TRANSVERSE BEAM EMITTANCE MEASUREMENT VIA SOLENOID SCAN TECHNIQUE

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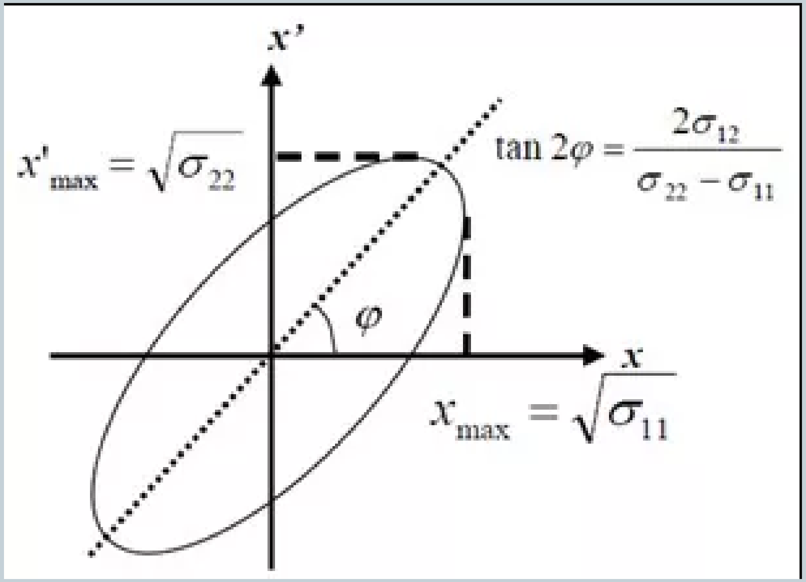
Emittance

For electron beams, the beam quality is the emittance

The emittance in the plane (x,x') can be expressed as the root mean square value (rms):

$$\varepsilon = \sqrt{\langle x'^2 \rangle \langle x^2 \rangle - \langle xx' \rangle^2}$$

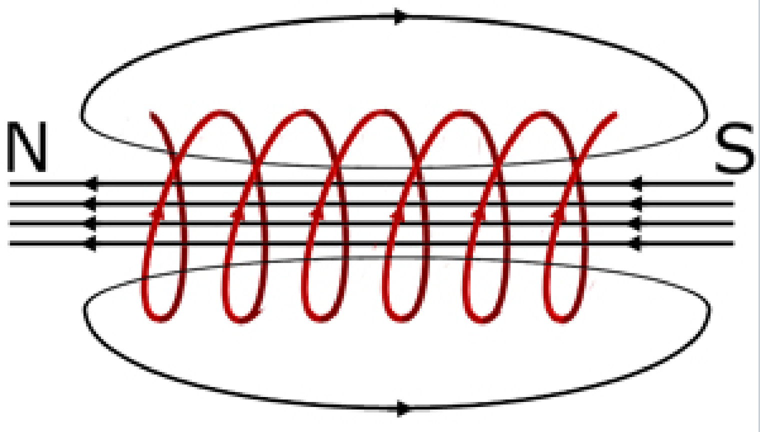


$<x^2>$ - the square of the rms beam size

$<x'^2>$ - the square of the divergence

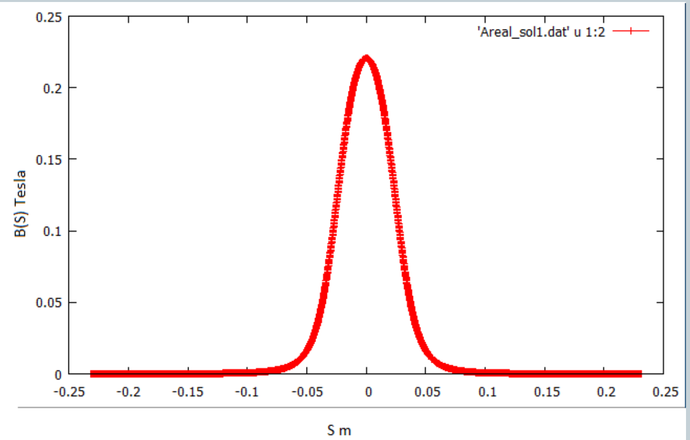
$<xx'>^2$ the cross-correlational terms

Solenoid magnet

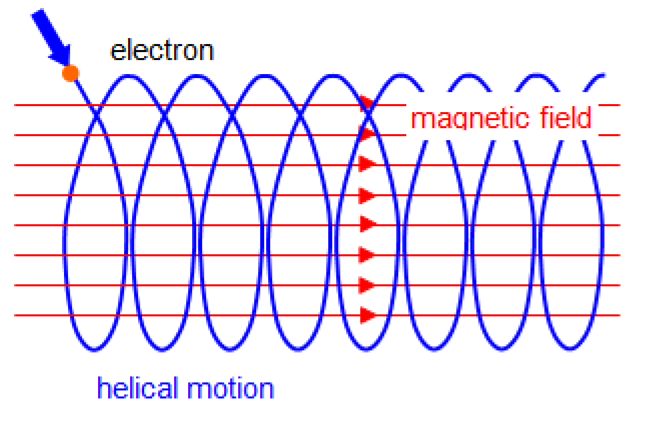


$$\oint \vec{B} \bullet d\vec{l} = \mu_0 InL$$

The field in the solenoid is uniform and axisymmetric

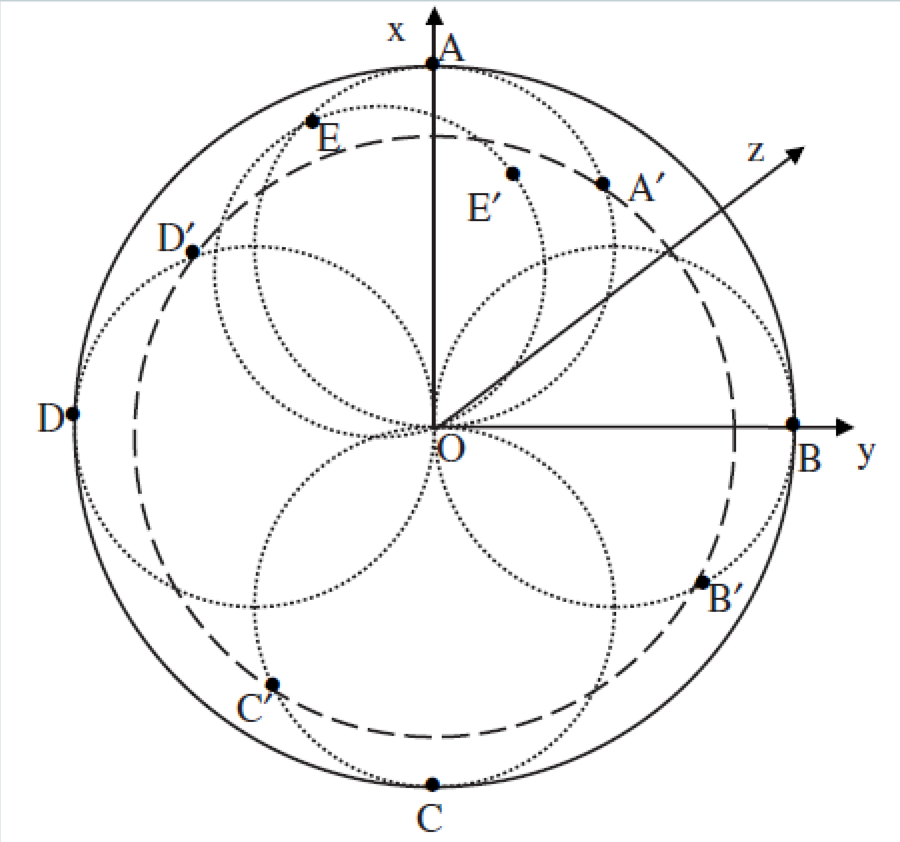


Generally, particles move in a helical trajectory



Electron beam behavior in solenoid magnet

The focusing of a charged particle beam in a solenoid.



The solid curve shows the periphery of the electron beam when it enters the solenoid.

The dashed curve shows the periphery of the electron beam after it travels some distance in the solenoid.

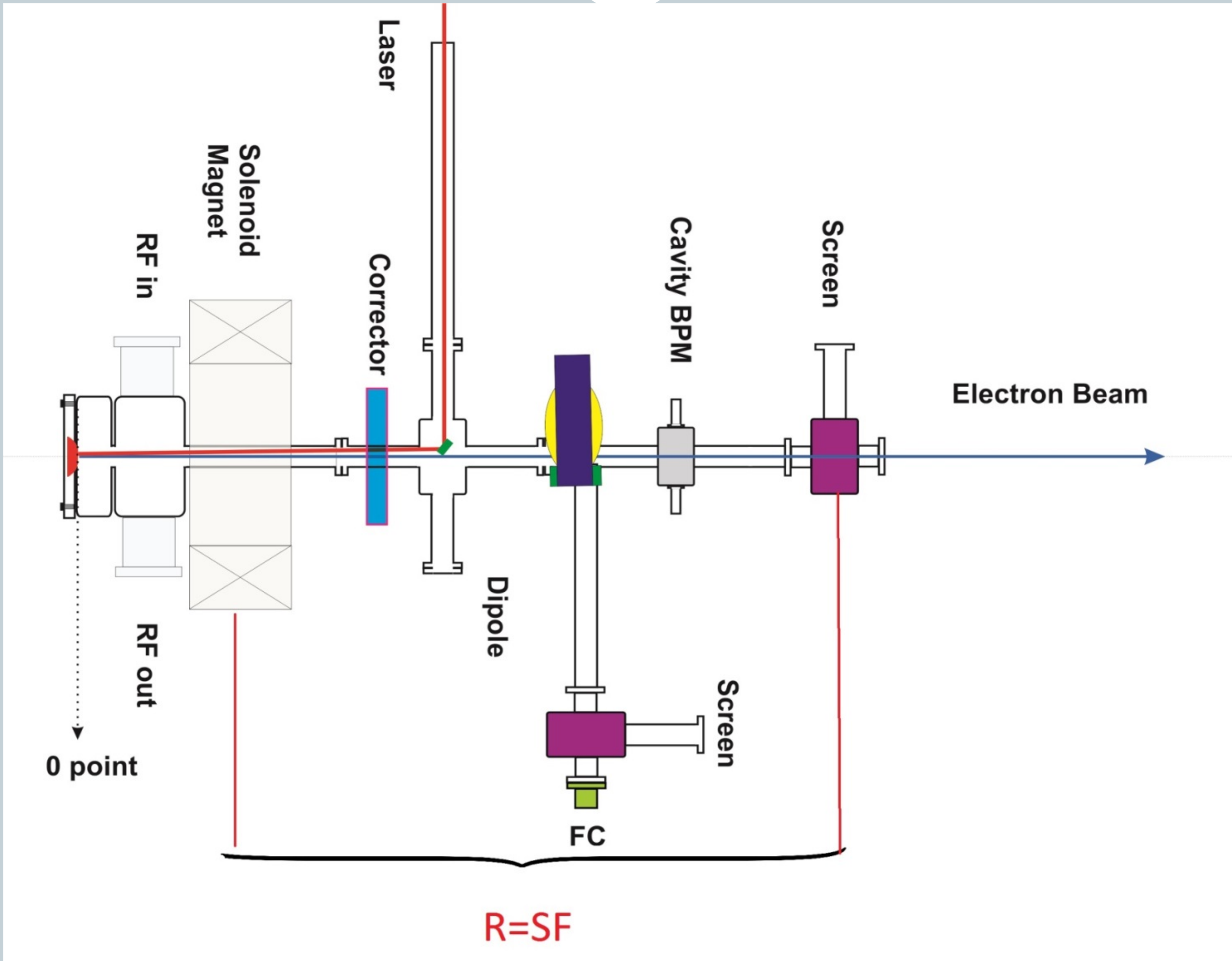
The dotted curves show the trajectories of individual electrons.

Research Objectives

Use solenoid-scan technique to measure transverse beam emittance

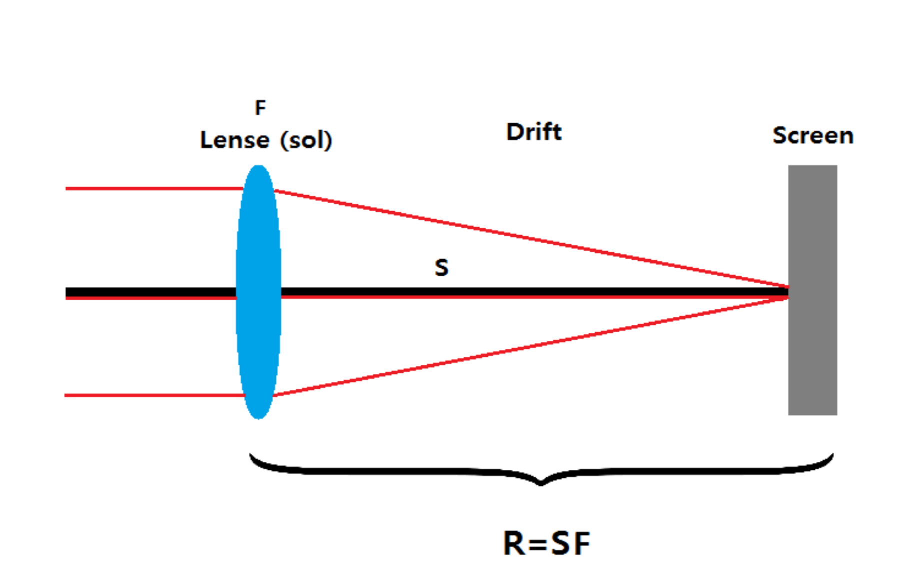
- Measure transverse beam size with the use of ASTRA simulation
- Plot square beam size as a function of field strength
- Do parabolic fit to get coefficients for function
- Calculate transverse beam emittance

Solenoid Scan



Thin lens approximation

Principle: to measure the beam size as a function of the solenoid strength



$$F = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}$$

$$S_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$K = kl = 1/f_{sol}$$

$$\frac{1}{f_{sol}} = \int \left(\frac{eB_s}{2p} \right)^2 ds$$

F is the transfer matrix of the solenoid

R is the transfer matrix between the solenoid and the beam size detector

$$R = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix}$$

Measurement of the Transverse Beam Emittance

The beam matrix is:

$$\Sigma_{beam} = R\Sigma_{beam,0}R^T$$

$$\Sigma_{beam} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

$\Sigma_{11} = \langle x^2 \rangle$ is the square of the rms beam size

$\Sigma_{22} = \langle x'^2 \rangle$ is the square of the divergence

$\Sigma_{12} = \Sigma_{21} = \langle xx' \rangle$ are the cross-correlational terms

Measurement of the Transverse Beam Emittance

The (11)-element of the beam transfer matrix after algebra is found to be:

$$\Sigma_{11}^{scr} = \left(S_{11}^2 \Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + S_{12}^2 \Sigma_{22_0}\right) + \left(2S_{11}S_{12}\Sigma_{11_0} + 2S_{12}^2\Sigma_{12_0}\right)K + S_{12}^2\Sigma_{11_0}K^2$$

Fitting function (parabolic):

$$\Sigma_{11}^{scr} = A(K - B)^2 + C = AK^2 - 2ABK + (C + AB^2)$$

Measurement of the Transverse Beam Emittance

Equating terms:

$$A = S_{12}^2 \Sigma_{11_0}$$

$$-2AB = 2S_{11}S_{12}\Sigma_{11_0} + 2S_{12}^2\Sigma_{12_0}$$

$$C + AB^2 = S_{11}^2\Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + S_{12}^2\Sigma_{22_0}$$

Solving for the beam matrix elements:

$$\Sigma_{11_0} = \frac{A}{S_{12}^2}$$

$$\Sigma_{12_0} = -\frac{A}{S_{12}^2}\left(B + \frac{S_{11}}{S_{12}}\right)$$

$$\Sigma_{22_0} = \frac{1}{S_{12}^2}\left[(AB^2 + C) + 2AB\left(\frac{S_{11}}{S_{12}}\right) + A\left(\frac{S_{11}}{S_{12}}\right)^2\right]$$

Measurement of the Transverse Beam Emittance

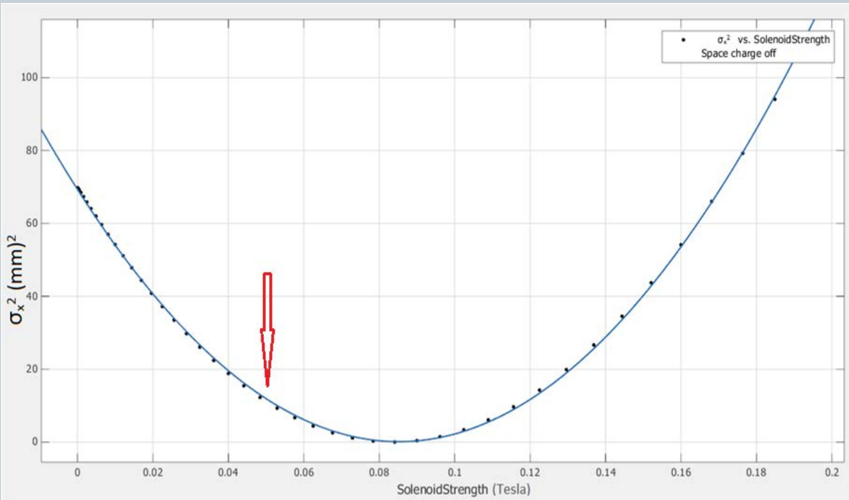
The emittance (here, horizontal) is given from the determinant of the beam matrix:

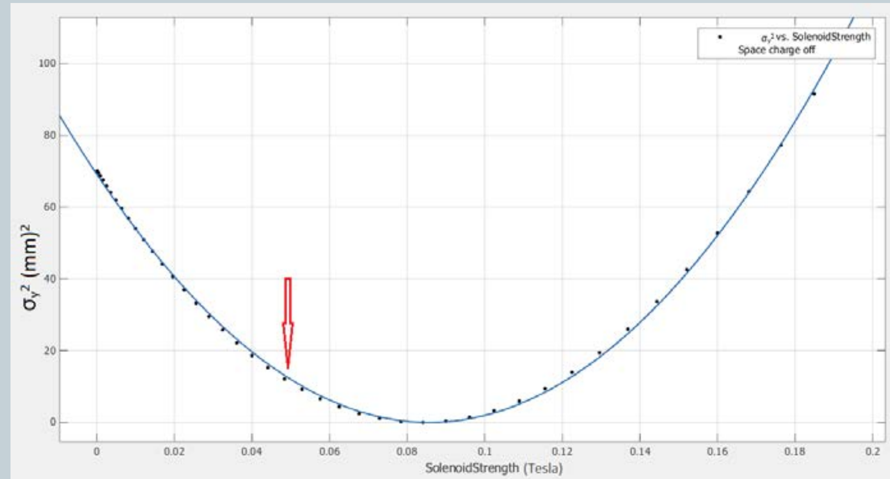
$$\varepsilon_x = \sqrt{det\Sigma^x_{beam}}$$

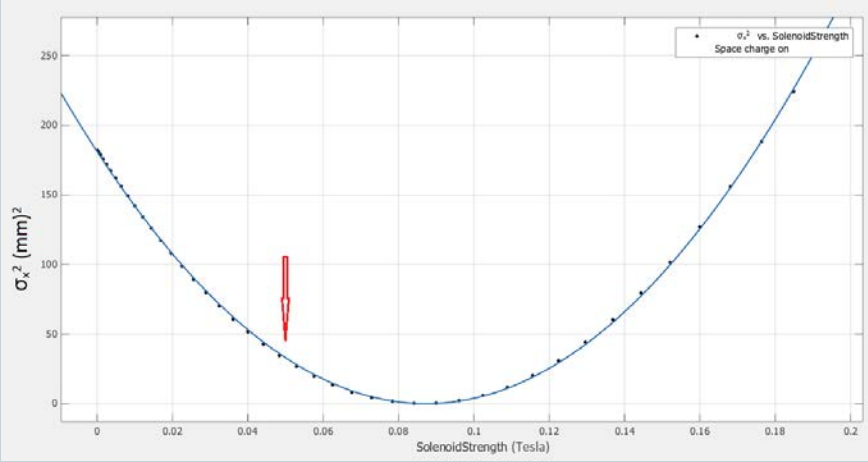
$$det\Sigma^x_{beam} = \Sigma_{11}\Sigma_{22} - \Sigma^2_{12} = \frac{AC}{S^4_{12}}$$

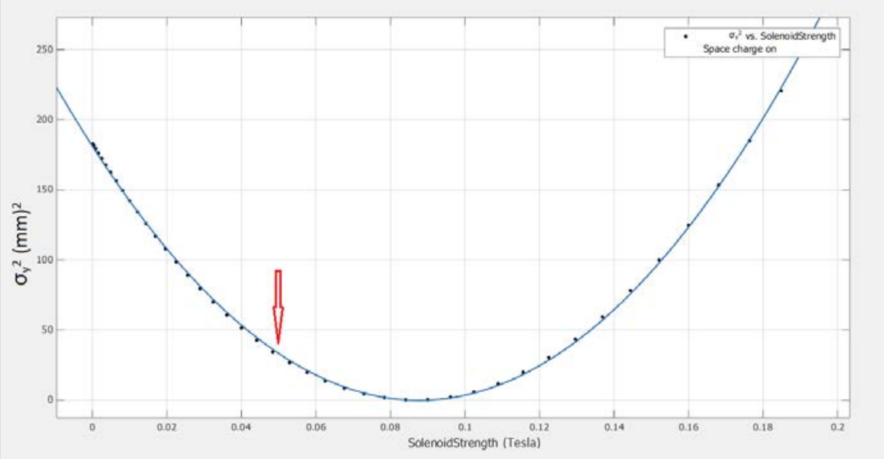
$$\varepsilon_x = \frac{\sqrt{AC}}{S^2_{12}}$$

The beam size is measured as a function of the solenoid strength.

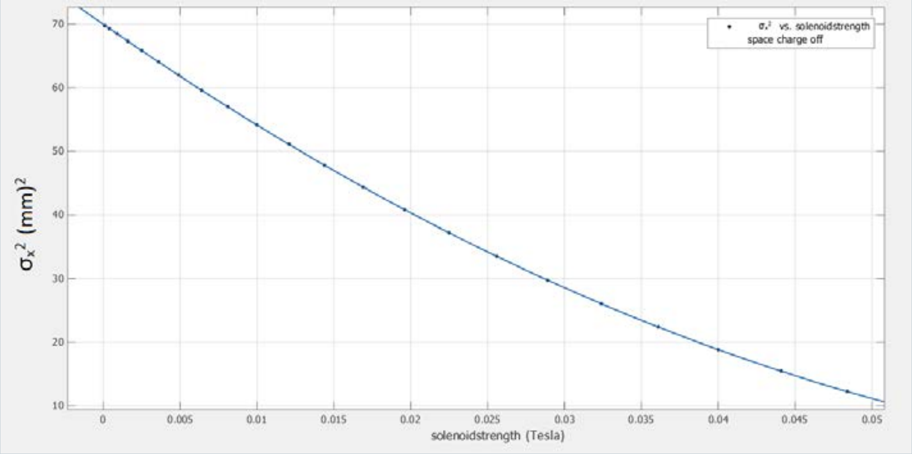


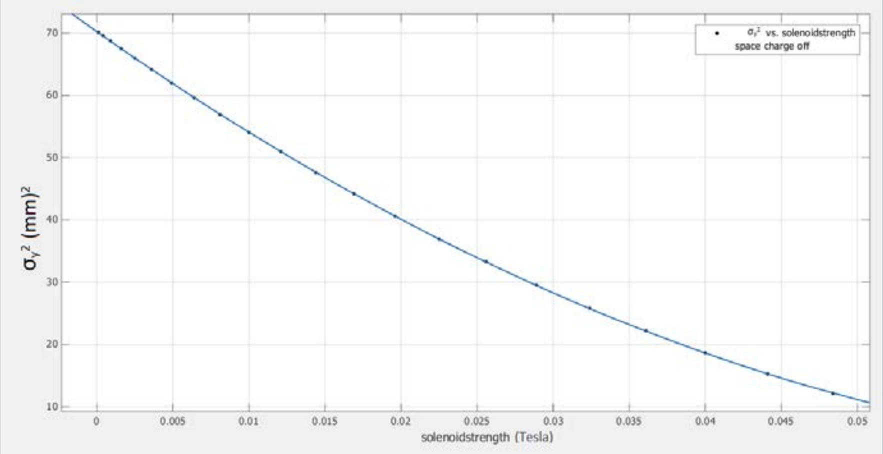


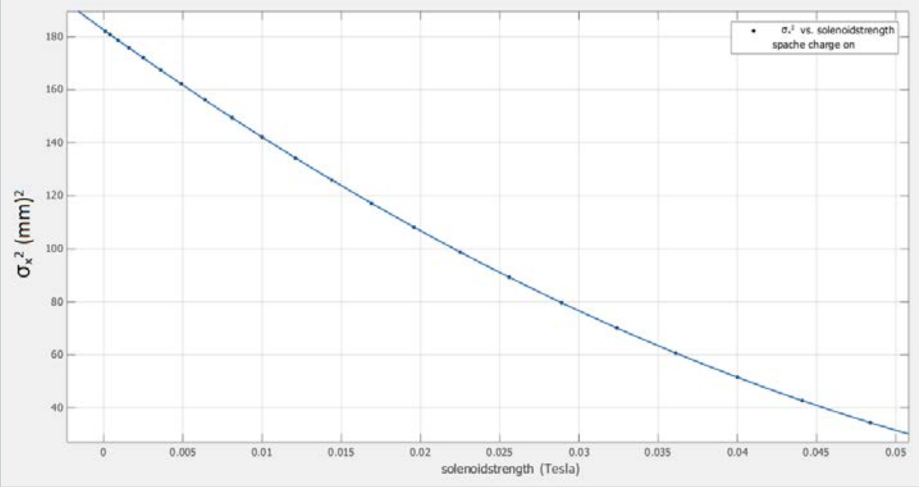


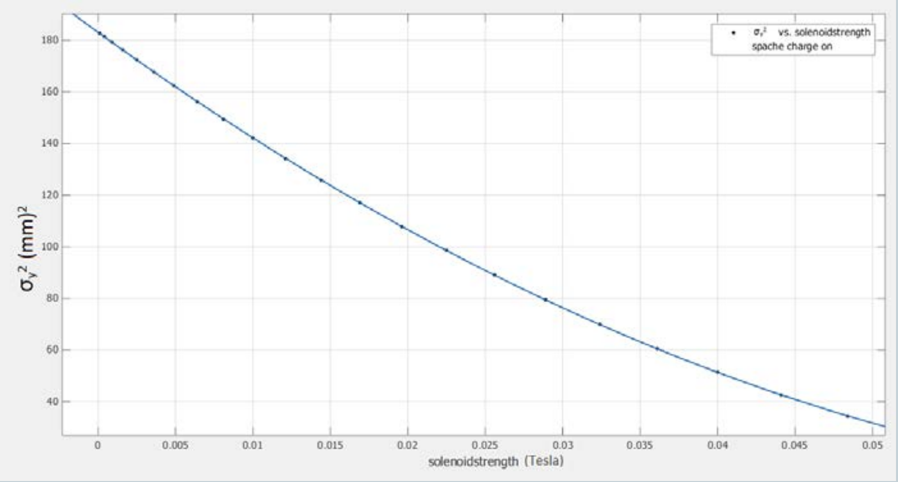


The beam size is measured as a function of the solenoid strength.









Measurements

A parabolic fitting function is used to find coefficients, which lead to the solution of beam matrix

$$A = S_{12}^2 \Sigma_{11_0}$$

$$-2AB = 2S_{11}S_{12}\Sigma_{11_0} + 2S_{12}^2\Sigma_{12_0}$$

$$C + AB^2 = S_{11}^2\Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + S_{12}^2\Sigma_{22_0}$$

$A = P_1$

$-2AB = P_2$

$C + AB^2 = P_3$

Results

σ_x^2	Space charge off	Space charge on
P_1	9542	2.376e+04
P_2	-1625	-4129
P_3	69.32	181.1

σ_y^2	Space charge off	Space charge on
P_1	9424	2.359e+04
P_2	-1615	-4135
P_3	69.18	181

σ_x^2	Space charge off	Space charge on
P_1	1.2e+04	2.559e+04
P_2	-1687	-4299
P_3	70	182.6

σ_y^2	Space charge off	Space charge on
P_1	1.083e+04	2.654e+04
P_2	-1723	-4356
P_3	70.24	183.1

Results

From the determinant of the beam matrix, the emittance was calculated.

$$\varepsilon_x = \frac{\sqrt{AC}}{S_{12}^2}$$

π mm mrad	Space charge off	Space charge on
ε_x	0.03	0.1
ε_y	0.0299	0.1

π mm mrad	Space charge off	Space charge on
ε_x	0.032	0.1
ε_y	0.033	0.1

Beam emittance from ASTRA simulations

Beam emittance obtained from ASTRA simulations

π mm mrad	Space charge off	Space charge on
ε_x	0.062	5.83
ε_y	0.063	5.84

Thank You