Mode Filtration and Enhancement of the Helical Undulator Radiation in Waveguide

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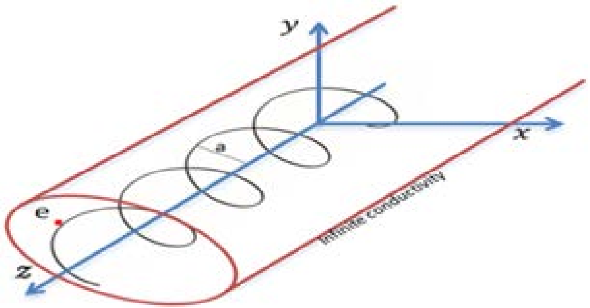
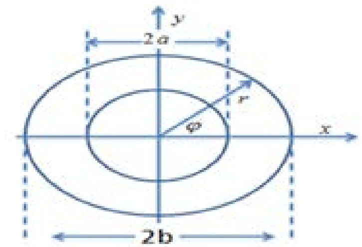
Introduction.

Charge and Current densities

$$\rho = Q\frac{\delta(r-a)}{r}\delta(\varphi-\omega_0 t)\delta(z-Vt)$$

$$\vec{j} = (\omega_0 a\vec{e}_\varphi + V\vec{e}_z)\rho$$





Field Configurations

$$E_{z,nm} = \frac{Q}{2\pi\varepsilon_0 b^2}\frac{J_n\left(j_{nm}\, a/b\right)}{J_{n-1}^2\left(j_{nm}\right)} J_n\left(j_{nm}\, r/b\right) e^{i(n\varphi-\omega_{nm}^j t + k_{nm}^j z)}$$

$$H_{z,nm} = \frac{Qa^2\omega_0}{2\pi b^3}\frac{i\,\nu_{nm}^3}{\nu_{nm}^2 - n^2}\frac{J_n'\left(\nu_{nm}\, a/b\right)}{f\left(\nu_{nm}\right)J_n^2\left(\nu_{nm}\right)}\times$$
$$\times J_n\left(\nu_{nm}\, r/b\right) e^{i(n\varphi-\omega_{nm}^\nu t + k_{nm}^\nu z)}$$

$$\boxed{k_{nm}^\lambda = \frac{\gamma_\parallel^2}{a}[n\beta_\parallel\beta_\perp + f(\lambda_{nm})sign(z-Vt)]}$$

$$\boxed{\omega_{nm}^\lambda = n\omega_0 + k_{nm}^\lambda\beta_\parallel c}$$

$$\boxed{f(\lambda_{nm}) = \sqrt{n^2\beta_\perp^2 - \gamma_\parallel^{-2}\lambda_{nm}^2 a^2/b^2}}$$

$$\boxed{\vec{E}_t^{TE} = -\frac{\omega_{nm}^\nu}{ck_{nm}^\nu}\vec{e}_z\times\vec{H}_t^{TM}}$$

$$\boxed{\vec{E}_t^{TM} = \frac{ik_{nm}^j b^2}{j_{mn}^2}\nabla_t E_z}$$

$$\boxed{\vec{H}_t^{TM} = \frac{\omega_{nm}^j}{ck_{nm}^j}\vec{e}_z\times E_t^{TM}}$$

$$\boxed{\vec{H}_t^{TE} = \frac{ik_{nm}^\nu b^2}{\nu_{nm}^2}\nabla_t H_z}$$

Introduction.

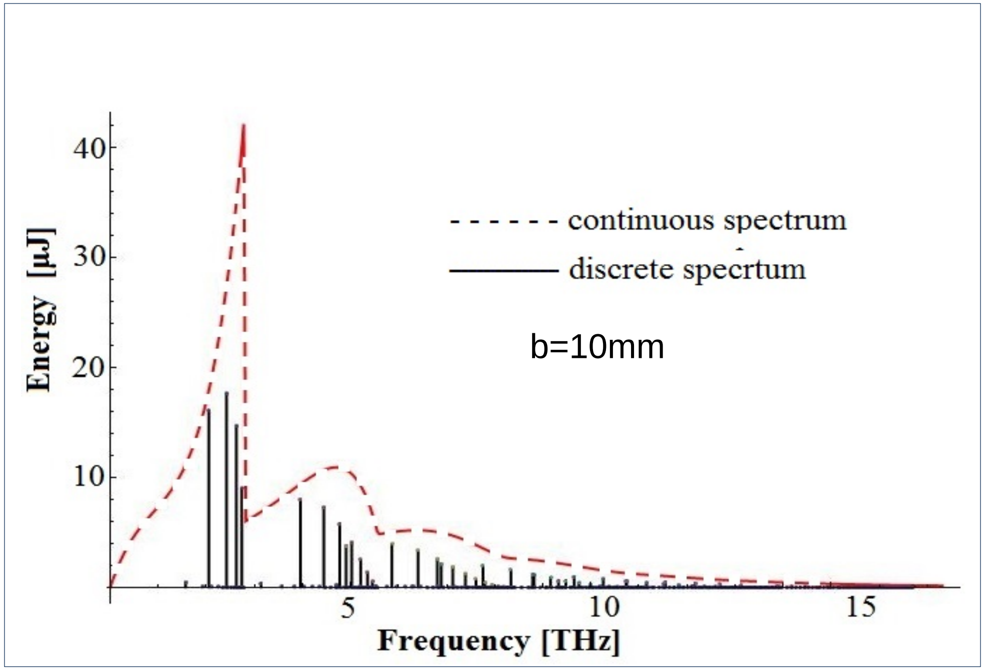
Radiated Energy Flow Along Waveguide

Real part of complex Poynting vector time-averaged energy flow along waveguide

$$P_z = \frac{1}{2}\int_0^b \int_0^{2\pi} \left[\vec{E} \times \vec{H}^*\right] \cdot \vec{e}_z r dr d\varphi$$

Helical undulator and beam parameters	
Period	8 cm
Number of periods	13
Peak field	0.1 T
Undulator constant K	0.75
Electron energy	12 MeV
Charge	100 pC

Free space radiation characteristics	
Expected rad.Freq. in 1st harmonic	2.78 THz
Particle total energy loss [μJ]	124.4
Maximum deflection angle[rad]	0.31
Central cone opening angle [rad]	0.45



- The results are in good agreement with the free space radiation continuous spectrum.
- TM modes is about 85% of total power, and TE - 15%

Condition of Non-vanishing Modes

From equation under root

$$f(\lambda_{nm}) = \sqrt{n^2\beta_\perp^2 - \gamma_\parallel^{-2}\lambda_{nm}^2 a^2/b^2} \quad \Rightarrow \quad n^2(\gamma_z\beta_\perp)^2 \geq \lambda_{nm}^2 a^2/b^2 \quad \rightarrow \quad n^2 \geq \frac{\lambda_{nm}^2}{4\pi^2}\frac{\lambda_u^2}{b^2}\frac{1}{(\gamma_z^2-1)}$$

- $n = 0$ indexes can be neglected → only propagating waves.
- For fixed $n \geq 1$ the eigenvalues λ_{nm} increase with increasing the *m*.
- Number of propagating modes depends on beam energy , undulator period and waveguide radius.

The number of non vanishing modes and each mode average energy[μJ] depending on waveguide radius for fixed $\gamma_z = 16.5$ (12 MeV) and $\lambda_u = 8cm$.

radius [mm]	n=1		n=2	
	modes	Energy[μJ]	modes	Energy[μJ]
30	14	4.2	28	1.2
20	9	6.6	18	1.8
10	4	14.3	8	3.7
5	2	30.5	4	9.2
3	1	57.4	2	19.9
2	0	-	1	41.2

✓ Radius decrease → number of propagating modes decreases.

✓ $b = 3mm$ → only one propagating mode for $n = 1$ with 50% of radiated total energy.

Condition of General Mode Enhancement

The behavior of discrete energy spectrum depending from charged particle energy for fixed undulator period $\lambda_u = 4.5cm$ and first index $n = 1$

radius [mm]	Number of non vanishing modes			
	100MeV	50MeV	25MeV	15MeV
5	24	14	7	4
4	23	11	5	3
3	17	8	4	2
2	11	5	2	1
1	5	2	1	0
0.5	2	1	0	0
0.3	1	0	0	0

- As it seen for a certain case there is possibility to choose parameters for which a significant part of radiation will modified in one mode.

The condition for undulator general mode enhancement is

$$\frac{\lambda_{1,1}^2}{4\pi^2}\Sigma(\lambda_u, b, \gamma_z) \le 1 < \frac{\lambda_{1,2}^2}{4\pi^2}\Sigma(\lambda_u, b, \gamma_z) \qquad \Sigma = \frac{\lambda_u^2}{(\gamma_z^2 - 1)b^2}$$

Energies that Satisfies the Enhancement Condition

Theoretically for every fixed undulator period and particle energy we can decrease the waveguide radius until reaching the point when there is only one mode for $n = 1$.

✓Actually the minimum value for radius which can be implemented from engineering point of view is a few millimeters.

The energy ranges $[MeV]$ **that satisfies the general mode enhancement condition for various undulator periods** λ_u $[cm]$ **and waveguide radiuses** b $[mm]$

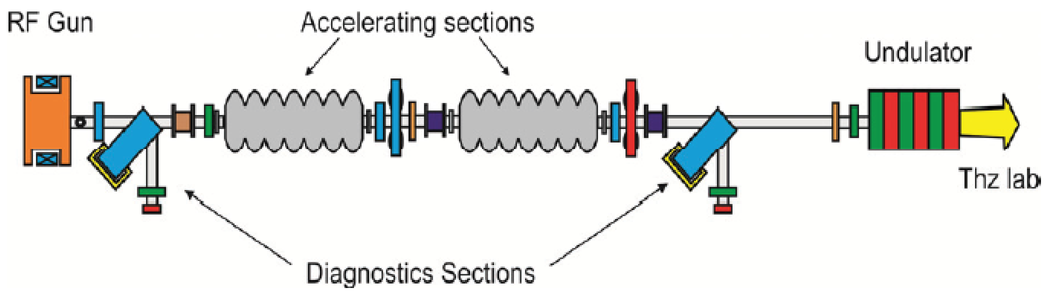
	$\lambda_u = 1$	$\lambda_u = 2$	$\lambda_u = 3$	$\lambda_u = 4$	$\lambda_u = 5$	$\lambda_u = 6$	$\lambda_u = 7$	$\lambda_u = 8$
$b=2$	1.7 - 3	3.5 - 6.3	6 - 10	9 - 15	12 - 22	17 - 29	21 - 38	27 - 48
$b=3$	1.2 – 2	2.4 - 4.2	4 - 7	6 - 10	8 - 15	11 - 19	14 - 25	18 - 32
$b=4$	1 - 1.5	1.8 - 3	3 - 5	4 - 8	6 - 11	8 - 14	11 - 19	14 - 24
$b=5$	0.8 - 1.3	1.5 - 2.5	2.4 - 4	3 - 6	5 - 8	7 - 11	9 - 15	11 - 19
$\omega_R [THz]$	0.25 - 1	0.6 - 2	1 - 3.5	2 - 6	3 - 9.5	5 - 15	6 - 19	9 - 29

✓ The undulator parameter K is kept always 1

✓ The last row shows enhanced resonant mode frequencies for $b = 2mm$ in given energy ranges.

Enhancements in THz region

The next stage of the AREAL development imply enhancement of energy up to 50MeV and the creation of the ALPHA experimental station based on the THz SASE FEL principle.



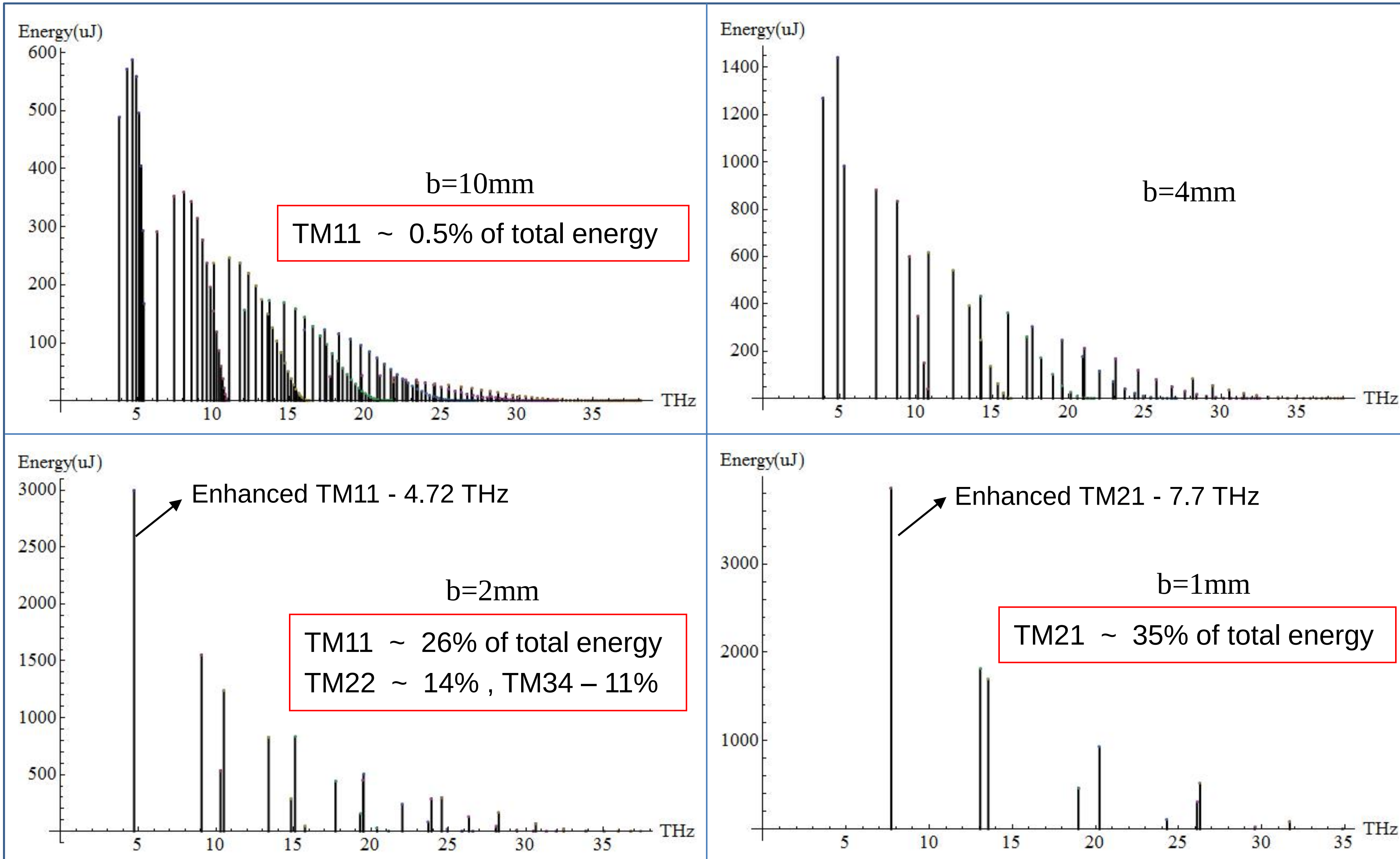
✓ The particle energy and charge are in the range of AREAL parameters.

✓Typical undulator parameters for THz radiation.

Undulator and charge specifications

	Undulator1	Undulator2
Period length	4.5 cm	7 cm
Parameter K	1.05	1.17
Number of Periods	40	26
Peak field	0.25 T	0.18 T
Particle charge	250 pC	250 pC
Particle Energy	15 Mev	21 Mev
Freq, 1^{st} harm.	5.45 THz	6 THz

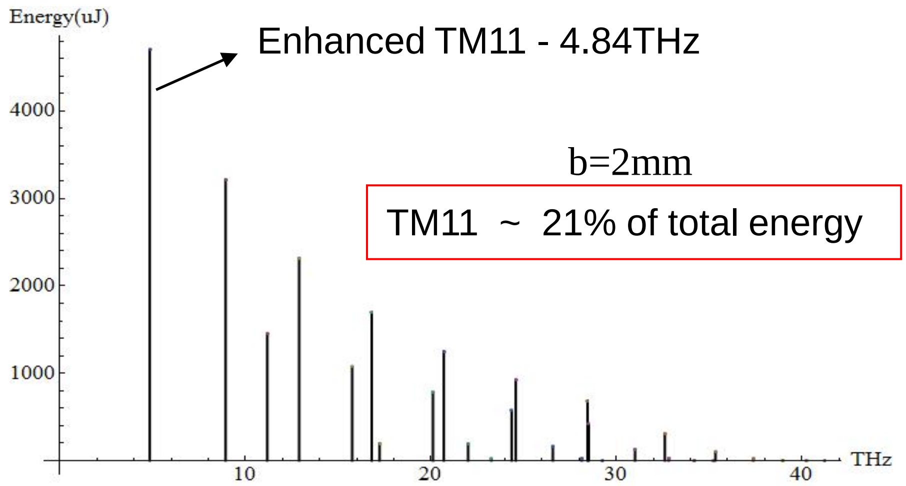
The Discrete Power Spectrums: Undulator 1.

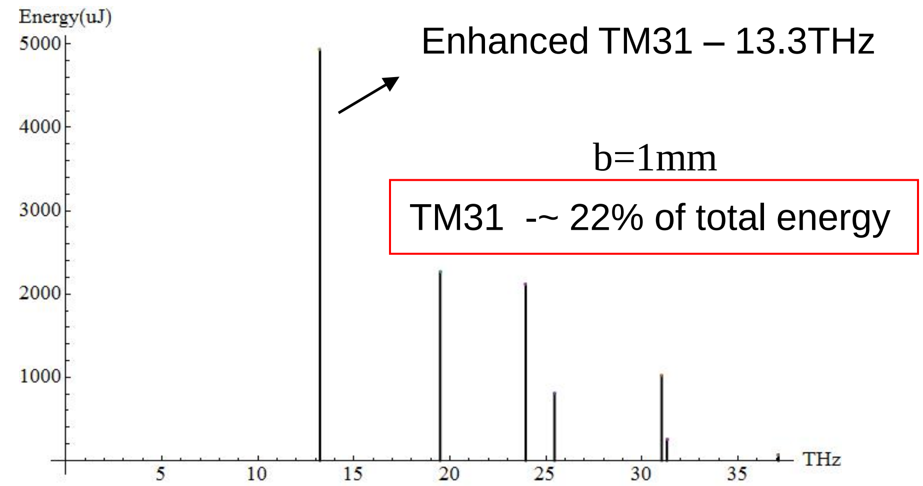


✓ Charged particle energy 15MeV, undulator period 4.5cm, field 0.25T.

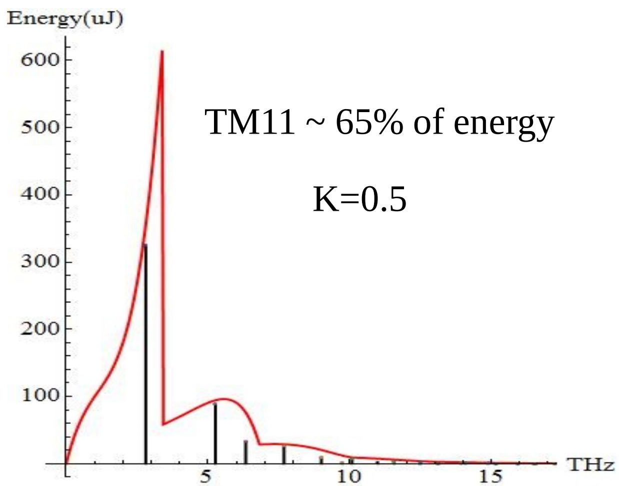
The Discrete Power Spectrums: Undulator 2.

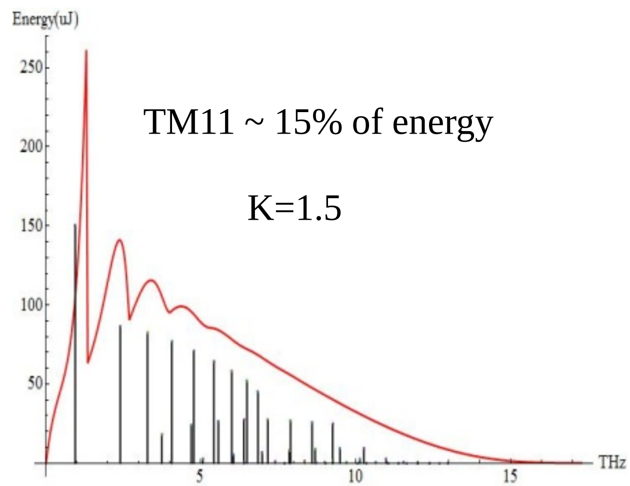
✓ Charged particle energy 21MeV, undulator period 7cm, field 0.18T

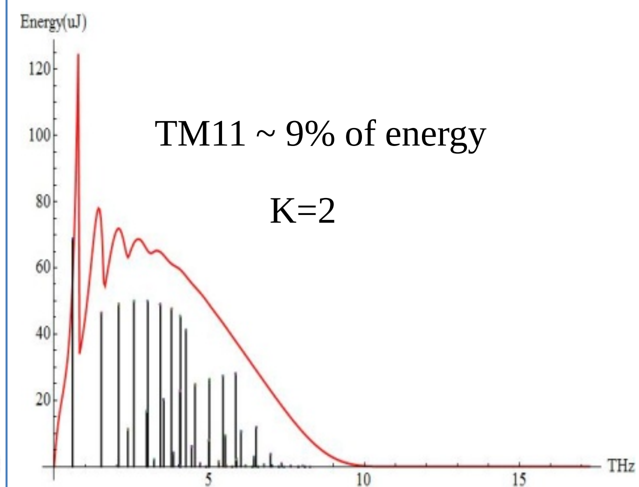




Discrete spectrum behavior depending on undulator parameter K







✓K – increase → the contribution of enhanced mode in power decreases.

Thank You For Your Attention