An Adiabatic Phase Matching Accelerator

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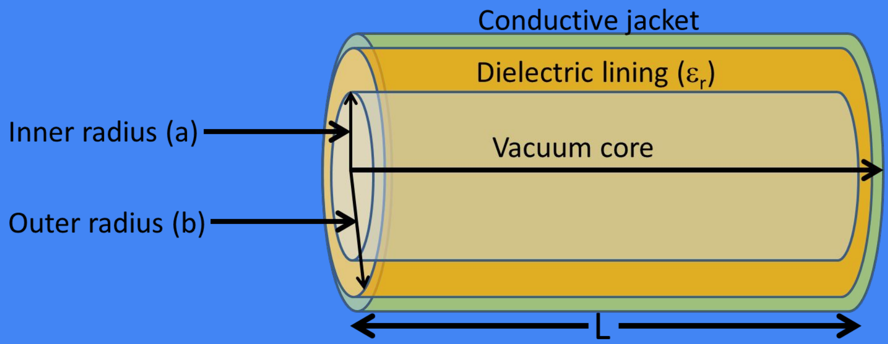
Ultrafast Beams and Applications

Overview & Motivation

- Conventional RF technologies operate with large wavelengths and moderately high fields e.g. S-band with 100 MV/m, 10 cm.
- Scaling to THz e.g. ~1 mm wavelengths requires scaling of the field also, the normalized vector potential $\propto E\,\lambda$.
- Phase slippage is more relevant at smaller wavelengths
- Achieving large fields is difficult from a laser-based approach in the sub-THz regime.
- Can we design a phase-matching accelerator?

Dielectric-lined Waveguides (DLW)



- Around since the 60s - telecom applications & research.
- Voss and Wieland (1982) - beam driven acceleration motivated research into their usage (along with plasmas)
- First experiments at SLAC in 90s - GV/m gradients.
- Strong dipole (deflection) mode - HE11; motivated a lot of research into beam breakup (BBU).

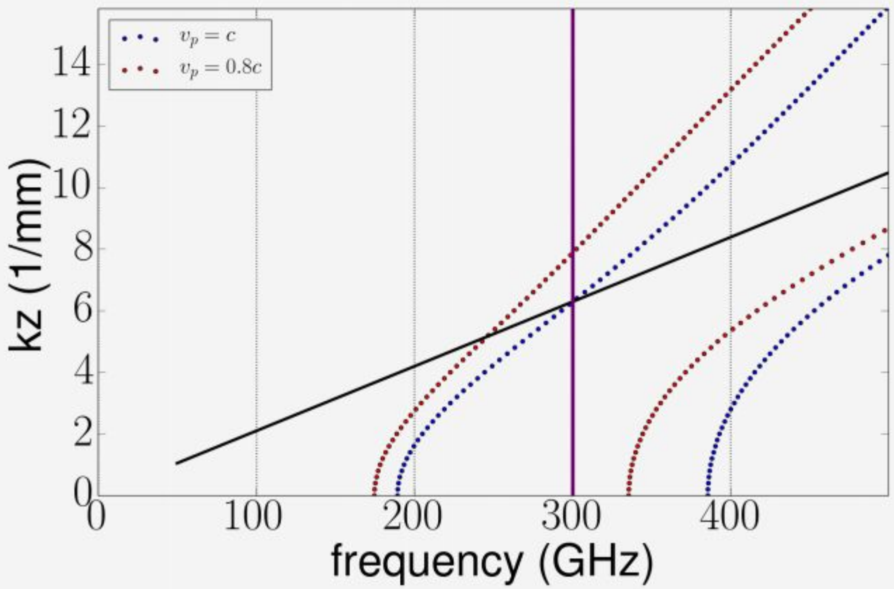
$$E_z = \begin{cases} B_1 J_0(k_1 r) e^{i(\omega t - k_z z)} & 0 \le r < a \\ B_2 F_{00}(k_1 r) e^{i(\omega t - k_z z)} & a \le r \le b \end{cases}$$

$$E_r = \begin{cases} \frac{-ik_z}{k_1} B_1 J_0'(k_1 r) e^{i(\omega t - k_z z)} & 0 \le r < a \\ \frac{-ik_z}{k_2} B_2 F_{00}'(k_2 r) e^{i(\omega t - k_z z)} & a \le r \le b \end{cases}$$

$$H_\phi = \begin{cases} \frac{-i\omega\epsilon_0}{k_1} B_1 J_0'(k_1 r) e^{i(\omega t - k_z z)} & 0 \le r < a \\ \frac{-i\omega\epsilon_r\epsilon_0}{k_2} B_2 F_{00}'(k_2 r) e^{i(\omega t - k_z z)} & a \le r \le b \end{cases}.$$

Phase velocities in DLWs

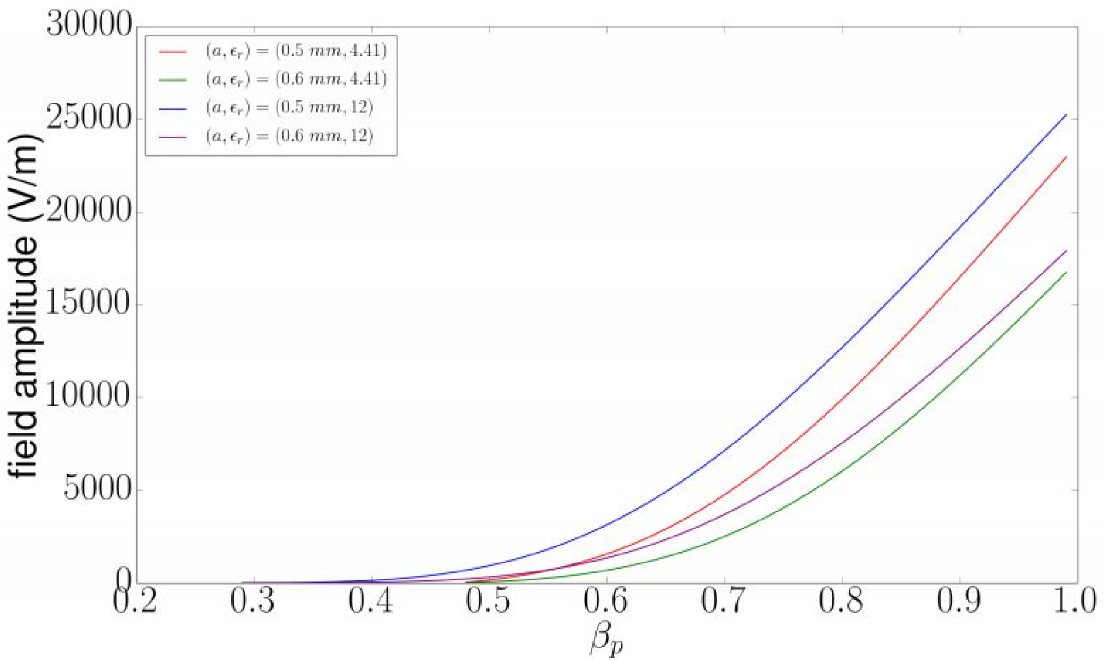
- The inner radius and dielectric thickness determine the phase velocity of the structure.
- Generally, thicker linings lead to slower phase velocities.
- Can we generate a tapered DLW to maintain phase matching with an accelerating low-energy bunch?



Dispersion curve for quartz DLWs with different thicknesses for same inner radius of 0.5 mm. The solution corresponding v p = c is illustrated in blue dots with corresponding dimension (a, b, r) = (0.5 mm, 0.590 mm, 4.41), the red dots correspond to a solution for v p = 0.8c with corresponding structure (a, b, r) = (0.5 mm, 0.612 mm, 4.41).

Field Amplitudes

- The longitudinal electric field is smaller for lower phase velocities.
- The scaling is worse for larger inner radii.
- Future design will look at inner radius taper also to increase low-energy field gradients.



Transverse Forces

- For phase velocities below c, the longitudinal compression region of the phase has a transverse defocussing force.

$$p_{\perp} = e\int \frac{\partial E_z}{\partial r} dz, \qquad = -eB_1 \frac{k_1 J_1(k_1 r)}{k_z} \sin(k_z z - \omega t).$$

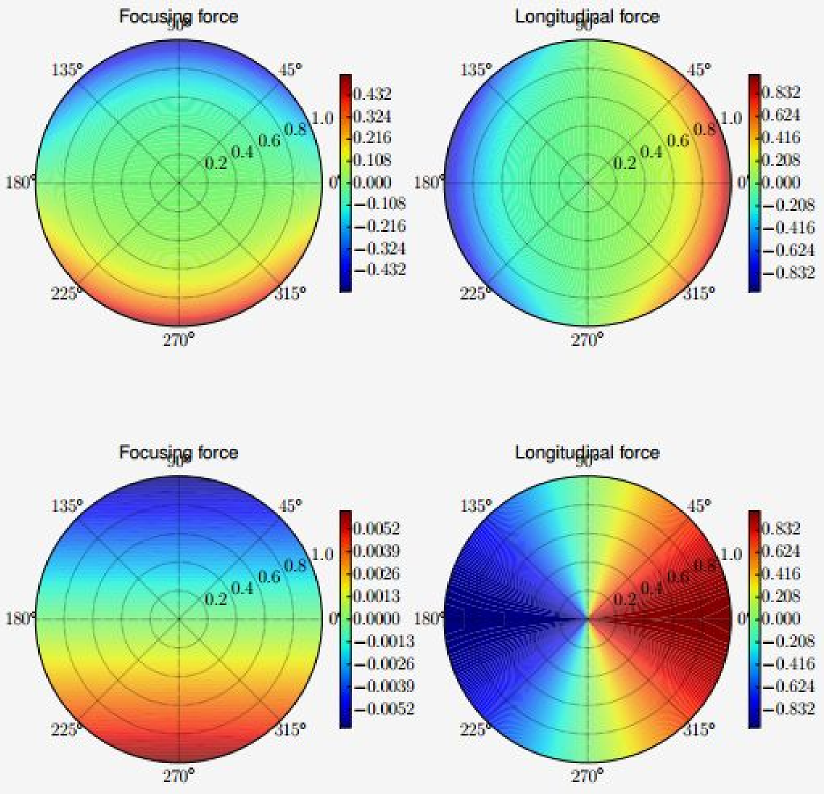


Figure 3: We show the longitudinal and transverse focussing forces in a contour plot in polar coordinates where the azimuthal coordinate represents the relative phase in the THz field and the radial coordinate represents the radial position in the DLW. The figures are normalized to the maximum longitudinal force specific to each case with phase velocities of $v_p = (0.6c, 0.8c, 0.999c)$.

Ansatz & Checking with Maxwell Equations

$$E_z = E_0 I_0(rk_1)\sin(\omega t - \int_0^z dz k_z + \psi)$$
$$E_r = \frac{E_0 k_z}{k_1} I_1(rk_1)\cos(\omega t - \int_0^z dz k_z + \psi)$$
$$H_\phi = \frac{\omega\epsilon_0 B_1}{k_1} I_1(rk_1)\cos(\omega t - \int_0^z dz k_z + \psi).$$

- Same fields but now E, k1, kz depend on z.

$$\frac{\partial}{\partial z}E_z = -\frac{1}{r}\frac{\partial}{\partial r}(rE_r), \tag{39}$$

$$\begin{aligned}\frac{\partial}{\partial z}E_z &= E_0[-k_z I_0\cos(...) + rk_1' I_0' \sin(...)]E_0' I_0 \sin(...)\\ &\simeq -k_z E_0 I_0(k_1 r)\cos(...),\end{aligned} \tag{40}$$

for

$$\begin{aligned}|\frac{rk_1' I_1}{k_z I_0}| &<< 1\\ |\frac{E_0'}{E_0 k_z} &<< 1.\end{aligned} \tag{41}$$

Maxwell cont.

- The fields pass Maxwell's equations for slow (adiabatic) taper rates.

$$-\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = -\frac{E_0 k_z \cos(\ldots)}{k_1 r}\frac{\partial}{\partial r}(rI_1(k_1 r)) \\ = -k_z E_0 I_0(k_1 r)\cos(\ldots). \tag{42}$$

We can also check the condition from

$$\frac{\partial}{\partial t}\frac{E_z}{c^2} = -\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi), \tag{43}$$

which leads to

$$\frac{\partial}{\partial t}\frac{E_z}{c^2} = \frac{\omega}{c^2}E_0 I_0(k_1 r)\cos(\ldots) \\ -\frac{1}{r}\frac{\partial}{\partial r}(rB_\phi) = \frac{\omega\epsilon_0\mu_0}{r}E_0 I_0 \cos(\ldots) \tag{44}$$

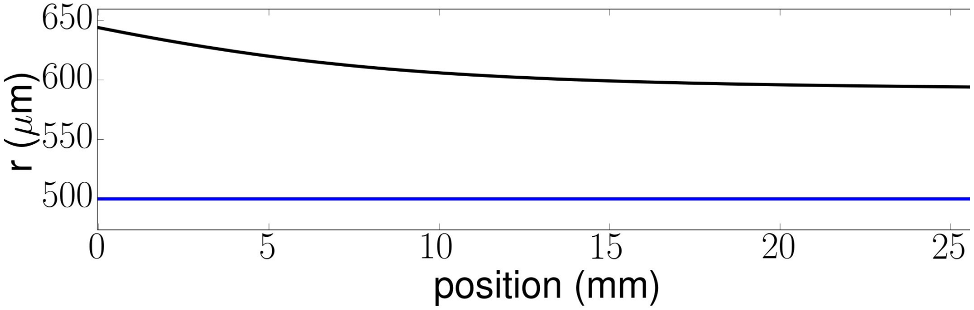
where we have used the identity $\frac{\partial}{\partial r} rI_1(k_1 r) = k_1 r I_0(k_1 r)$. Then the fields can be approximated as:

$$E_z = E_0\left(1 + \frac{(k_1 r)^2}{4}\right)\sin\left(\omega t - \int_0^z dz k_z + \psi\right) \\ E_r = \frac{r k_z E_0}{2}\left(1 + \frac{(k_1 r)^2}{8}\right)\cos\left(\omega t - \int_0^z dz k_z + \psi\right) \\ H_\theta = \frac{r\omega\epsilon_0}{2}\left(1 + \frac{(k_1 r)^2}{8}\right)\cos\left(\omega t - \int_0^z dz k_z + \psi\right) \tag{45}$$

Simulations with ASTRA

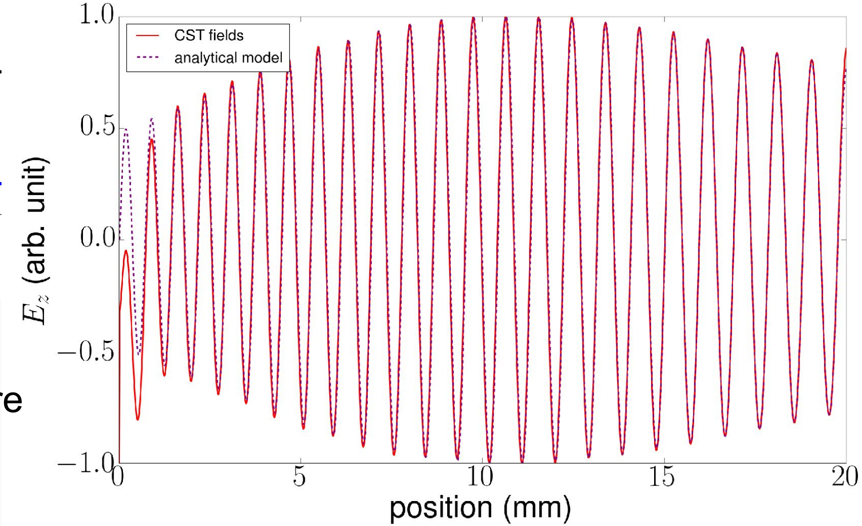
- Analytical fields are now implemented in ASTRA and include all effects.
- Following looks at a test case with a starting energy of 205 keV (0.7c).
- 1 T magnetic field based on alternating permanent magnet design.

Test Case 1: Cold beam injection 0.7c (205 keV), 10x10x10 um bunch into 1 mm, 100 MV/m structure.

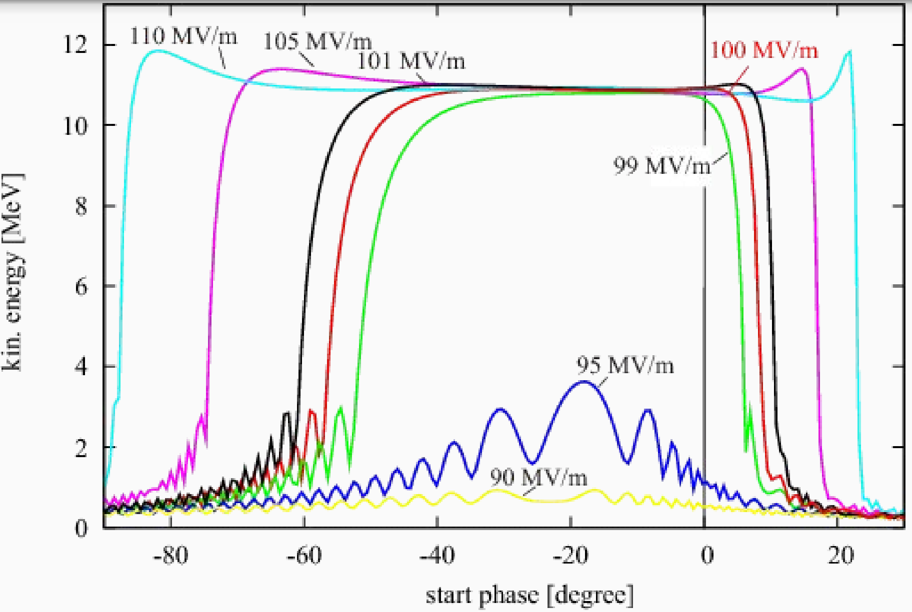


$$\left|\frac{E_0'}{E_0 k_z}\right| = \left|\frac{E_0' v_p}{E_0 \omega}\right|$$

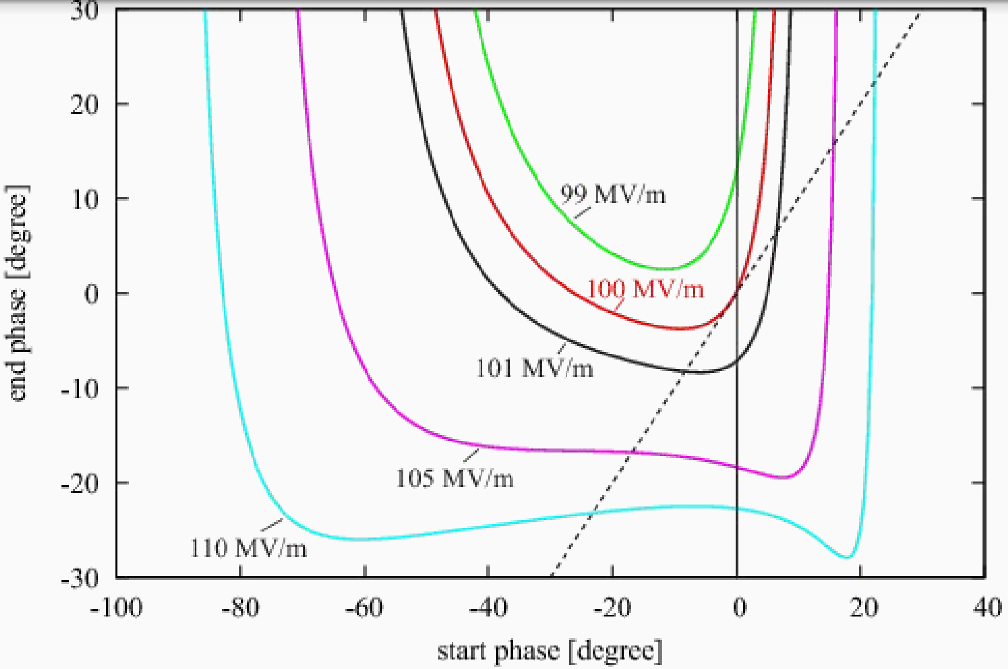
= .0017 at beginning of structure and 10^-7 at end of structure.



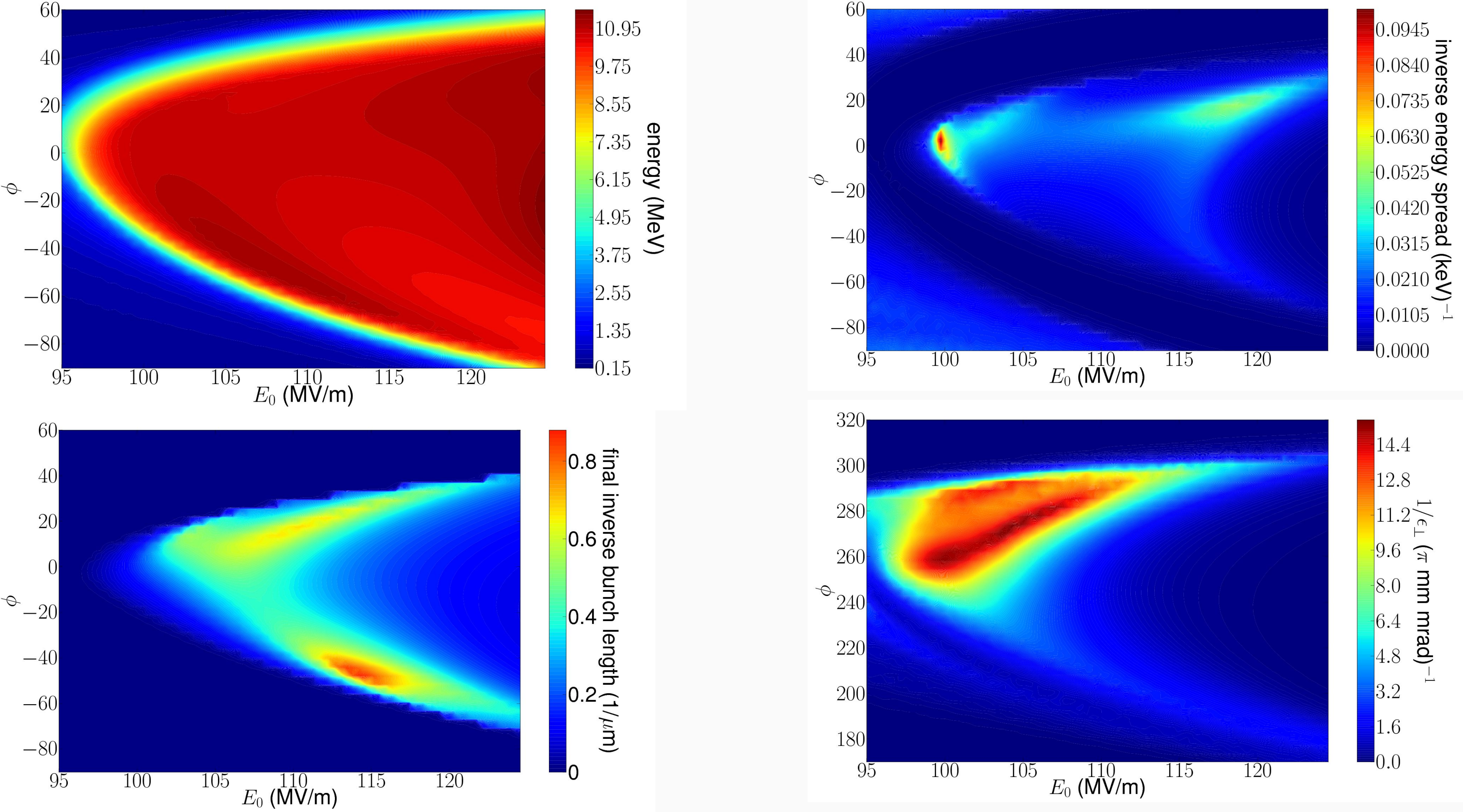
Phase scan



110 MV/m
105 MV/m
101 MV/m
100 MV/m
99 MV/m
95 MV/m
90 MV/m
kin. energy [MeV]
start phase [degree]

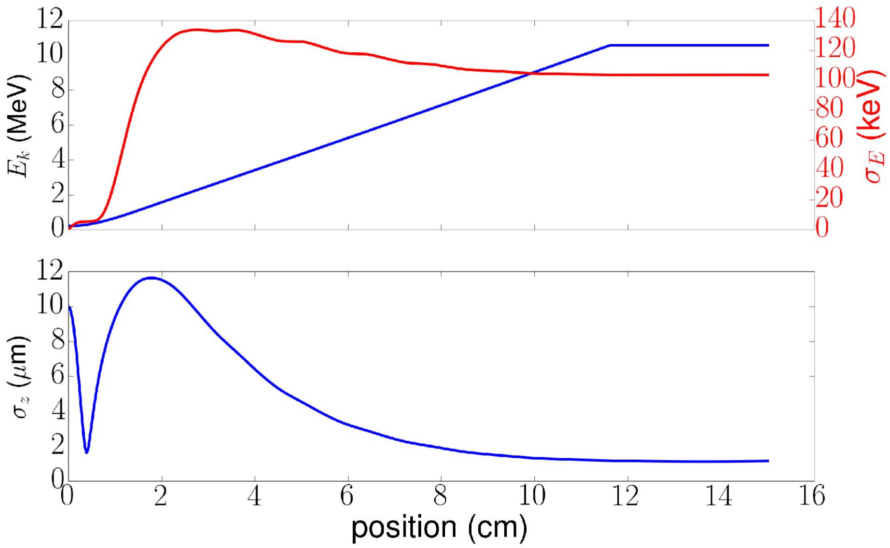


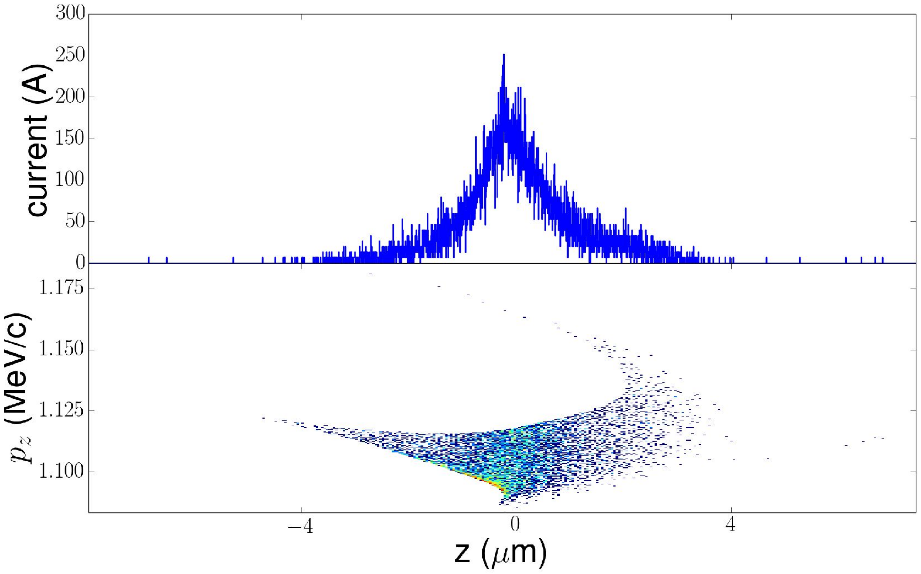
99 MV/m
100 MV/m
101 MV/m
105 MV/m
110 MV/m
end phase [degree]
start phase [degree]



energy (MeV)
inverse energy spread (keV)$^{-1}$
final inverse bunch length (1/μm)
$1/\epsilon_{\perp}$ (π mm mrad)$^{-1}$
E_0 (MV/m)
ϕ

Example trajectory





Looking forward

- Currently looking at a field enhancement gun as injector - large optimization space.
- Understanding the trade-off between different taper designs.
- Currently there are no substantial ways to make the required mJs of 300 GHz radiation; what about higher frequencies where less energy is required? Could potentially use a nanoscribe but we have an injector problem !

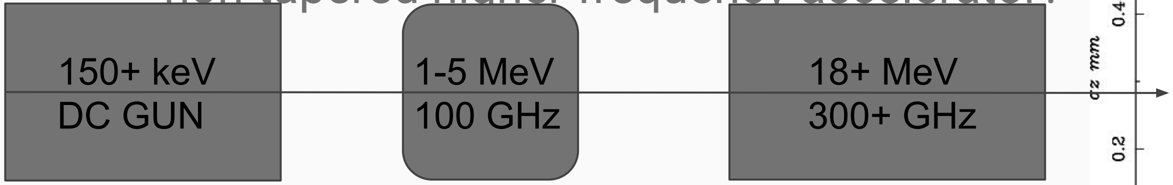
Շնորհակալություն

Philippe Piot - Idea initially picked up together in 2014

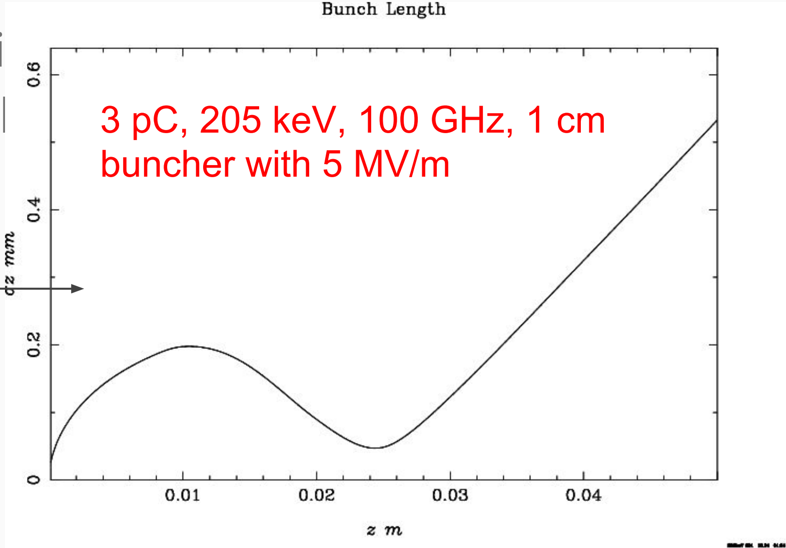
Franz Kaertner - Support to pursue this work.

DC gun + tapered-linac buncher + accelerator

- DC guns generally provide low-energy spread, low-brightness beams with long bunch lengths.
- Since the taper is only over 3-5 cm (dependi
generate a low-frequency accelerator+bunc
non-tapered higher frequency accelerator?

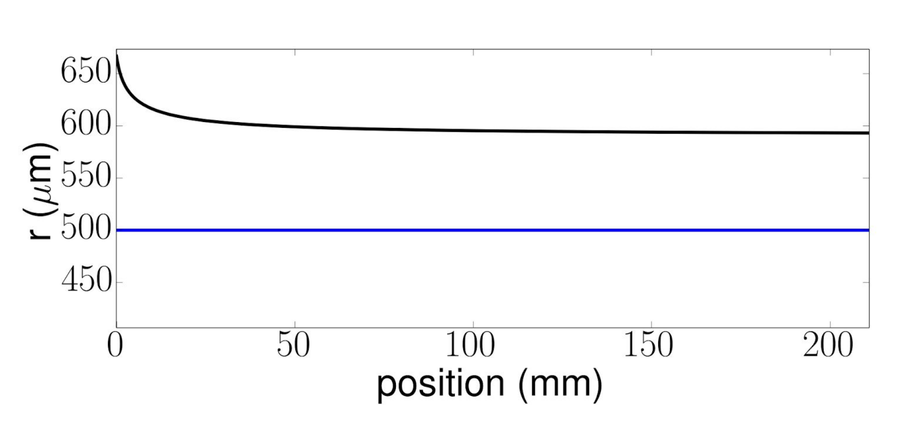


Diagnostics?



Fabrication

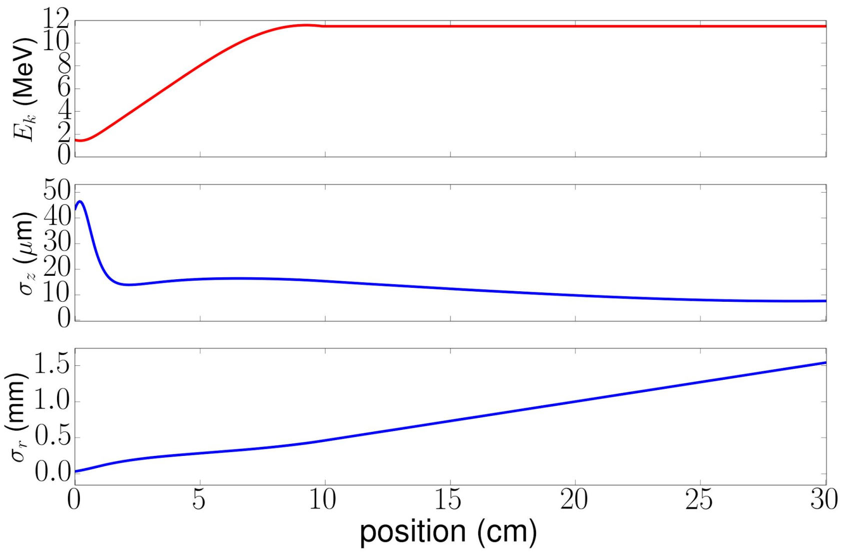
- Initial design takes fixed inner radius and outer taper for fabrication concerns and should be manufacturable with a high precision lathe; companies offer precisions to < 1 um.
- In optics, people employ tapered fibers (<mm) which are generally fabricated by st
g aspect ratios.
- Also interested
ay be more difficult

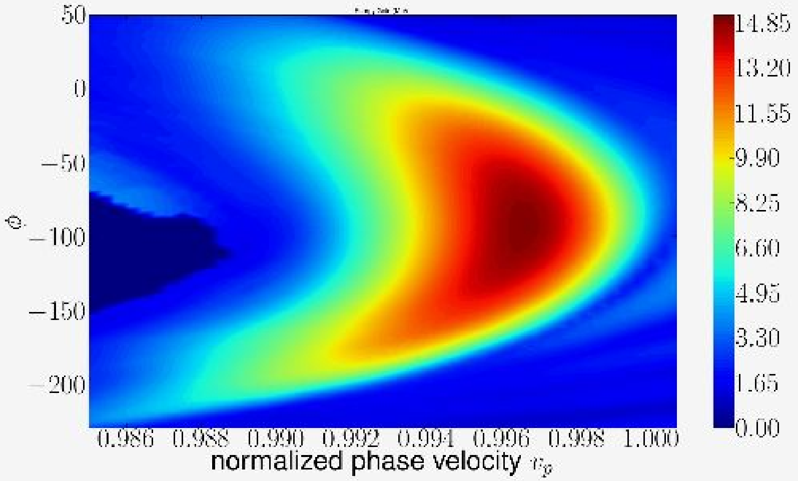


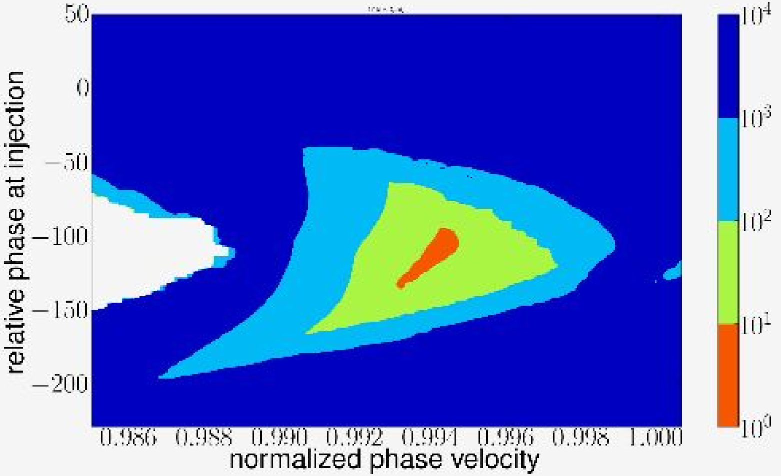
On non-tapered LINACs

Accelerating the Butterfly Bunch

- Non-tapered acceleration of butterfly bunch (1.5 MeV) no B field.

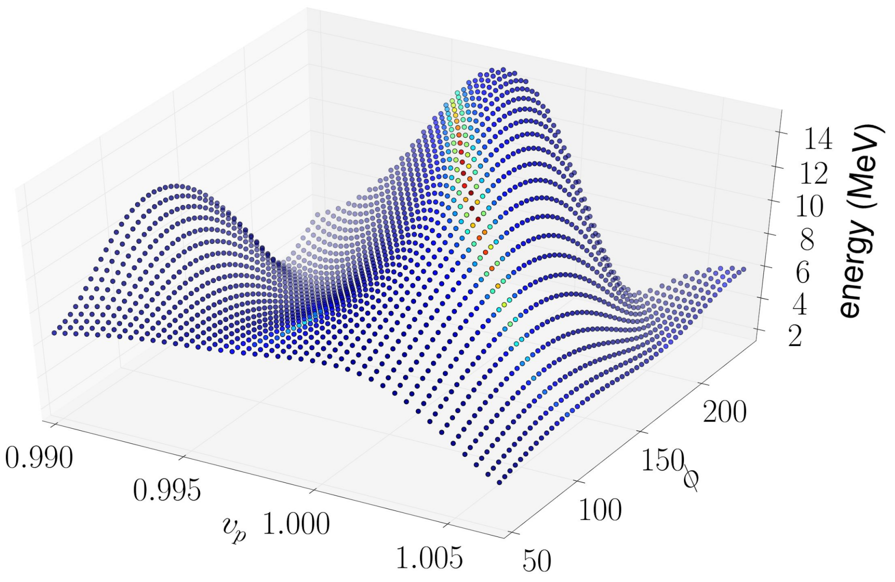


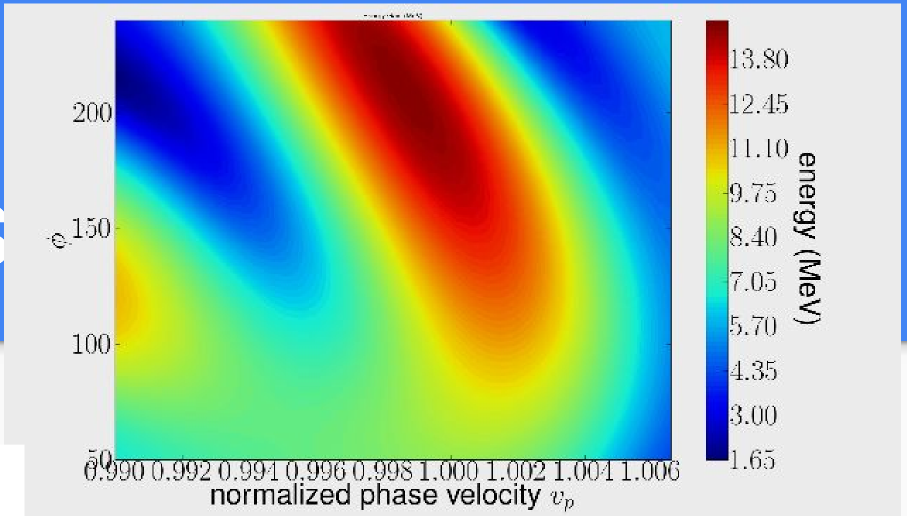


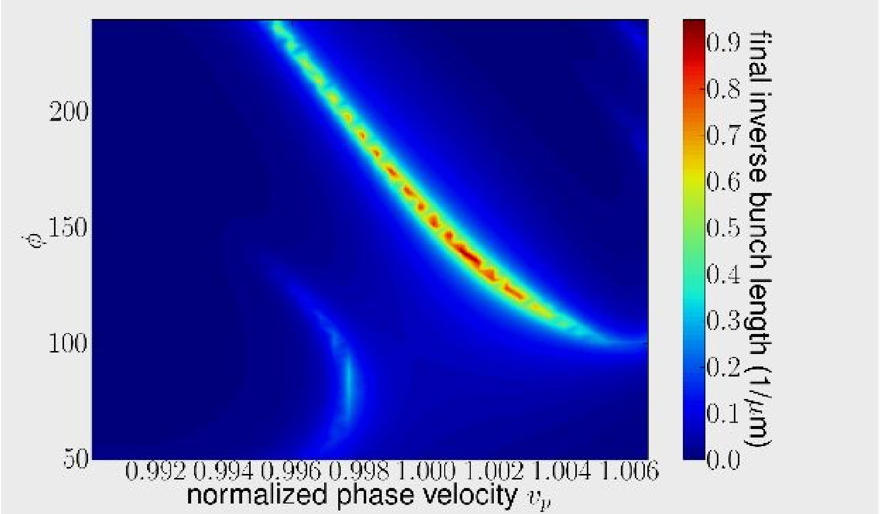


Acceleration with ARES

5 MeV 100 fC off-crest for some bunch compression.







Tunability - frequency

- We should expect high frequency devices to be sensitive.
- Can look to tune with frequency:

*Can also consider temperature tuning with some low-absorption plastics where heat expansion coef. 10^4

$$k_z = \frac{\partial k}{\partial \omega}\omega + k_z^0, \tag{28}$$

and let us consider the example from from Fig. 8, where $k_z^0 = \frac{\omega_0}{c} = \frac{2\pi 300GHz}{c}$, then,

$$k_z = \frac{\omega}{v_g} + \frac{\omega_0}{c}(1 - \frac{c}{v_g}), \tag{29}$$

introducing a dimensionless scaling factor $\omega = \alpha\omega_0$, simplifying and reorganizing leads to

$$k_z = \frac{\omega_0}{c}(n(\alpha - 1) + 1). \tag{30}$$

The phase velocity is given by

$$v_p = \frac{\omega}{k_z} = \frac{\alpha c}{n(\alpha - 1) + 1}, \tag{31}$$

finally, introducing the normalized phase velocity for simplicity $\beta_p = \frac{v_p}{c}$, we can solve for α in terms of β_p; after some algebra, the result is given by

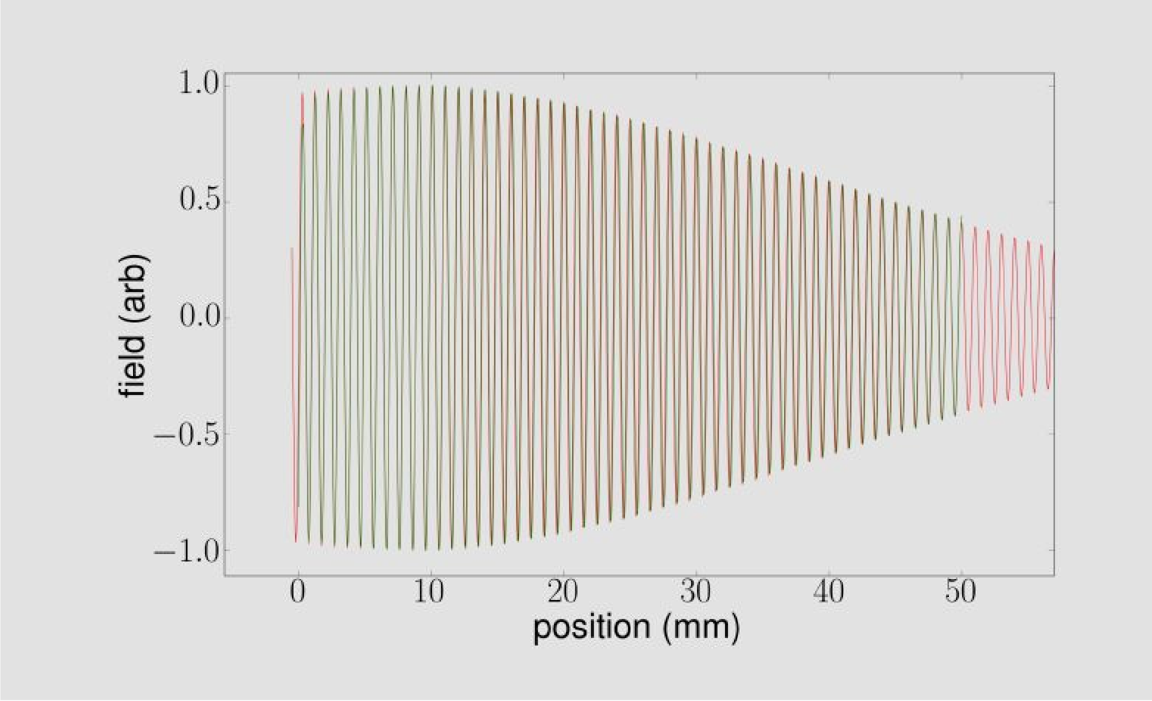
$$\alpha = \frac{n - 1}{n - 1/\beta_p}. \tag{32}$$

As an example, a phase velocity change of 0.01c (e.g. from $v_p = c$ to $v_p = .99c$) can be achieved by a frequency change of $\alpha = 1.009$; i.e. a frequency shift from $f = 300\ GHz$ to $f = 302.7\ GHz$. We note smaller n generally has larger impact on v_p for a given frequency shift.

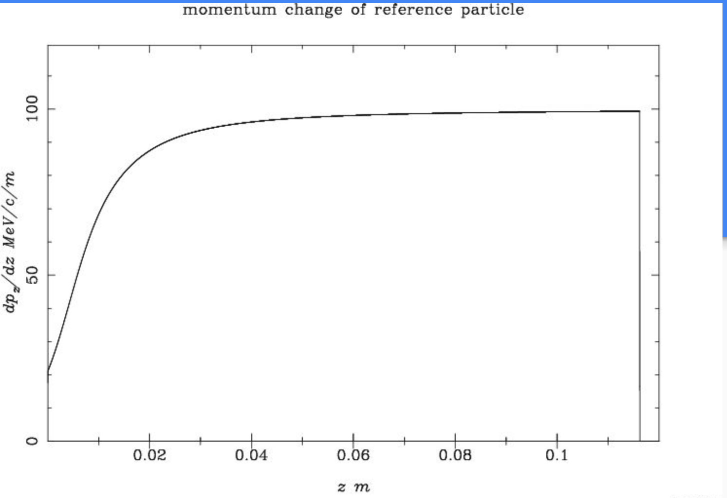
Control over the THz-generation frequency would be a valuable asset to control the accelerating conditions inside the structure.

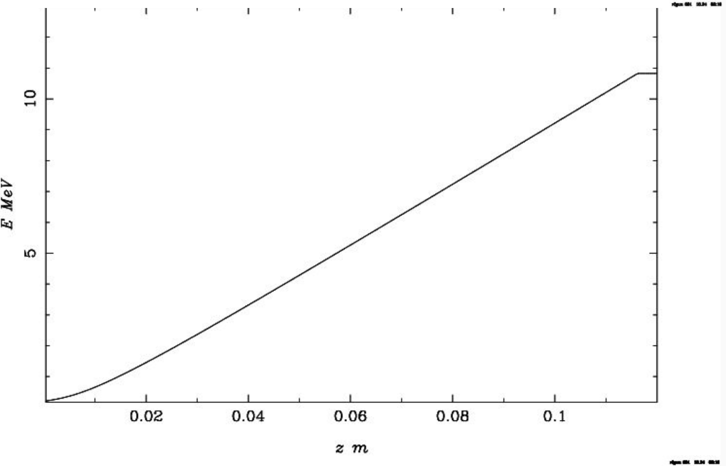
Checking transients with CST

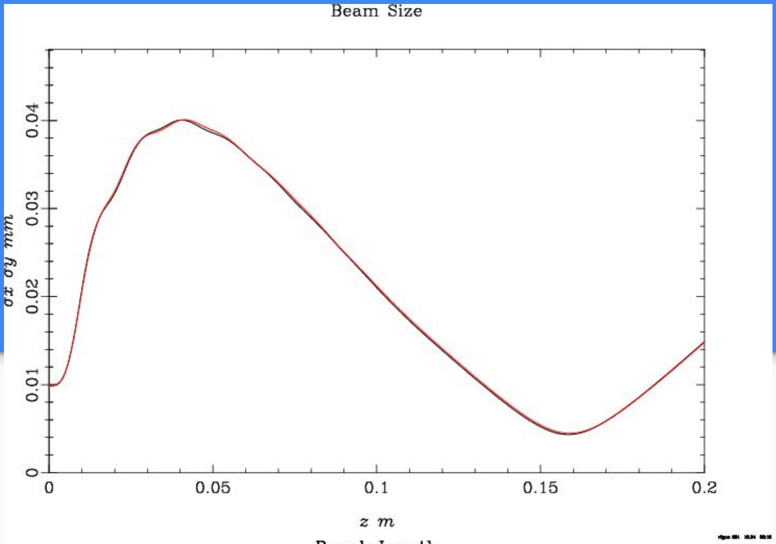
- Found good agreement for a variety of cases, linear, quadratic, hybrid and applicable test cases.

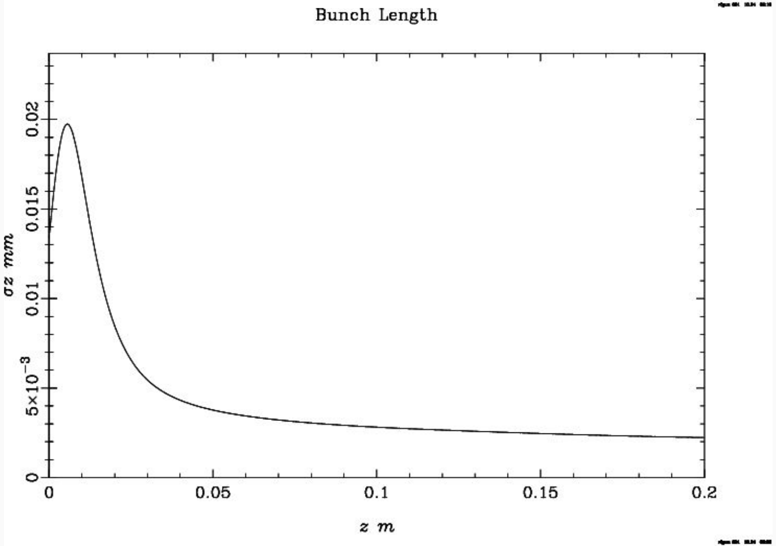


results


momentum change of reference particle
dp_z/dz MeV/c/m
100
50
0
0.02
0.04
0.06
0.08
0.1
z m


E MeV
10
5
0.02
0.04
0.06
0.08
0.1
z m


Beam Size
σx σy mm
0.04
0.03
0.02
0.01
0
0
0.05
0.1
0.15
0.2
z m


Bunch Length
σz mm
0.02
0.015
0.01
5×10⁻³
0
0
0.05
0.1
0.15
0.2
z m