TRANSVERSE BEAM EMITTANCE MEASUREMENT VIA SOLENOID SCAN TECHNIQUE

PRESENTER - T. MELKUMYAN
Contents

• Introduction
• Beam emittance
• Solenoid magnet
• Solenoid Scan
• Measurements
• Results
• Conclusion
Emittance

For electron beams, the beam quality is the emittance

The emittance in the plane \((x, x')\) can be expressed as the root mean square value (rms):

\[
\varepsilon = \sqrt{\langle x'^2 \rangle \langle x^2 \rangle - \langle xx' \rangle^2}
\]

\(<x^2>\) - the square of the rms beam size

\(<x'^2>\) - the square of the divergence

\(<xx'\rangle^2\) the cross-correlational terms
Solenoid magnet

\[ \int \vec{B} \cdot d\vec{l} = \mu_0 I n L \]

Generally, particles move in a helical trajectory.

The field in the solenoid is uniform and axisymmetric.
Electron beam behavior in solenoid magnet

The focusing of a charged particle beam in a solenoid.

The solid curve shows the periphery of the electron beam when it enters the solenoid.

The dashed curve shows the periphery of the electron beam after it travels some distance in the solenoid.

The dotted curves show the trajectories of individual electrons.
Research Objectives

Use solenoid-scan technique to measure transverse beam emittance

- Measure transverse beam size with the use of ASTRA simulation
- Plot square beam size as a function of field strength
- Do parabolic fit to get coefficients for function
- Calculate transverse beam emittance
Thin lens approximation

Principle: to measure the beam size as a function of the solenoid strength

\[ F = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \]

\[ S_{\text{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \]

\[ K = k l = 1/f_{\text{sol}} \]

\[ \frac{1}{f_{\text{sol}}} = \int \left( \frac{eB_s}{2p} \right)^2 ds \]

F is the transfer matrix of the solenoid

\[ R = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix} \]

R is the transfer matrix between the solenoid and the beam size detector
Measurement of the Transverse Beam Emittance

The beam matrix is:

\[ \Sigma_{\text{beam}} = R \Sigma_{\text{beam},0} R^T \]

\[ \Sigma_{\text{beam}} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \]

\[ \Sigma_{11} = \langle x^2 \rangle \]

is the square of the rms beam size

\[ \Sigma_{22} = \langle x'^2 \rangle \]

is the square of the divergence

\[ \Sigma_{12} = \Sigma_{21} = \langle xx' \rangle \]

are the cross-correlational terms
Measurement of the Transverse Beam Emittance

The (11)-element of the beam transfer matrix after algebra is found to be:

\[ \Sigma_{11}^{scr} = \left( S_{11}^2 \Sigma_{110} + 2S_{11}S_{12}\Sigma_{120} + S_{12}^2 \Sigma_{220} \right) \\
+ \left( 2S_{11}S_{12}\Sigma_{110} + 2S_{12}^2 \Sigma_{120} \right)K + S_{12}^2 \Sigma_{110}K^2 \]

Fitting function (parabolic):

\[ \Sigma_{11}^{scr} = A(K - B)^2 + C = AK^2 - 2ABK + (C + AB^2) \]
Equating terms:

\[ A = S_{12}^2 \Sigma_{110} \]

\[ -2AB = 2S_{11}S_{12} \Sigma_{110} + 2S_{12}^2 \Sigma_{120} \]

\[ C + AB^2 = S_{11}^2 \Sigma_{110} + 2S_{11}S_{12} \Sigma_{120} + S_{12}^2 \Sigma_{220} \]

Solving for the beam matrix elements:

\[ \Sigma_{110} = \frac{A}{S_{12}^2} \]

\[ \Sigma_{120} = -\frac{A}{S_{12}^2} \left( B + \frac{S_{11}}{S_{12}} \right) \]

\[ \Sigma_{220} = \frac{1}{S_{12}^2} \left[ (AB^2 + C) + 2AB \left( \frac{S_{11}}{S_{12}} \right) + A \left( \frac{S_{11}}{S_{12}} \right)^2 \right] \]
Measurement of the Transverse Beam Emittance

The emittance (here, horizontal) is given from the determinant of the beam matrix:

\[ \varepsilon_x = \sqrt{\det \Sigma_{beam}^x} \]

\[ \det \Sigma_{beam}^x = \Sigma_{11} \Sigma_{22} - \Sigma_{12}^2 = \frac{AC}{S_{12}^4} \]

\[ \varepsilon_x = \frac{\sqrt{AC}}{S_{12}^2} \]
The beam size is measured as a function of the solenoid strength.
The beam size is measured as a function of the solenoid strength.
A parabolic fitting function is used to find coefficients, which lead to the solution of beam matrix

\[
A = S_{12}^2 \Sigma_{110} \\
-2AB = 2S_{11} S_{12} \Sigma_{110} + 2S_{12}^2 \Sigma_{120} \\
C + AB^2 = S_{11}^2 \Sigma_{110} + 2S_{11} S_{12} \Sigma_{120} + S_{12}^2 \Sigma_{220}
\]
# Results

<table>
<thead>
<tr>
<th>$\sigma_x^2$</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>9542</td>
<td>2.376e+04</td>
</tr>
<tr>
<td>$P_2$</td>
<td>-1625</td>
<td>-4129</td>
</tr>
<tr>
<td>$P_3$</td>
<td>69.32</td>
<td>181.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_y^2$</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>9424</td>
<td>2.359e+04</td>
</tr>
<tr>
<td>$P_2$</td>
<td>-1615</td>
<td>-4135</td>
</tr>
<tr>
<td>$P_3$</td>
<td>69.18</td>
<td>181</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_x^2$</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1.2e+04</td>
<td>2.559e+04</td>
</tr>
<tr>
<td>$P_2$</td>
<td>-1687</td>
<td>-4299</td>
</tr>
<tr>
<td>$P_3$</td>
<td>70</td>
<td>182.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_y^2$</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1.083e+04</td>
<td>2.654e+04</td>
</tr>
<tr>
<td>$P_2$</td>
<td>-1723</td>
<td>-4356</td>
</tr>
<tr>
<td>$P_3$</td>
<td>70.24</td>
<td>183.1</td>
</tr>
</tbody>
</table>
From the determinant of the beam matrix, the emittance was calculated.

\[ \varepsilon_x = \frac{\sqrt{AC}}{S_{12}^2} \]

<table>
<thead>
<tr>
<th>π mm mrad</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_x )</td>
<td>0.03</td>
<td>0.1</td>
</tr>
<tr>
<td>( \varepsilon_y )</td>
<td>0.0299</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>π mm mrad</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_x )</td>
<td>0.032</td>
<td>0.1</td>
</tr>
<tr>
<td>( \varepsilon_y )</td>
<td>0.033</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Beam emittance obtained from ASTRA simulations

<table>
<thead>
<tr>
<th>$\pi \text{ mm mrad}$</th>
<th>Space charge off</th>
<th>Space charge on</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x$</td>
<td>0.062</td>
<td>5.83</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>0.063</td>
<td>5.84</td>
</tr>
</tbody>
</table>
Thank You