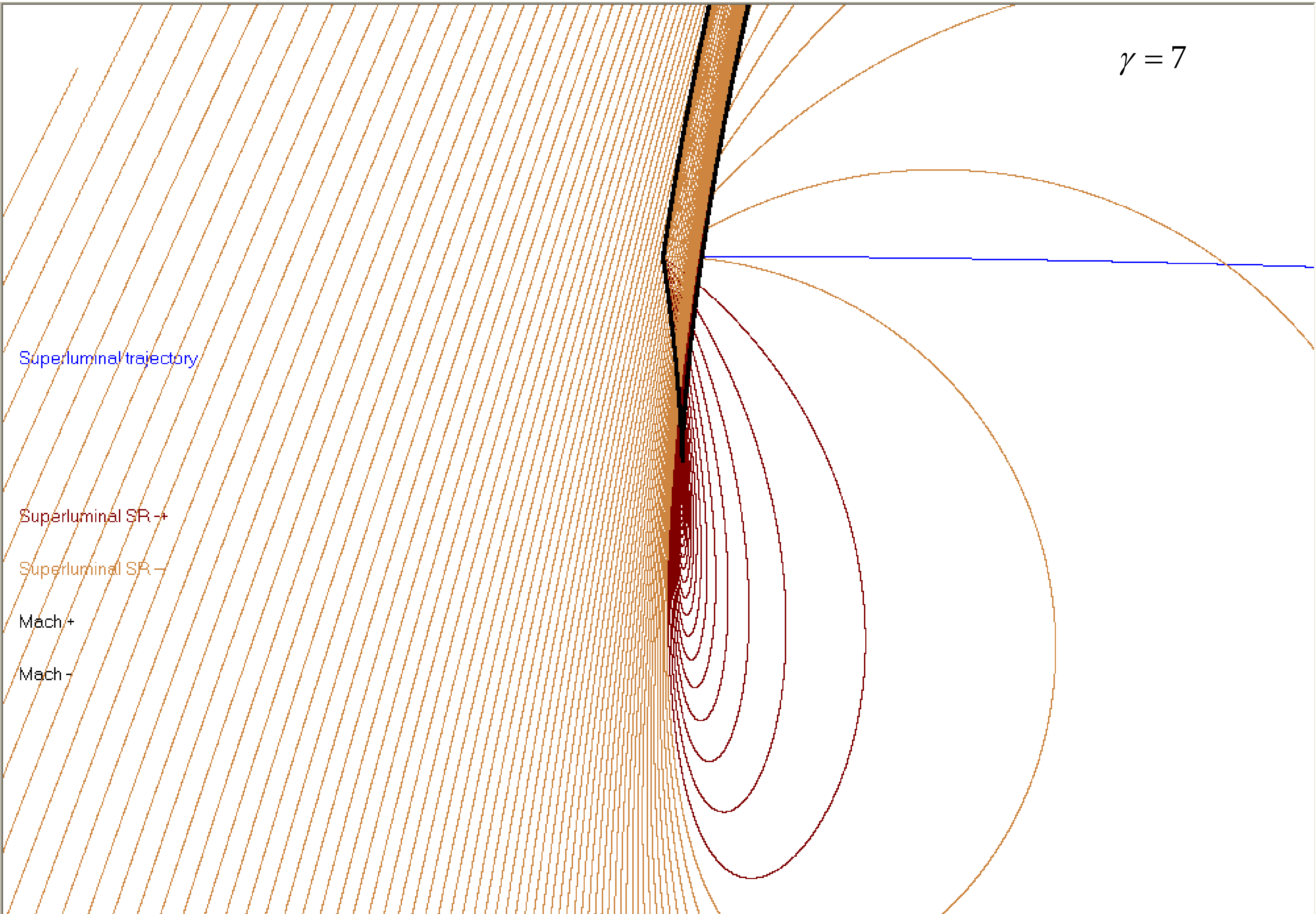

$\gamma = 7$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-


$\gamma = 7$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-
Mach +

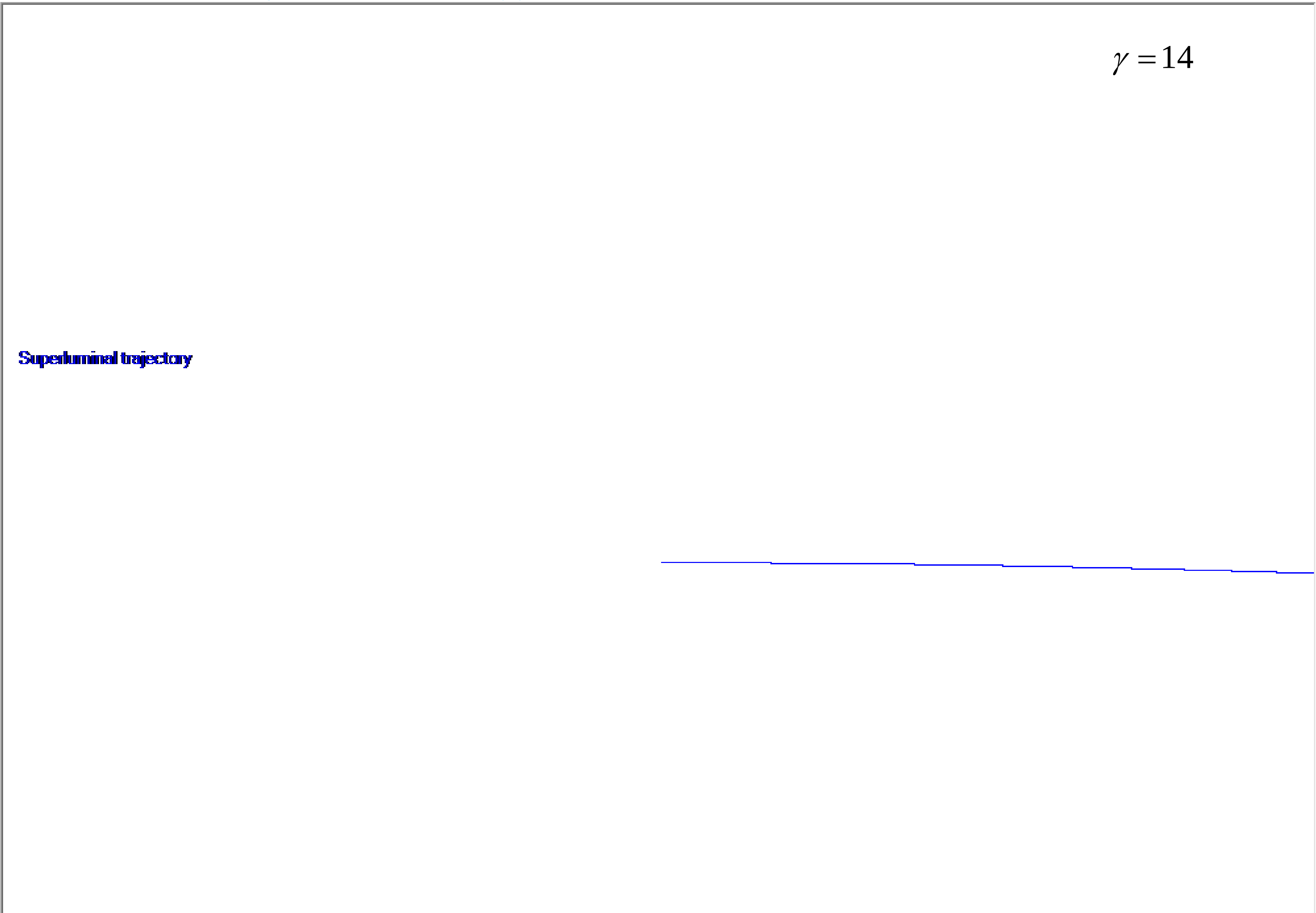

$\gamma = 7$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +-
Mach +
Mach -

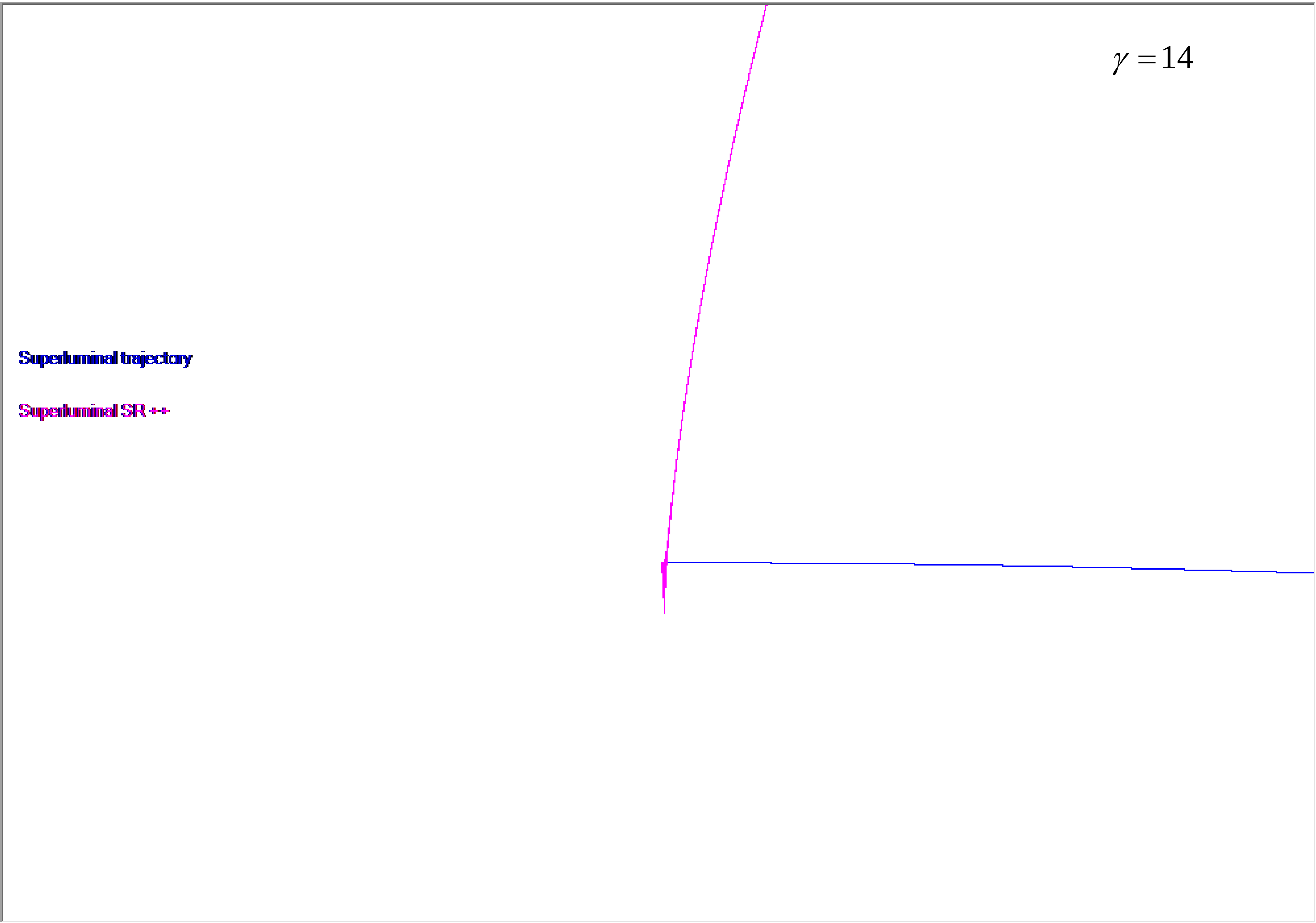

$\gamma = 7$
Superluminal trajectory
Superluminal SR -+

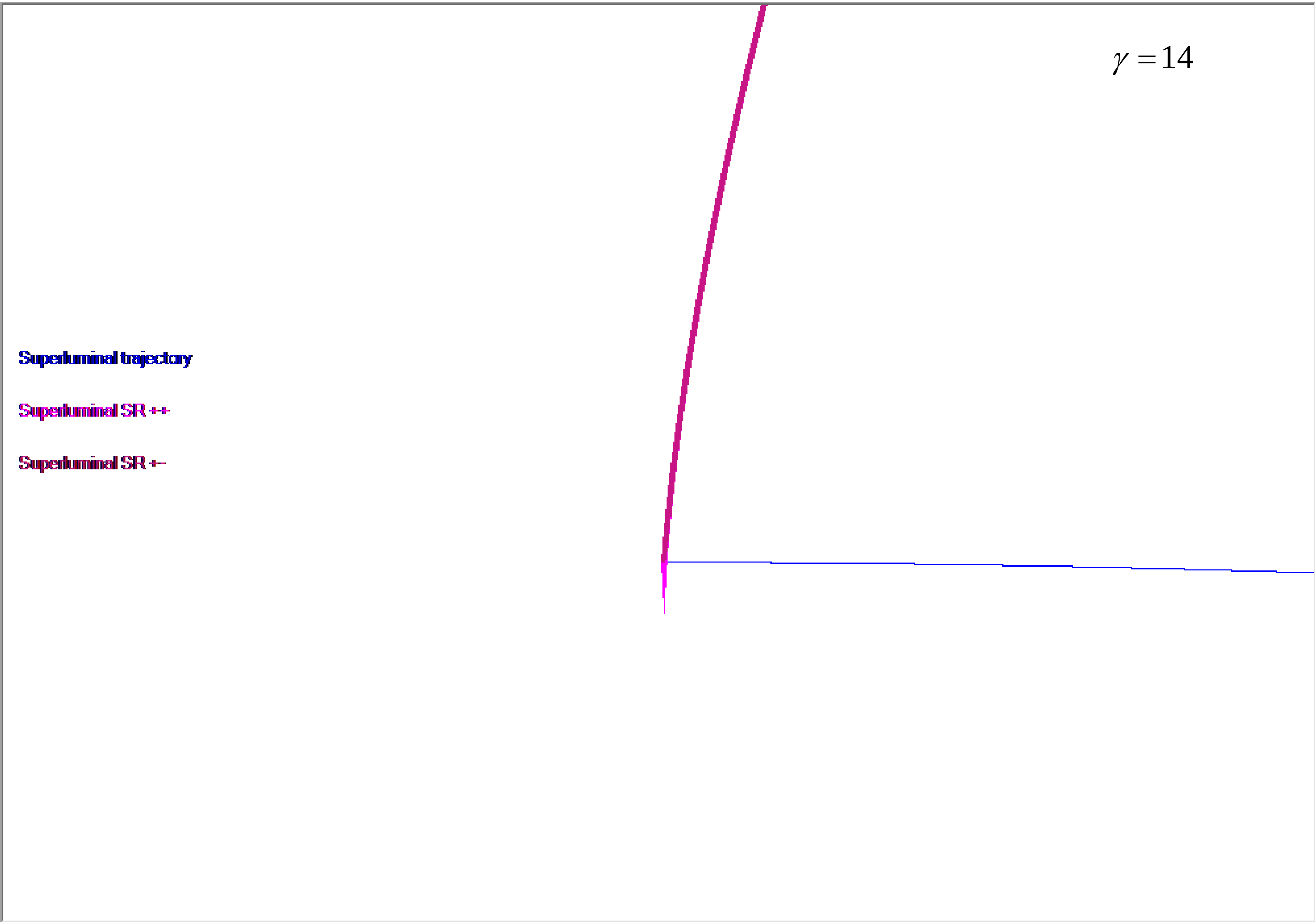

$\gamma = 7$
Superluminal trajectory
Superluminal SR +
Superluminal SR –

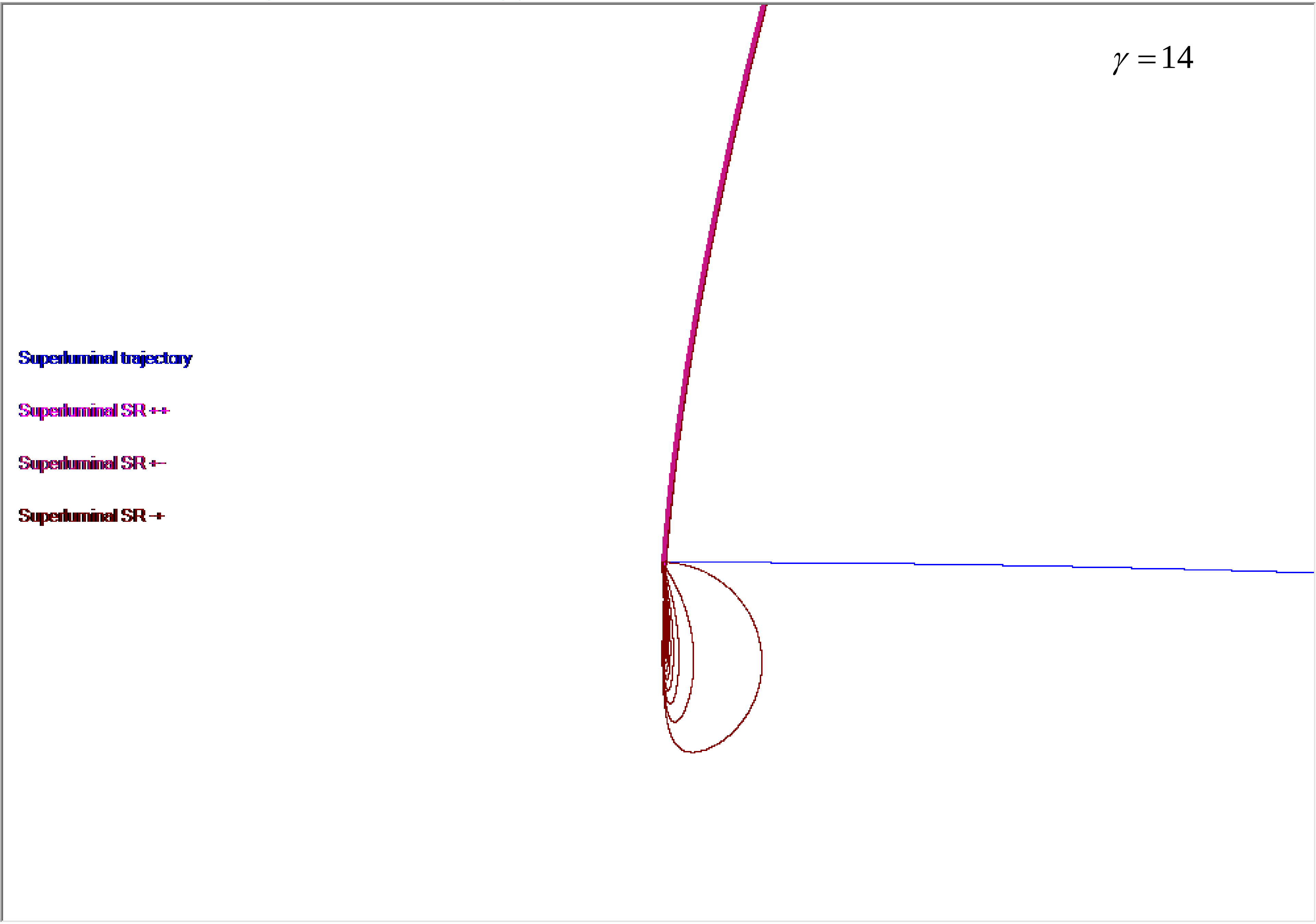

$\gamma = 7$
Superluminal trajectory
Superluminal SR +
Superluminal SR –
Mach +
Mach -

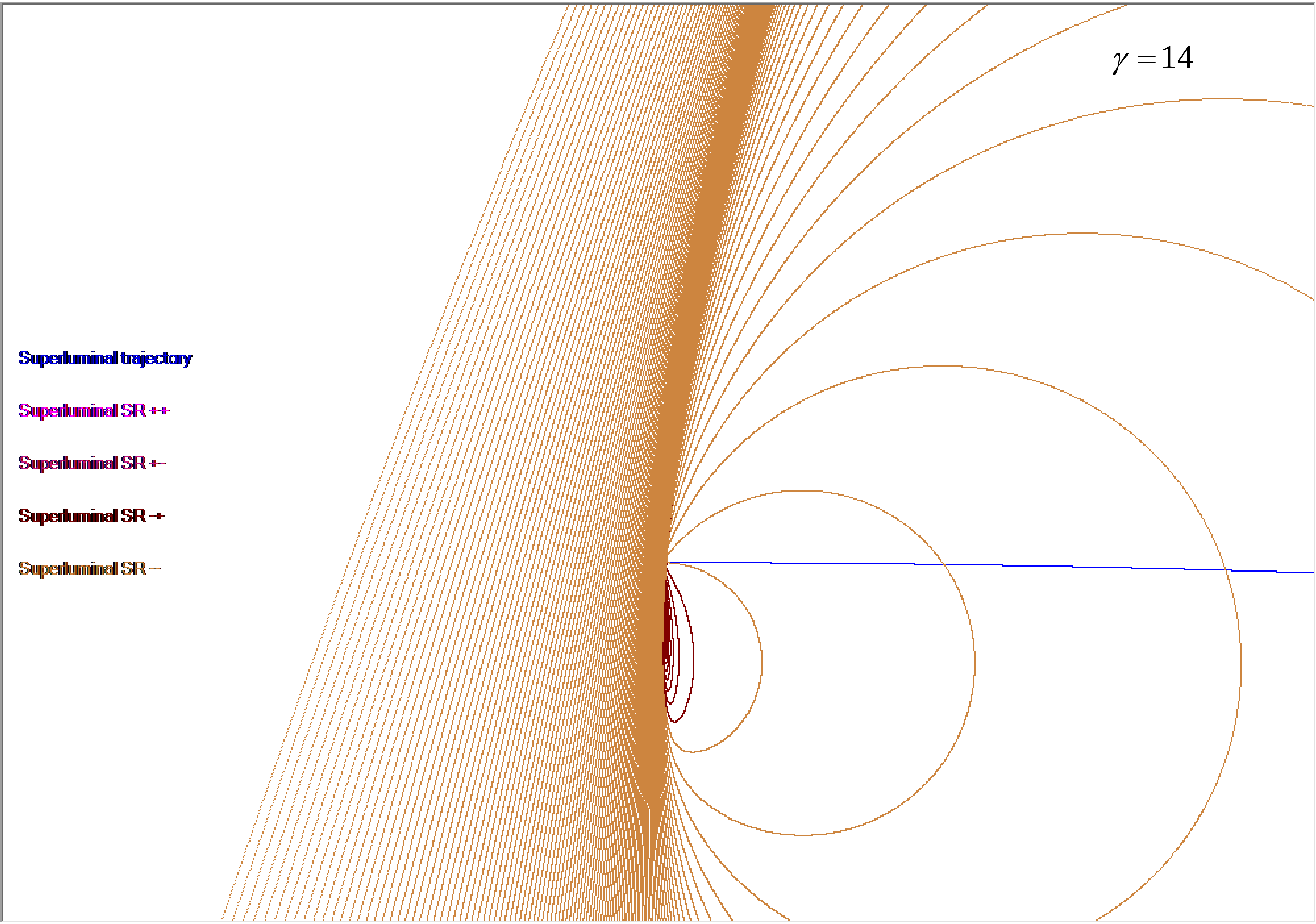
$\gamma = 14$

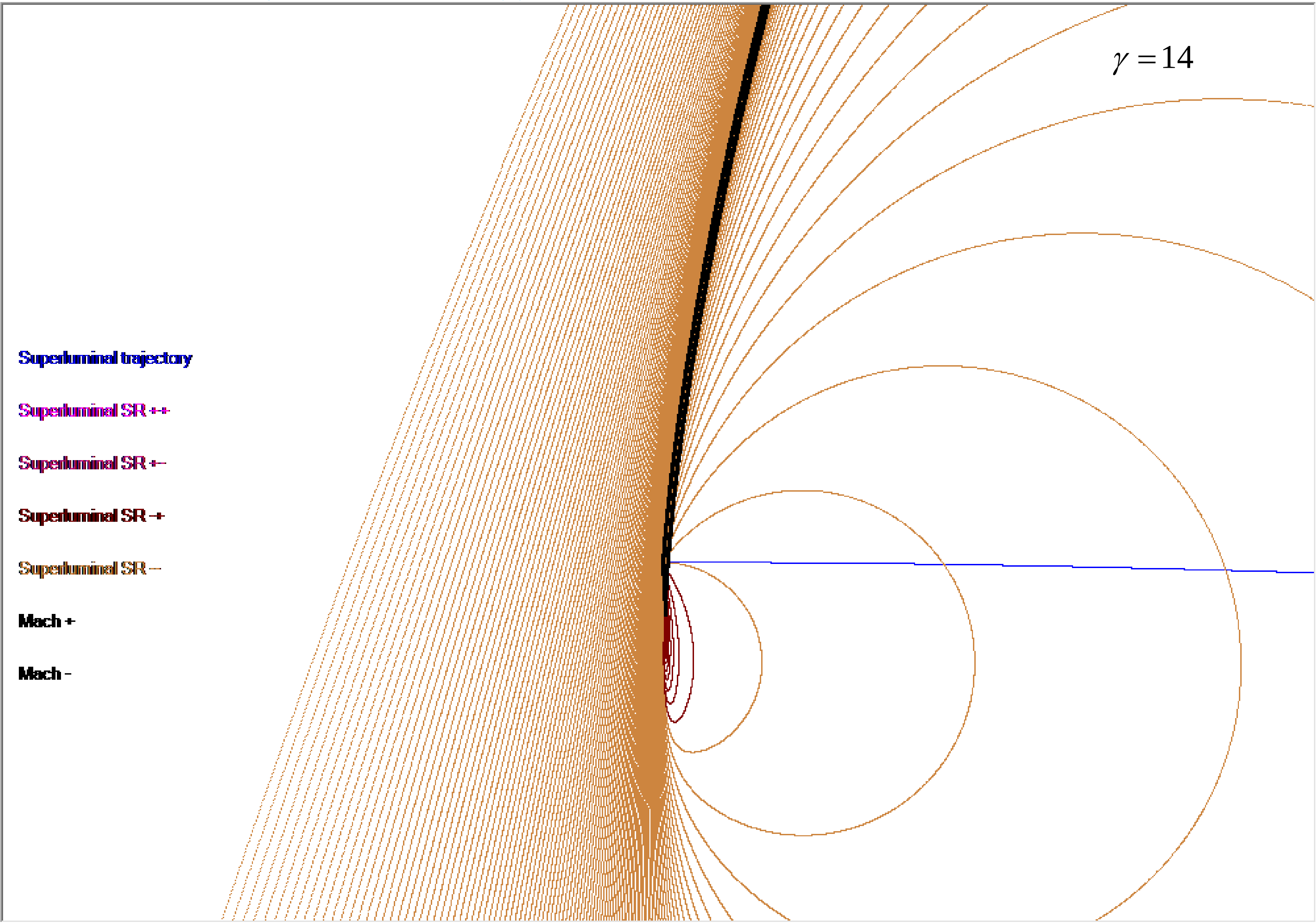



$\gamma = 14$
Superluminal trajectory
Superluminal SR ++


$\gamma = 14$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−


$\gamma = 14$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−
Superluminal SR −+


$\gamma = 14$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−
Superluminal SR −+
Superluminal SR −−


$\gamma = 14$
Superluminal trajectory
Superluminal SR ++
Superluminal SR +−
Superluminal SR −+
Superluminal SR −−
Mach +
Mach -

Conclusion

The main aim of the presentation is to emphasize the fine spatial structure of synchrotron radiation. In presence of vacuum chamber, especially walls placed outer from the beam orbit a lot of faster than lights "spots" contribute to beam self-produced field. The structure of the fields is so fine that for real bunches in accelerators it is thinner than interparticle distances. It means that these effects should be involved at least into dynamic consideration. We hope that new generation of accelerator physicists will continue this very interesting topic.

Thank you for attention