Workshop UBA17

The THz Radiation in Laminated Structures

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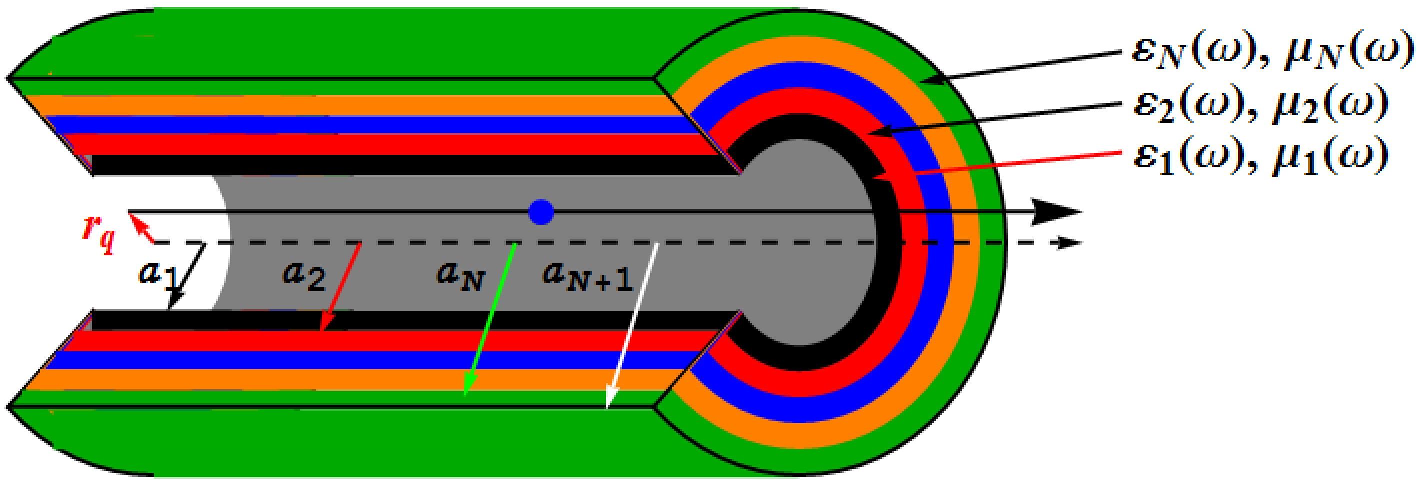
Cylindrical Laminated Structures

The object of research is a cylindrical layered structure: an infinite cylindrical waveguide with a multilayered wall.



A matrix method for constructing the radiation fields of a charged particle flying at an arbitrary constant velocity parallel to the axis of the waveguide is developed.

The radiation fields are constructed using the matrix

$Q^{(m)} = \prod_{i=1}^{N} Q_i^{(m)}$ of dimension 4 × 4:

$$Q^{(m)} = \begin{Bmatrix} Q_{11}^{(m)} & Q_{12}^{(m)} & Q_{13}^{(m)} & Q_{14}^{(m)} \\ Q_{21}^{(m)} & Q_{22}^{(m)} & Q_{23}^{(m)} & Q_{24}^{(m)} \\ Q_{31}^{(m)} & Q_{32}^{(m)} & Q_{33}^{(m)} & Q_{34}^{(m)} \\ Q_{41}^{(m)} & Q_{42}^{(m)} & Q_{43}^{(m)} & Q_{44}^{(m)} \end{Bmatrix}, \qquad Q_i^{(m)} = \begin{Bmatrix} Q_{i,11}^{(m)} & Q_{i,12}^{(m)} & Q_{i,13}^{(m)} & Q_{i,14}^{(m)} \\ 0 & Q_{i,22}^{(m)} & Q_{i,23}^{(m)} & Q_{i,24}^{(m)} \\ Q_{i,31}^{(m)} & Q_{i,32}^{(m)} & Q_{i,33}^{(m)} & Q_{i,34}^{(m)} \\ Q_{i,41}^{(m)} & Q_{i,42}^{(m)} & 0 & Q_{i,44}^{(m)} \end{Bmatrix}$$

consisting of a product of matrices $Q_i^{(m)}$ each of which contains geometric (a_i, a_{i+1}) and electromagnetic (ε_i, μ_i) parameters of the material filling the i-th layer; m = 0,1,2, ... is the order of the multipole.

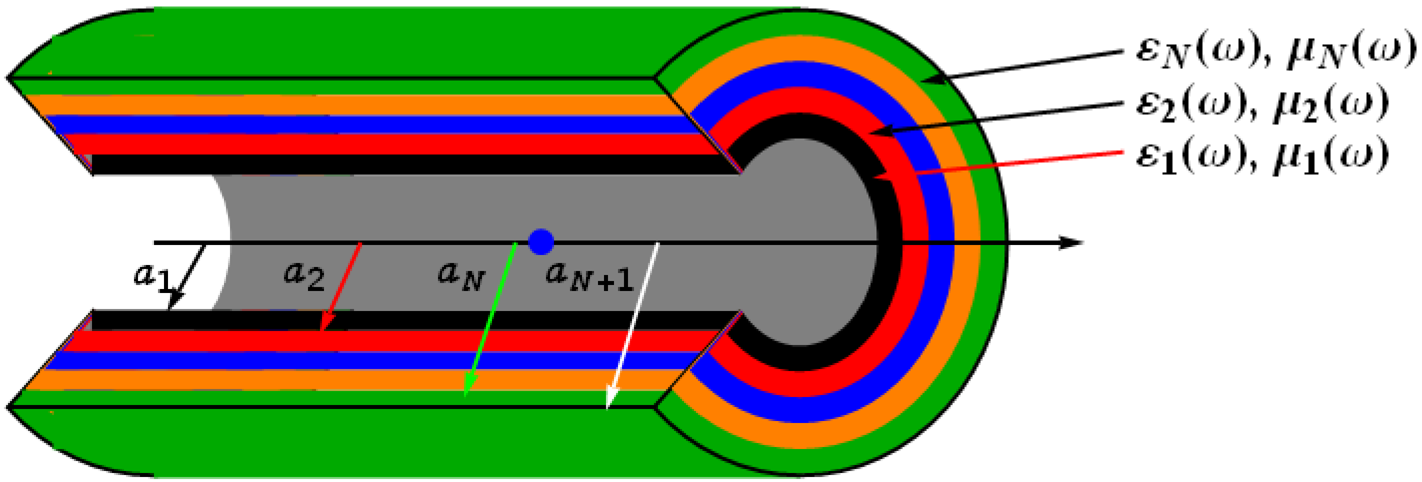
In the general case of motion of a particle with the offset to the axis, the longitudinal impedance of the structure can be written in the form of a multipole expansion

$$Z_{||}(\omega, r_q) = \sum_{m=0}^{\infty} Z_{||}^{(m)}(\omega, r_q)$$

Longitudinal monopole impedance of multilayer tube

For the axial motion of a particle $(r_q = 0)$ only the first term, the monopole term of the expansion (m = 0), contributes to the longitudinal impedance:

$$Z_{||}(\omega, r_q = 0) = Z_{||}^{(0)}(\omega, 0)$$



$$Z_{||}^{(0)}(\omega,0) = \gamma^{-1} I_0(\lambda r) D_2 + \frac{E_z^{(p,m=0)}}{q} \longrightarrow$$

Contribution of the charge field

$$\boldsymbol{E_z^{(p,m)} \to 0 \ at \ \gamma \to \infty}$$

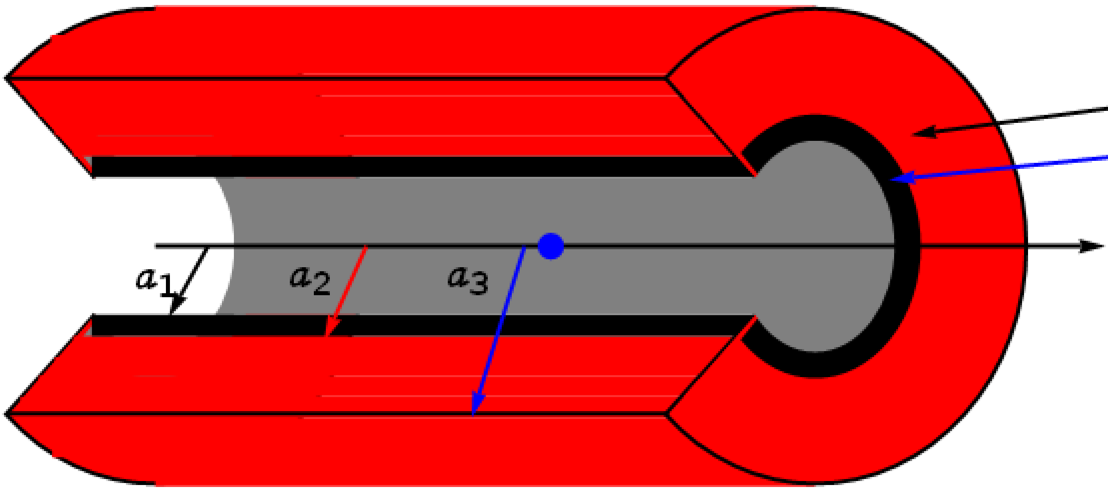
$$D_2 = -\frac{jZ_0}{2\pi a_1} \frac{1}{I_0(\varsigma_1)} \frac{\beta}{I_0'(\varsigma_1)\beta^2 - \gamma^{-1} I_0(\varsigma_1)\alpha} \qquad Q^{(m)} = \prod_{i=1}^{N} Q_i^{(m)}$$

$$\alpha = \frac{Q_{33}\beta^2 K_0'(\varsigma_2) + Q_{32}\gamma^{-1} K_0(\varsigma_2)}{Q_{23}\beta^2 K_0'(\varsigma_2) + Q_{22}\gamma^{-1} K_0(\varsigma_2)} \qquad \varsigma_1 = \lambda a_1 \qquad \varsigma_2 = \lambda a_{N+1} \qquad \lambda = k/\gamma$$

$$Q^{(0)} = \begin{Bmatrix} Q_{11}^{(0)} & 0 & 0 & Q_{14}^{(0)} \\ 0 & Q_{22}^{(0)} & Q_{23}^{(0)} & 0 \\ 0 & Q_{32}^{(0)} & Q_{33}^{(0)} & 0 \\ Q_{41}^{(0)} & 0 & 0 & Q_{44}^{(0)} \end{Bmatrix} \qquad Q_i^{(0)} = \begin{Bmatrix} Q_{i,11}^{(0)} & 0 & 0 & Q_{i,14}^{(0)} \\ 0 & Q_{i,22}^{(0)} & Q_{i,23}^{(0)} & 0 \\ 0 & Q_{i,32}^{(0)} & Q_{i,33}^{(0)} & 0 \\ Q_{i,41}^{(0)} & 0 & 0 & Q_{i,44}^{(0)} \end{Bmatrix}$$

TM field

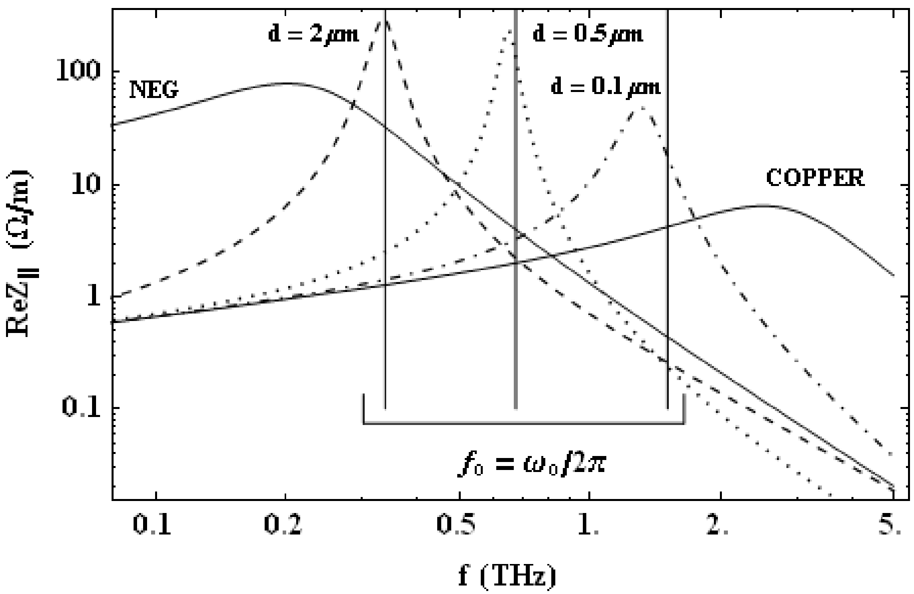
TWO-LAYER ROUND TUBE



$$\boldsymbol{\mu_{1,2} = \mu_0}$$

$$\boldsymbol{\varepsilon_{1,2} = \varepsilon_0 + j\frac{\sigma_{1,2}}{\omega}}$$

$$\boldsymbol{\sigma_2 \gg \sigma_1}$$



Real part of copper-NEG tube impedances for various cover thicknesses.

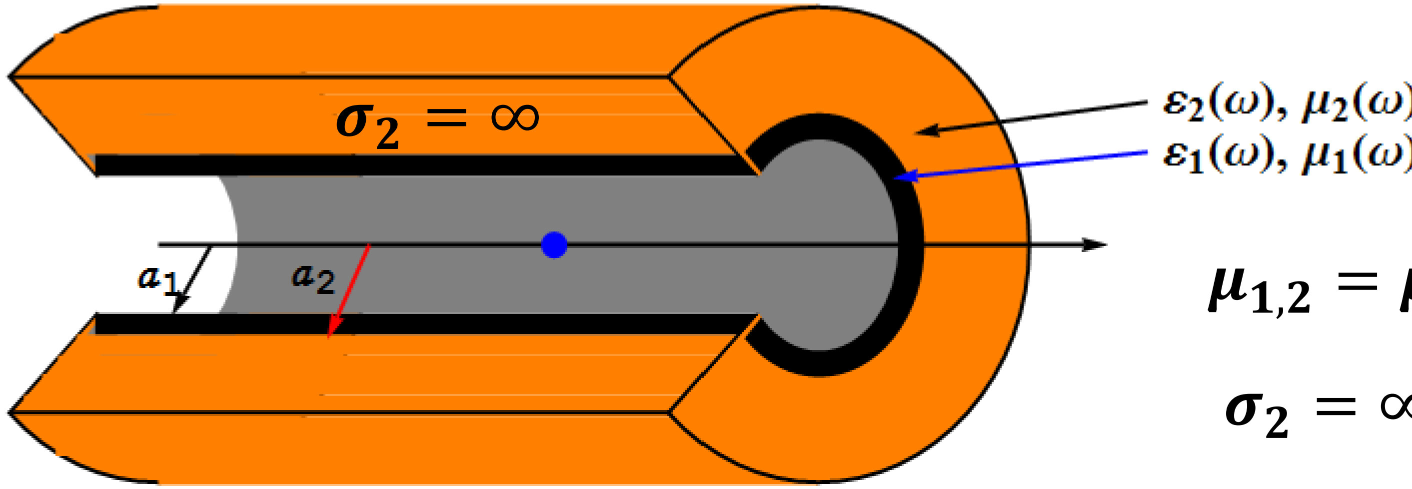
$$\sigma_1 = 3 \times 10^4 \Omega^{-1} m^{-1}$$

$$\sigma_2 = 58 \times 10^6 \Omega^{-1} m^{-1}$$

$$a_1 = 2cm$$

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{2}{a_1 d}}$$

TWO-LAYER ROUND TUBE
perfecty conducting outer layer



$$\boldsymbol{\mu_{1,2} = \mu_0}$$

$$\boldsymbol{\sigma_2 = \infty}$$

$$Z_{\parallel}^{0}(\omega) = \frac{R}{1 + jQ\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)}$$

$$R = Z_0 c / \pi a_1^2 A$$

$$Q = \omega_0 / A$$

$$A = \frac{2c}{\sqrt{3}a}\left(\varsigma + \varsigma^{-1}\right)$$

$$\varsigma = d\sigma_1 Z_0 / \sqrt{3}$$

$$\boldsymbol{\omega_0 = c\sqrt{\frac{2}{a_1 d}}}$$

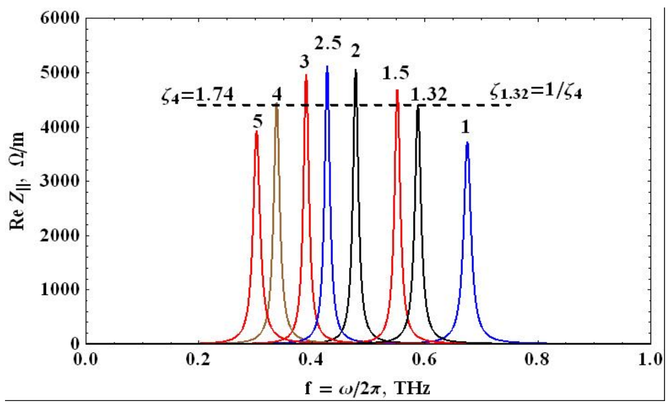
$$\frac{d}{a_1} \ll 1$$

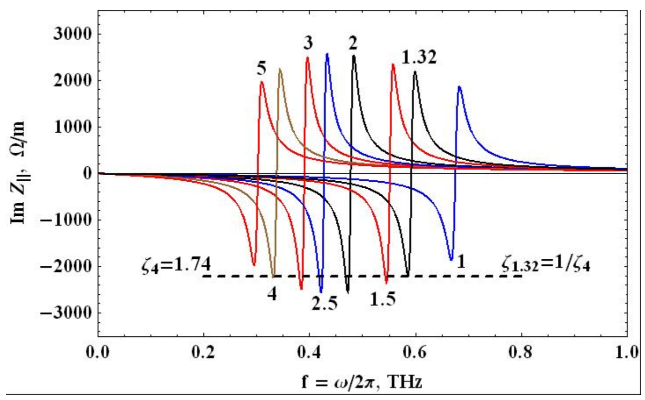
$$\boxed{\zeta \sim 1}$$

Maximal Q-factor

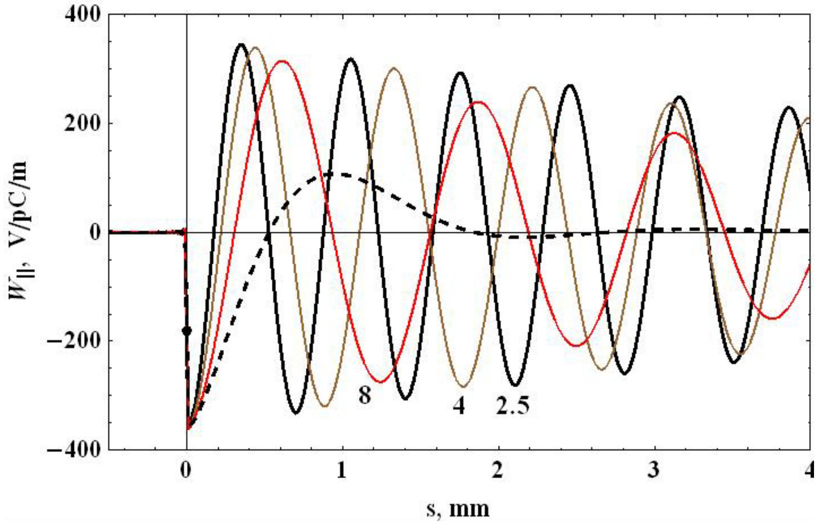
$\boxed{\text{at } \zeta = 1}$

TWO-LAYER ROUND TUBE
perfecty conducting outer layer





Real (left) and imaginary (right) parts of the longitudinal impedance of two-layer cylindrical structure. On the graphs, the thicknesses of the inner layer in microns are shown.

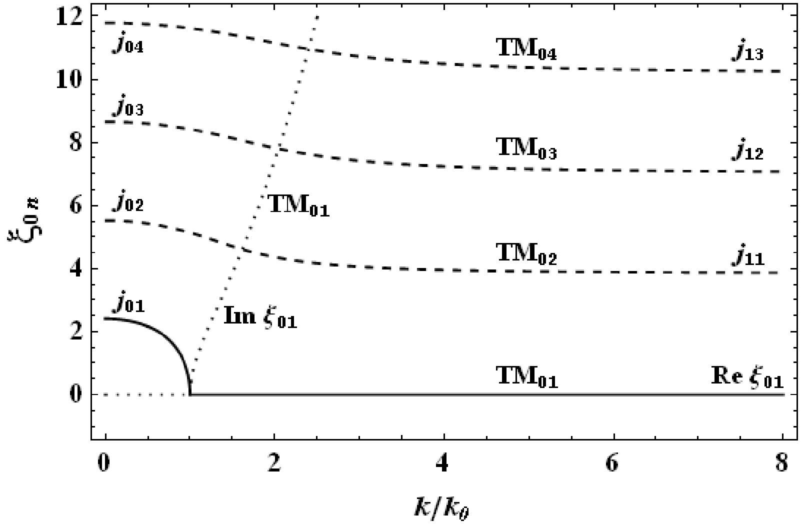


Longitudinal wake potential of the two-layer cylindrical waveguide for different values of the thickness of the inner layer and the fixed conductivity $\sigma = 2 \times 10^3\ \Omega^{-1} m^{-1}$

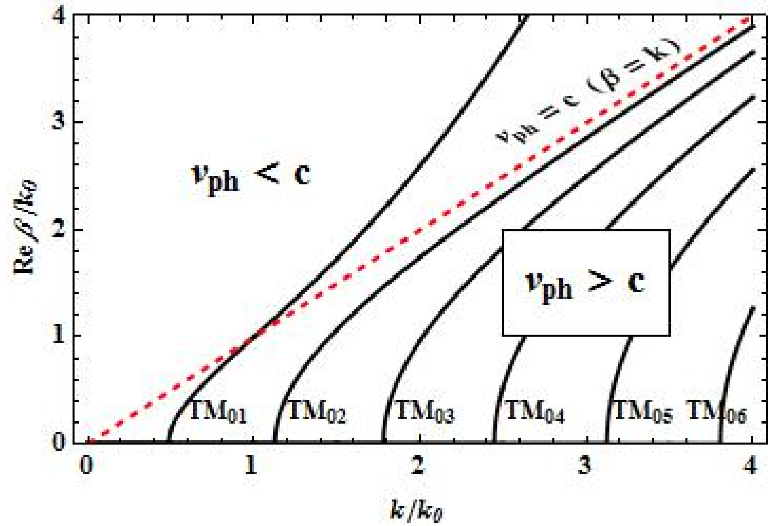
$$a_1 = 1\ cm$$

Dispersion relation. Synchronous mode

$$\frac{1}{\xi_{0n}}\frac{J_1(\xi_{0n})}{J_0(\xi_{0n})}=\frac{1}{k^2ad}$$

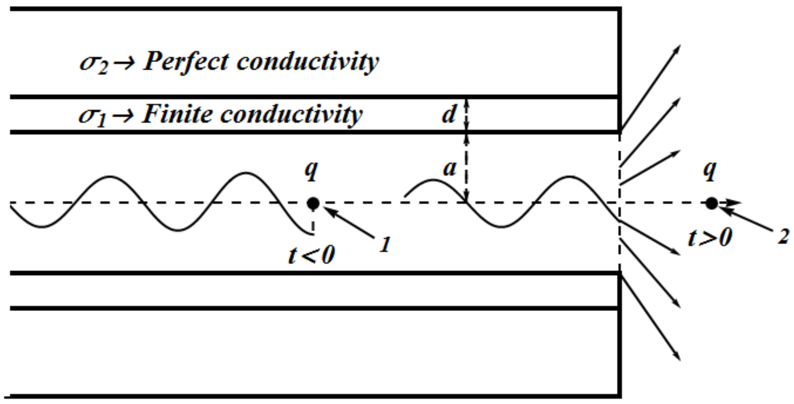


Real (solid) and imaginary (dotted) parts of TM_{01} transverse eigenvalues versus frequency. Shown also the purely real transverse eigenvalues for the high axially symmetric TM modes (dashed).

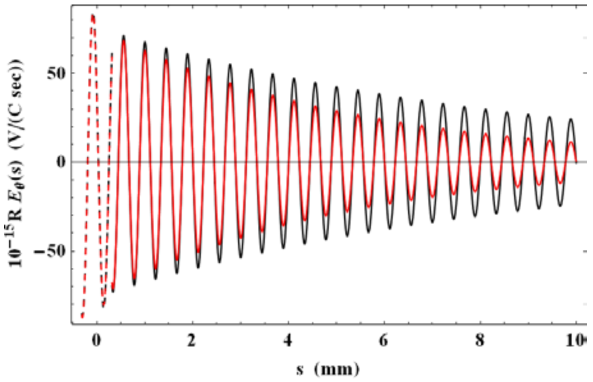


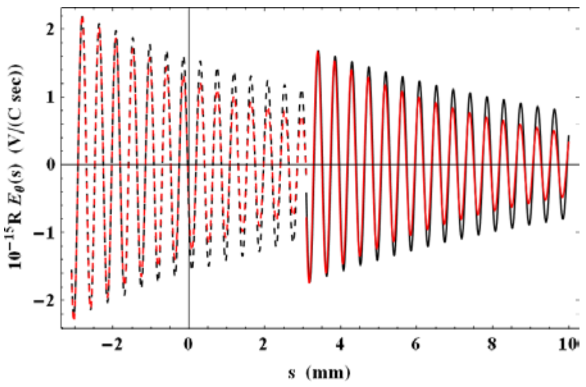
Dispersion curves for fundamental TM_{01} and high order TM_{0n} (n>1) modes.

Wake-Field Radiation from the Open End of the Internally Coated Metallic Tube



Geometry of the problem: particle in waveguide 1; and out of waveguide 2;

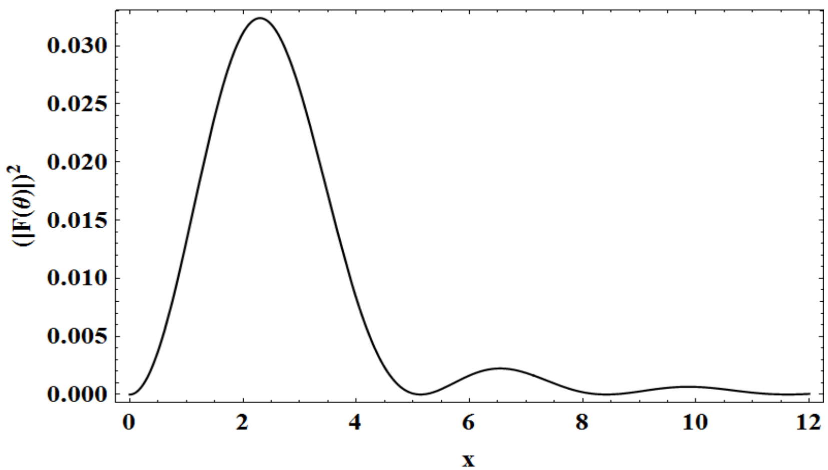




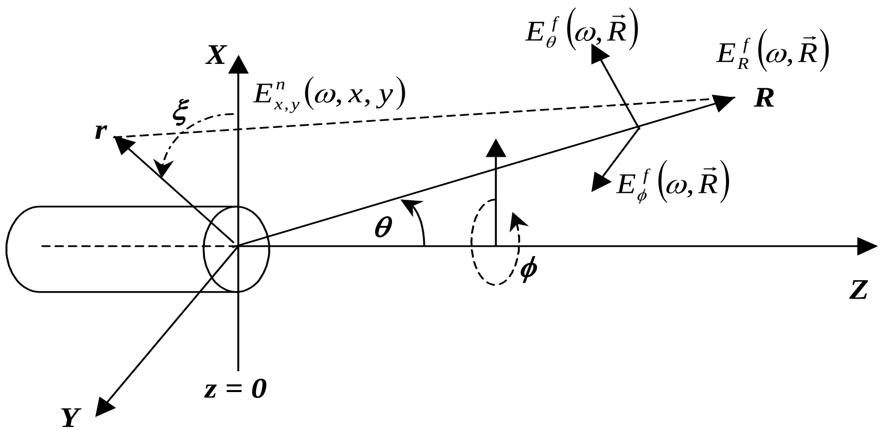
Spatial-temporal distribution of the radiation field at fixed angles of observation; $\vartheta = \frac{\pi}{100}$ (left), $\vartheta = \frac{\pi}{10}$ (right); first pulse (dotted), second pulse (solid); $\zeta = 1$ (black), $\zeta = 3$ or $\zeta = 1/3$ (red).

Wake-Field Radiation from the Open End of the Internally Coated Metallic Tube.
Angular distribution

$$F(x) = \frac{J_2(x)}{x}, \quad x = \frac{\omega_0}{c} a \sin\theta, \quad x = \sqrt{\frac{2a}{d}} \sin\theta \qquad x_{max} = 2.3$$

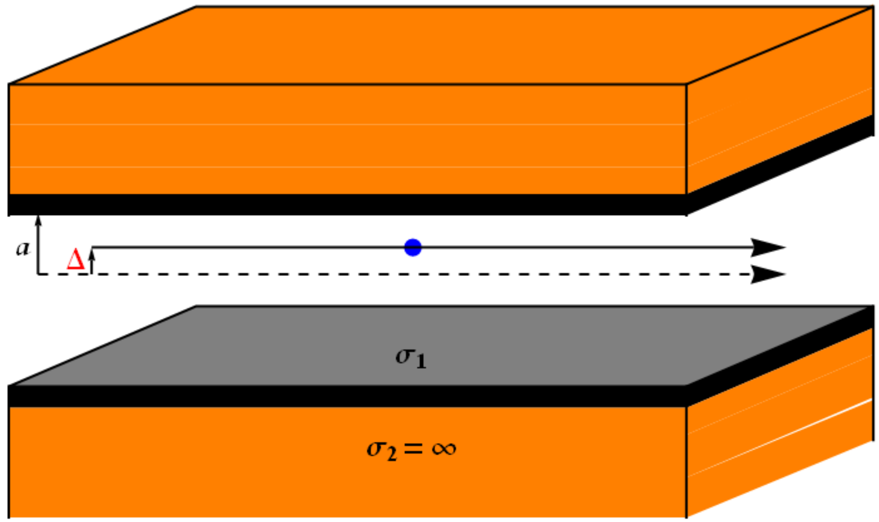


The radiation pattern in power (at $ct > R + a_1$ R- the distance from the open end to the observation point).



Near field – Far field transformation

Impedance and wake of two-layer metallic flat structure

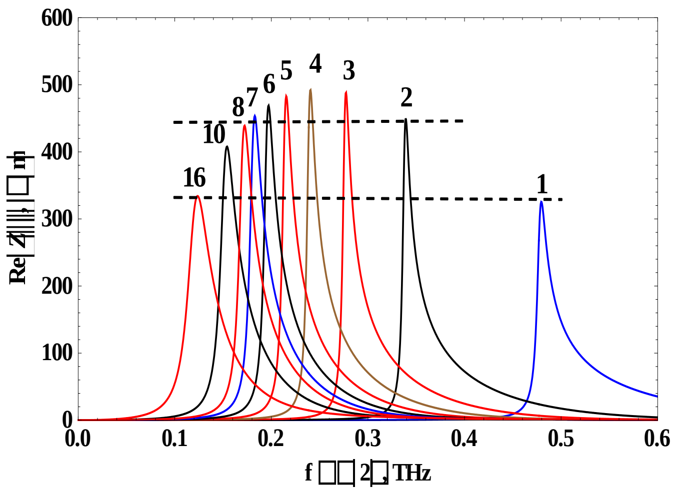


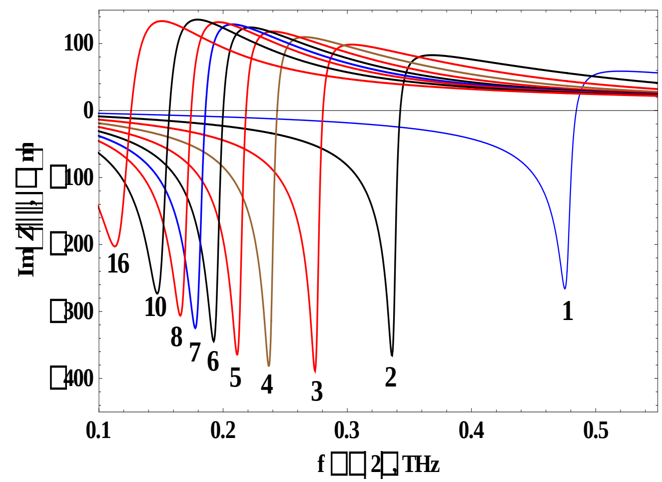
$$Z_{||}^{(0)}(k) = j\,\frac{qk}{8\pi^2\varepsilon_0 c}\int_{-\infty}^{\infty} k_x(k_x^2 - K^2)\,\frac{Sh(2\,Kd)}{S_{1c}\,S_{2\,c}}\,dk_x\,, \qquad \mathbf{\Delta = 0}$$

$$\left.\begin{matrix} S_{1c} \\ S_{2c} \end{matrix}\right\} = \left\{\begin{matrix} K \\ k_x K_1^2 \end{matrix}\right\} Ch(Kd)Ch(k_x a) \pm \left\{\begin{matrix} k_x \\ K k^2 \end{matrix}\right\} Sh(Kd)Sh(k_x a)$$

$$K_1^2 = K^2 - k_x^2 - k^2 \qquad K = \sqrt{k_x^2 + (1 - \varepsilon\mu c^2)k^2}$$

Impedance of two-layer metallic flat structure





Longitudinal impedance of the two-layer flat structure for different values of the thickness of the inner layer and the fixed conductivity $\sigma = 2\times\ 10^3\ \Omega^{-1}m^{-1}$; half-height of structure $a = 1\ cm$.

cylinder

$$\omega_{rez} = c\sqrt{2/r_0 d}$$

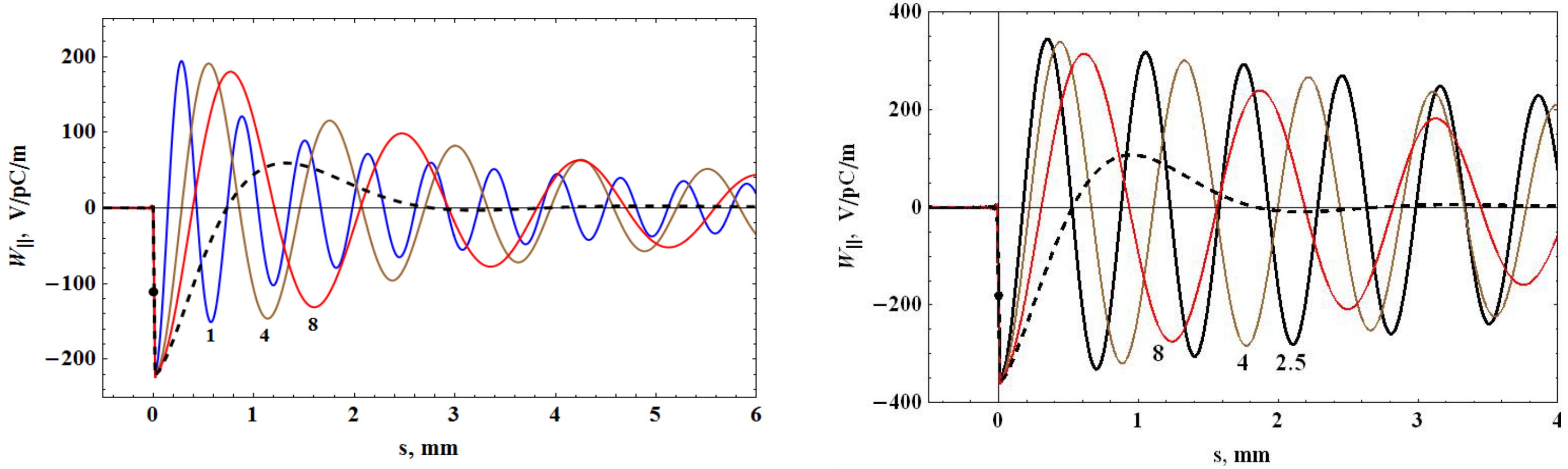
$$\zeta = Z_0\sigma_1 d\ /\sqrt{3} = 1$$

plane

$$\omega_{rez} = c\sqrt{1/ad}$$

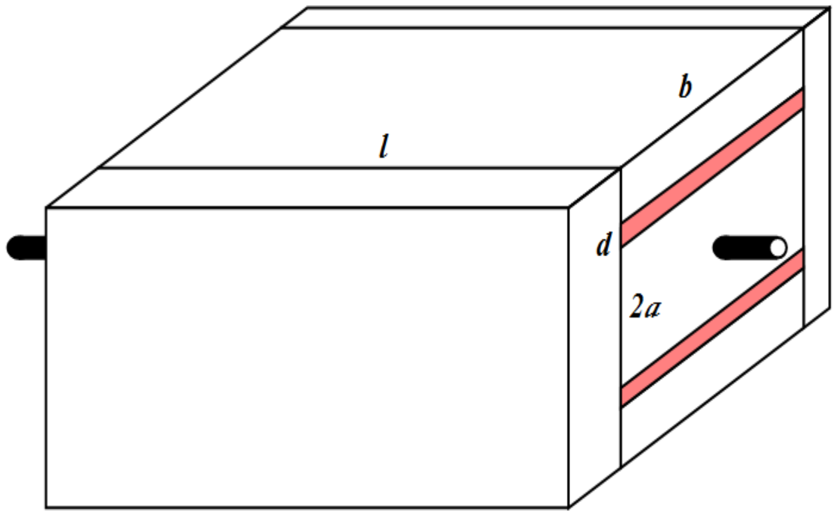
$$\zeta = Z_0\sigma_1 d\ /3 = 1$$

Wake potential of two-layer metallic flat structure



Longitudinal wake potential of the two-layer flat (left) and cylindrical (right) structures for different values of the thickness of the inner layer and the fixed conductivity $\sigma = 2 \times 10^3\ \Omega^{-1} m^{-1}$; half-height (radius) of structure $1\ cm$.

Experimental testing



Tested resonator; $b = 65$**mm;** $l = 200\ mm$**;** $\boldsymbol{a = 10, 15, 20\ mm}$

$$\boldsymbol{f_{rez} = \frac{c}{2\pi}\sqrt{\frac{1}{ad}} < \ 15\ \mathrm{GHz} \ \ \rightarrow \ \ d\ \sim 1\ mm}$$

$$\boldsymbol{\zeta = \frac{1}{3} Z_0 \sigma_1 d \sim 1 \rightarrow \ \sigma_1 \sim 2\ \Omega^{-1} m^{-1}}$$

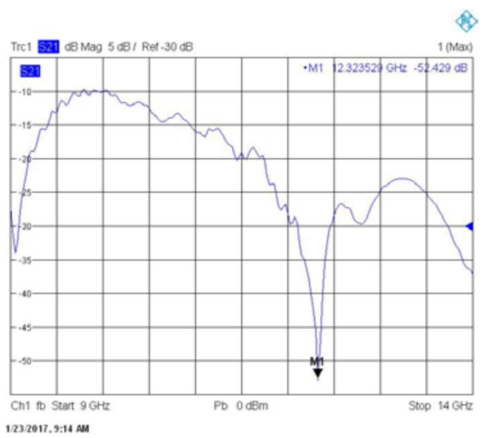
Experimental testing

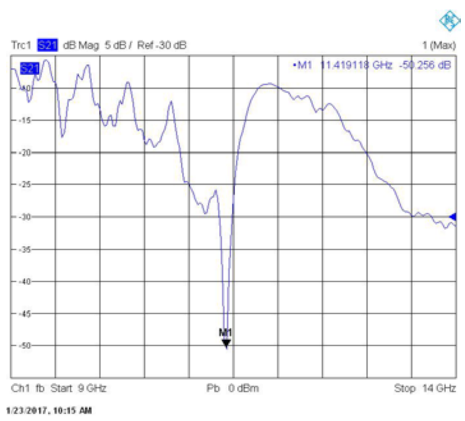
Choosing parameters

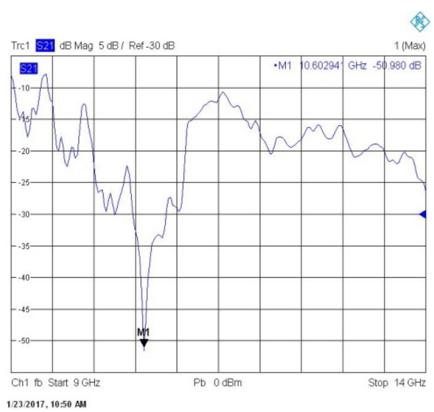
Thickness of inner layer $d = 1\ mm$, material of inner layer – Germanium (Ge)

$$\varepsilon_{Ge} = 16\varepsilon_0 + j\,2/\omega\ [\Omega^{-1}m^{-1}]$$

Measurement results



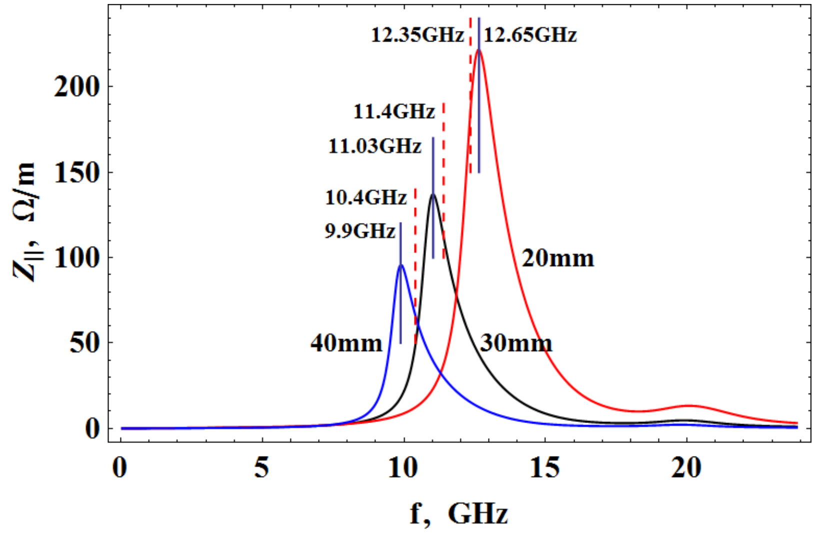




Insertion loss S21 for copper cavity with inner Ge plates for the cavities with half-apertures 1cm, 1.5cm 2cm. The Ge layer thickness is 1 mm.

Experimental testing

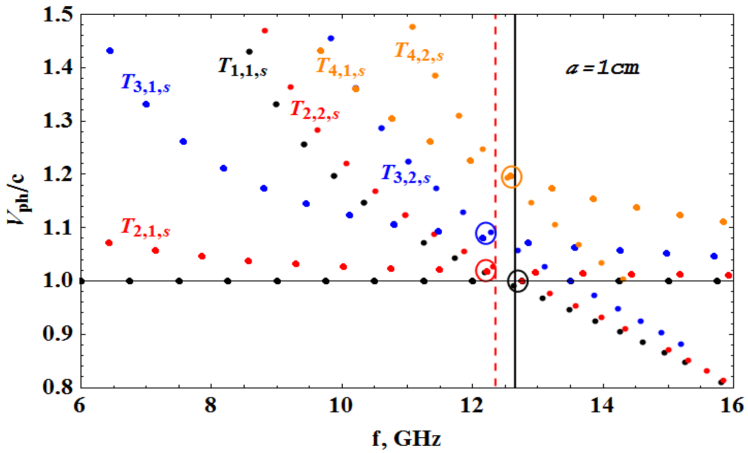
Comparison with calculated resonance frequencies

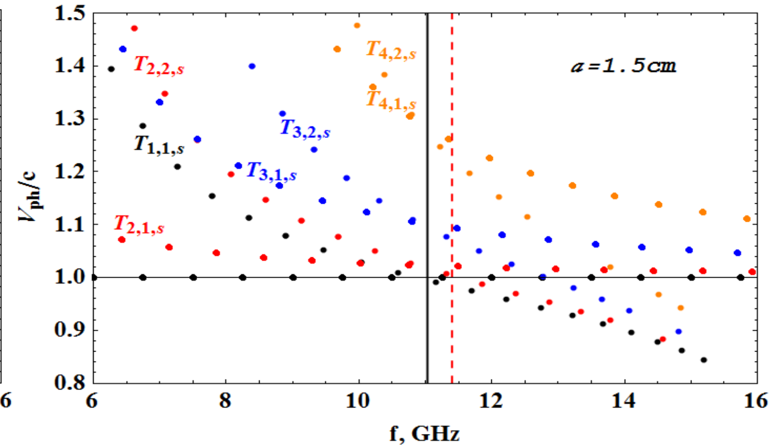


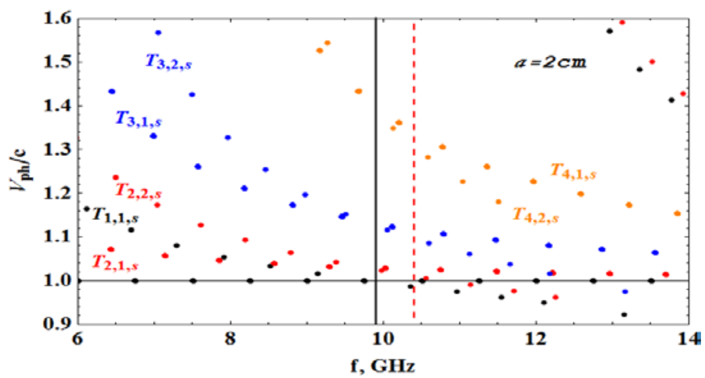
The longitudinal impedance of perfect conductor-Ge infinite parallel plates (solid lines) and measured resonant frequencies of copper cavity with Ge inner layers at the top and bottom walls (dashed).

Processing of measurement data

Dispersion curves for tested resonator

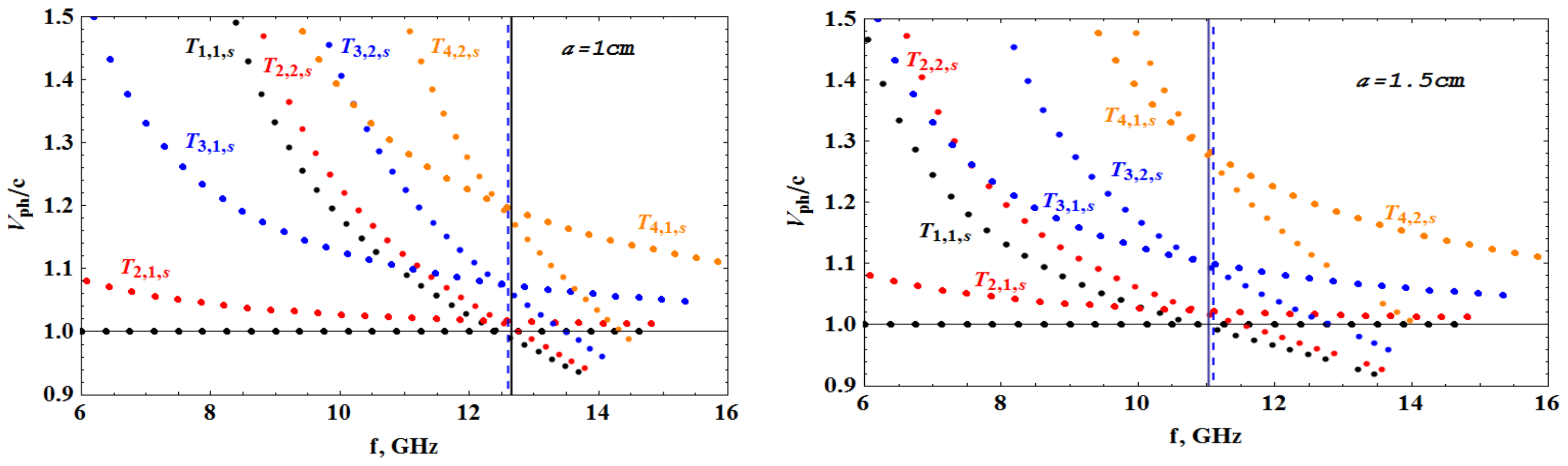






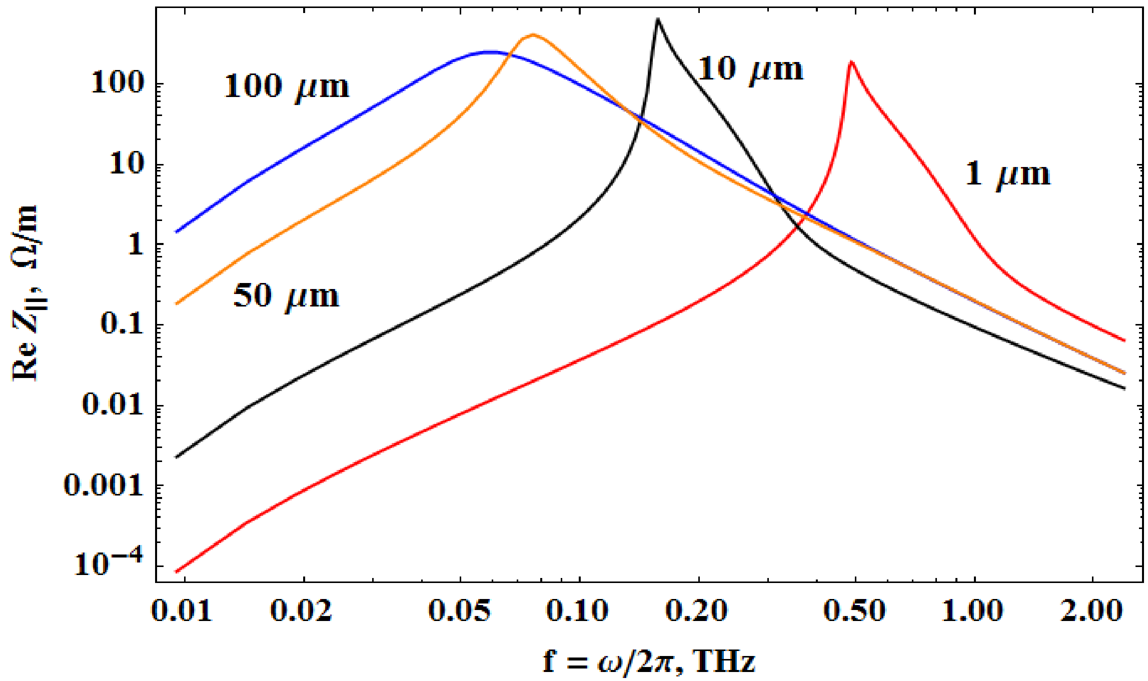
Dispersion curves for resonators with different heights; $l = 20cm$
Dashed, red – measured
Solid black - calculated

Dispersion curves for resonator, suggested for testing



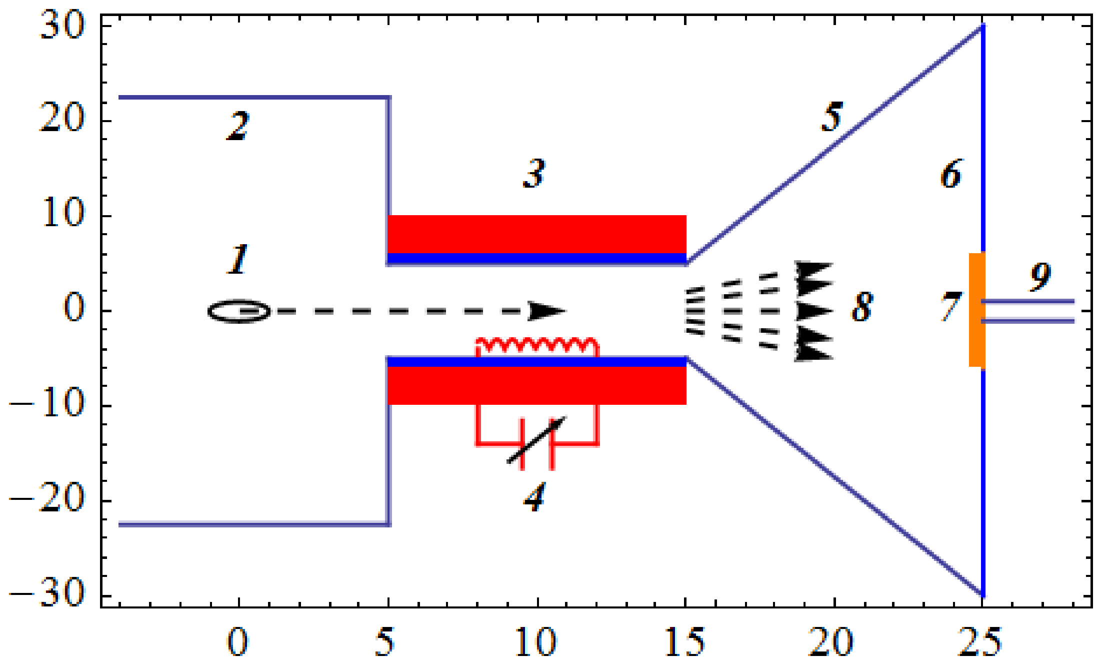
Dispersion curves for resonators with two different heights; $l = 40cm$
Solid black – calculated
Dotted blue – predicted measured

Planning experiment with layered by cadmium sulfide rectangular resonator



Longitudinal impedance of two-layer plates (cadmium sulfide – ideal conductor) at different thicknesses of the layer; $a = 1cm, \sigma = 600\Omega^{-1}m^{-1}$.

Planning experiment of fixation of wakefield radiation from the open end of waveguide



Scheme of the experiment; 1 – beam, 2-beam pipe, 3 –waveguide, 4 - frequency measuring device, 5 – horn, 6 –screen, 7 - device for measuring radiation intensity distribution, 8 – radiation, 9 - beam pipe