Impedance Computation in the Frequency Domain

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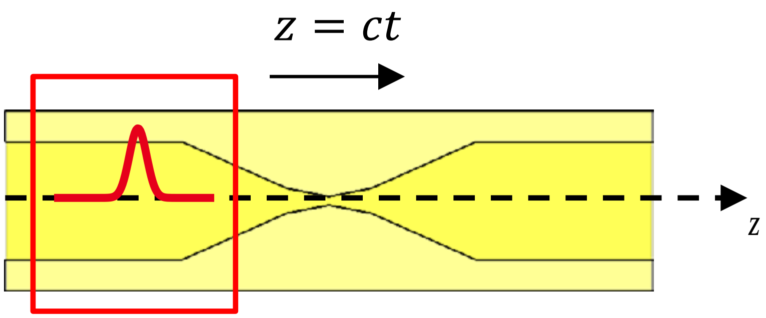
Contents

Motivation

- Conventional wakefield computation

$$W_{\parallel}(r,s) = \frac{1}{Q}\int dz\, E_z\big(r,z,t(s,z)\big) \qquad t(s,z) = \frac{s+z}{c}$$

- Solve Maxwell's grid equations in the time domain
  - FIT/Cartesian grids/ Dispersion-free methods
  - Co-moving computational window
  - Indirect integration
  - Wall losses by convolution of surface imp.



- Impedance by Fourier transform

$$Z_{\parallel}(r,\omega) = -\frac{1}{c}\frac{1}{\tilde{\lambda}(\omega)}\int ds W_{\parallel}(r,s)\exp\left(-i\frac{\omega}{c}s\right)$$

Motivation

- Long range wakefields
  - Low frequency, long bunches, bunch trains, wall heating
- Approximation of geometry
  - Geometrical details smaller than bunch length, smooth tapering etc.
- Dispersive problems
  - Surface impedance, dielectrics
  - Free-space and waveguide boundary conditions
- Radiation effects
  - Curved beam trajectories (and CSR)
  - Accelerated beams
- Coupler and Pickup signals

Frequency Domain Formulation

- The frequency domain problem

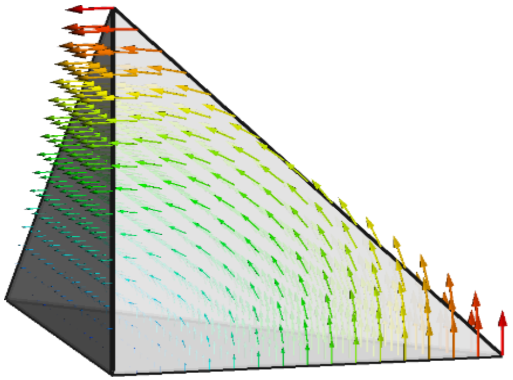
$$\boxed{\nabla \times \mu^{-1} \nabla \times E - k_0^2 \varepsilon E = -jk_0 Z_0 J_s \qquad J_s(x,y,z,\omega) = \rho(x,y) e^{-i\frac{\omega}{v}z}}$$

- Weak FEM formulation: find $E \in H(curl)$ such that:

$$\int dV \mu^{-1}\, \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h = -jk_0 Z_0 \int dV\, J_s \cdot v_h$$

$$\underbrace{+ \oint_S \; dS\;\, n \cdot [v_h \times \mu^{-1}\, \nabla \times E]}_{\textbf{boundary term}}$$

$\forall\, \boldsymbol{v_h} \in \boldsymbol{H(curl)}$



Frequency Domain Formulation

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- Treatment of boundary surfaces

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon \, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV \, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} \boldsymbol{dS\, n \cdot [v_h \times \mu^{-1} \nabla \times E]}}_{\textbf{resistive wall}} + \underbrace{\int_{S_{WG}} dSn \cdot [v_h \times \mu^{-1} \nabla \times E]}_{\textbf{in \& outgoing pipes}}$$

- Resistive wall boundary

$$\oint_{S_{SIBC}} dS \;\, n \cdot [v_h \times \mu^{-1} \nabla \times E] = \cdots = j\omega \boldsymbol{Y_S(\omega)} \oint_{S_{SIBC}} dS \;\, v_h \cdot [n \times n \times E]$$

Simple modification of the system matrix on SIBC surfaces

No fitting of the surface impedance function or ADE/convolution is needed

Frequency Domain Formulation

- Treatment of boundary surfaces

$$\int dV \mu^{-1}\, \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h =$$

$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \underbrace{\int_{S_{SIBC}} dS\, n \cdot [v_h \times \mu^{-1}\, \nabla \times E]}_{\textbf{resistive wall}} + \underbrace{\int_{S_{WG}} \boldsymbol{dSn} \cdot [\boldsymbol{v_h} \times \boldsymbol{\mu^{-1}}\, \boldsymbol{\nabla} \times \boldsymbol{E}]}_{\textbf{in \& outgoing pipes}}$$

- Beam pipe boundaries

$$n \times \nabla \times E = n \times \nabla \times E^{inc} + \sum_m a_m^{TE} \gamma_m^{TE} e_m^{TE} + \sum_m a_m^{TM} \frac{-k_0^2}{\gamma_m^{TM}} e_m^{TM}$$

$$a_m^{TE} = \int_{S_{WG}} dS e_m^{TE} \cdot [E - E^{inc}]$$

Reflection coefficients for each mode

$$a_m^{TM} = \int_{S_{WG}} dS e_m^{TM} \cdot [E - E^{inc}]$$

Frequency Domain Formulation

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- Beam pipe boundary conditions

$$\int dV \mu^{-1} \nabla \times E \cdot \nabla \times v_h - k_0^2 \int dV \varepsilon\, E \cdot v_h + \sum_m \boldsymbol{P_m^{TE}(E)} + \sum_m \boldsymbol{P_m^{TM}(E)} =$$

$$-jk_0 Z_0 \int dV\, J_s \cdot v_h + \oint_{S_{WG}} \boldsymbol{dS\ n \cdot \left[v_h \times \mu^{-1} \nabla \times E^{inc}\right]} + \sum_m \boldsymbol{U_m^{TE}} + \sum_m \boldsymbol{U_m^{TM}}$$

with $P_m^{TE}(E) = -\gamma_m^{TE} \left(\int_{S_{WG}} dS\ v_h \cdot e_m^{TE} \right) \left(\int_{S_{WG}} dS\ e_m^{TE} \cdot E \right), \qquad P_m^{TM}(E) = \cdots$

and matrix representation (TE):

$$\boxed{P_m^{TE}(E) \ \rightarrow\ \mathbf{P}_m^{TE} \cdot \mathbf{e} = -\gamma_m^{TE} \mathbf{R}^T \cdot \mathbf{M}_m^{TE} \cdot \mathbf{R} \cdot \mathbf{e}}$$

$$\underbrace{[\mathbf{R}\,]_{ij} = \int_{S_{WG}} dS\ \varphi_i^{2D} \cdot \varphi_j^{3D}}_{\text{3D-to-2D projection matrix}}$$

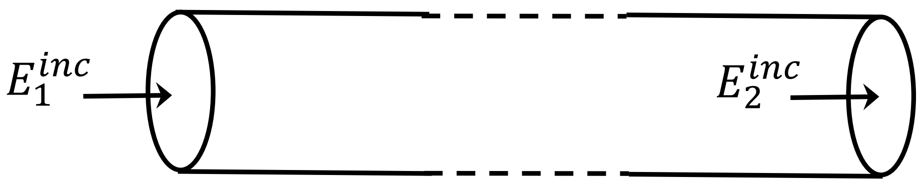
$$\boxed{\mathbf{M}_m^{TE} = \mathbf{e}_m^{TE} \otimes \mathbf{e}_m^{TE}}$$ dense modal dyadic

Frequency Domain Formulation

- Beam pipe boundary excitation
  - For an ultra-relativistic bunch (same idea for β < 1):

$$\nabla_t \cdot E^{inc} = \frac{1}{\varepsilon_0} \rho(x, y) e^{-ik_0 z_0}$$

$$\nabla \times E^{inc} = 0$$



2D-electrostatic problem at both ends of the pipe

  - Modal contribution to the RHS

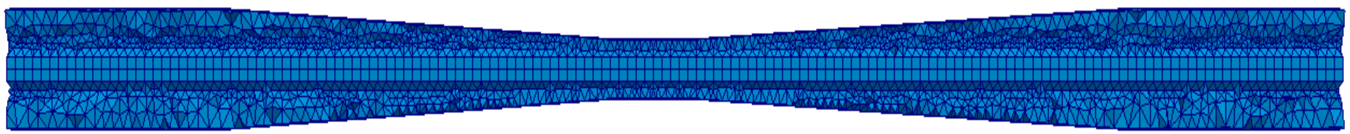
$$U_m^{TE}\left(E^{inc}\right) = -\gamma_m^{TE} \left(\int_{S_{WG}} dS\, v_h \cdot e_m^{TE} \right) \left(\int_{S_{WG}} dS\, e_m^{TE} \cdot E^{inc} \right)$$

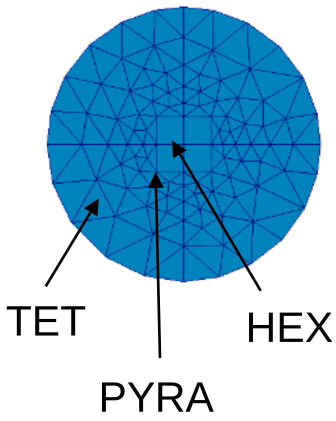
$$\boxed{U_m^{TE}\left(E^{inc}\right) \;\; \rightarrow \cdot\, \mathbf{U}_m^{TE} \cdot \mathbf{e}^{inc} = -\gamma_0^{TE} \mathbf{R}^T \cdot \mathbf{M}_m^{TE} \cdot \mathbf{R}^{2D} \cdot \mathbf{e}^{inc}}$$

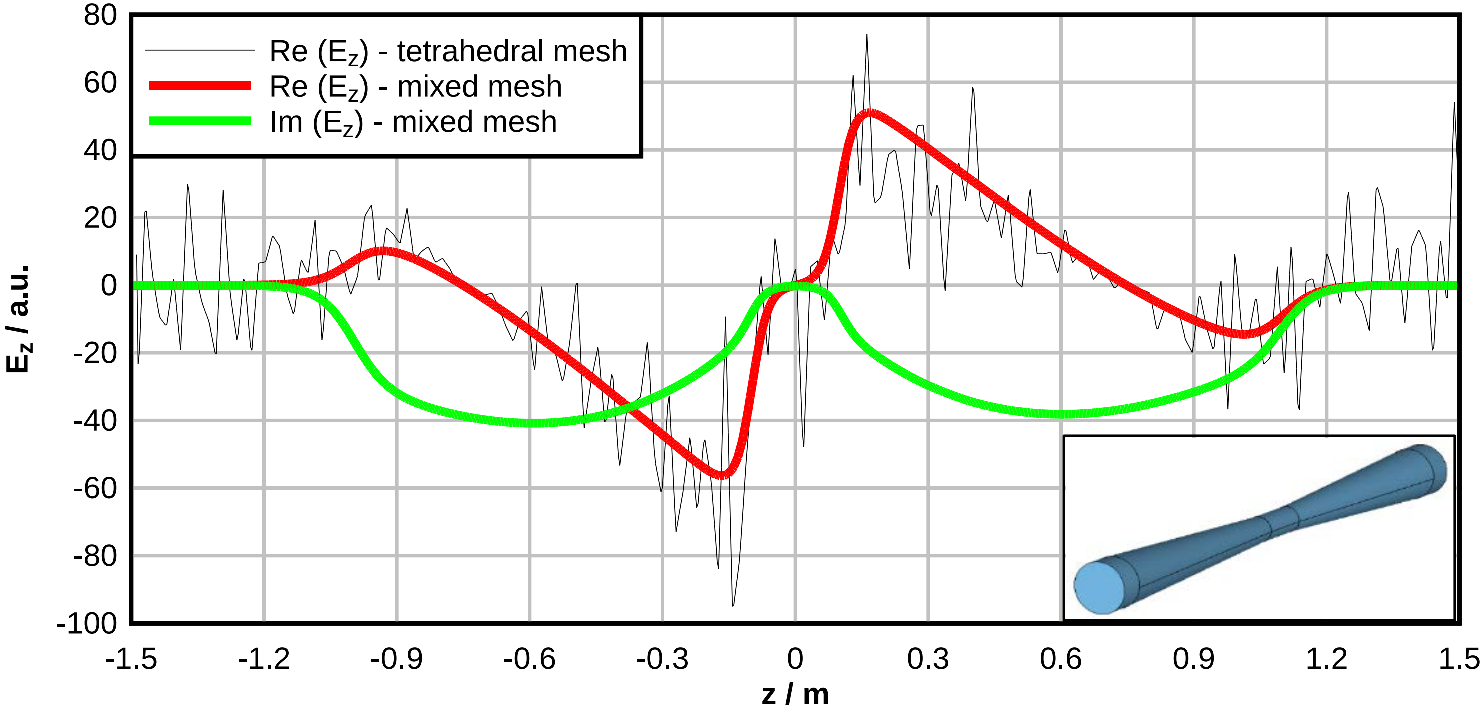
…do this for all waveguide modes supported in the pipe

Frequency Domain Formulation

- Collimator – hybrid meshes







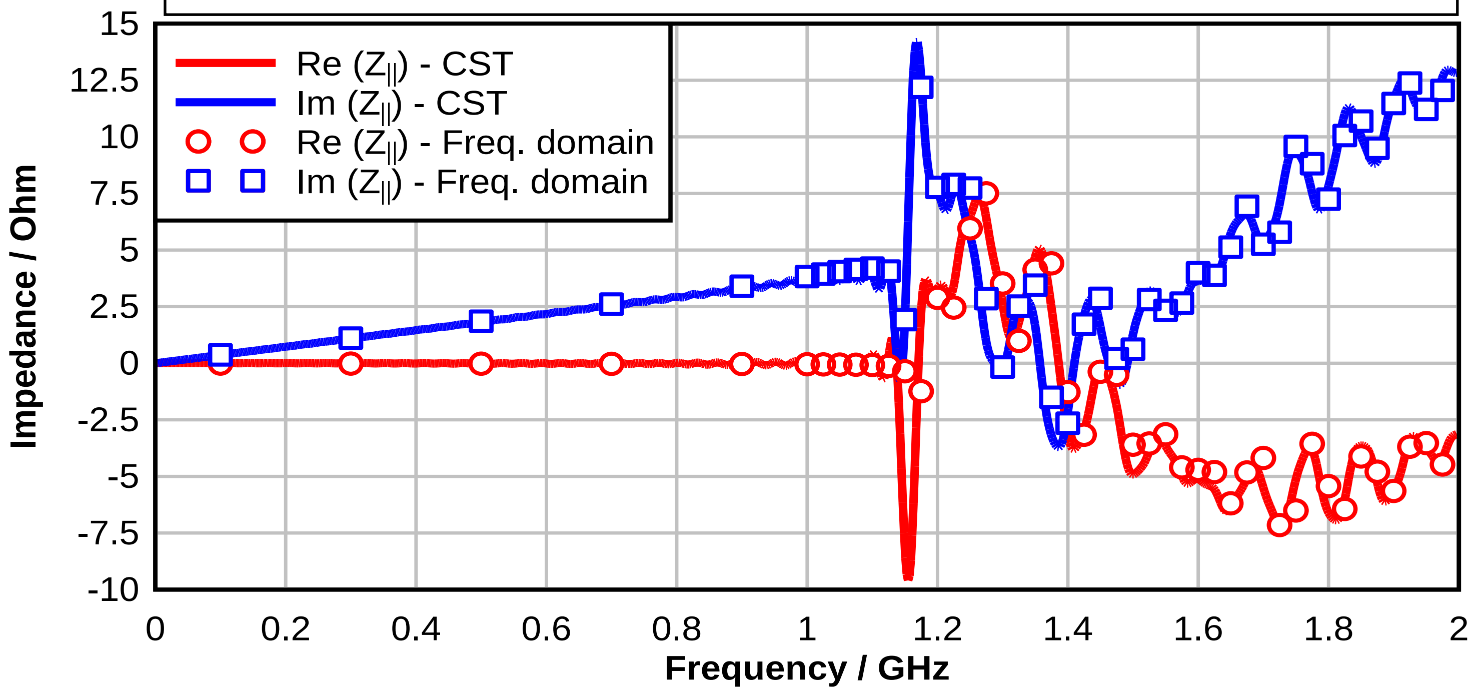
Longitudinal wakefield on axis at 100MHz

Frequency Domain Formulation

- Collimator – impedance

E_z – 1GHz

E_z – 1.5GHz

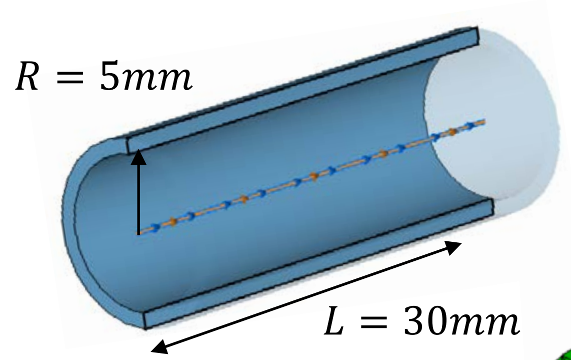


Comparison with CST PS

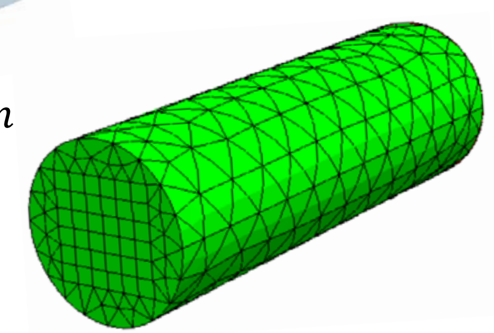
Resistive Wall Impedance

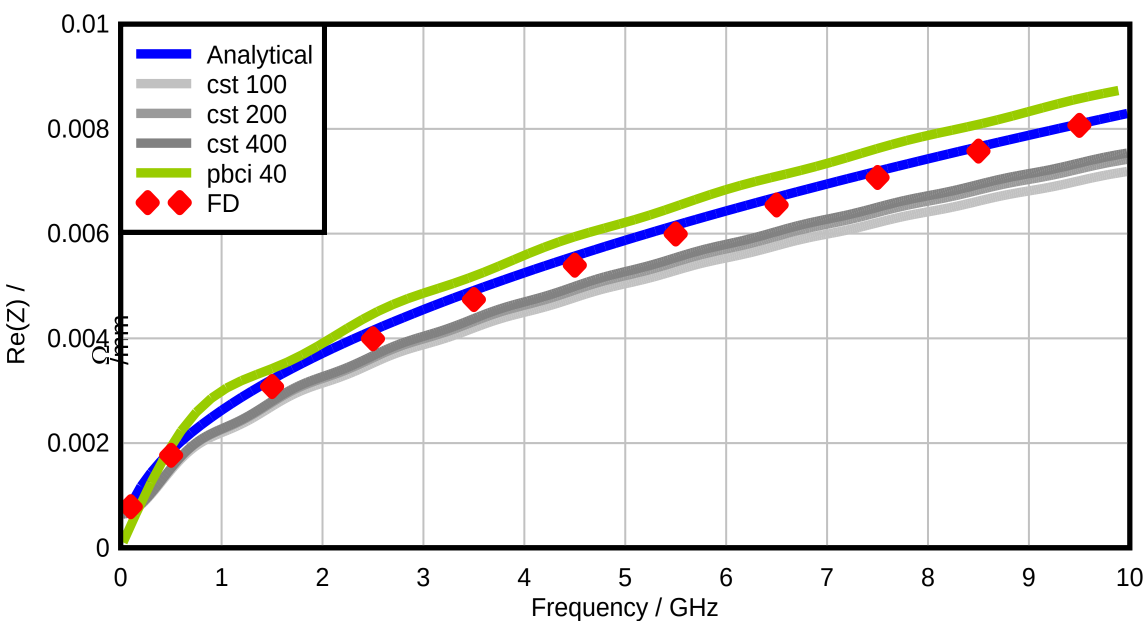
- Resistive wall round pipe
  - Analytical solution

$$Z(\omega) = L\frac{1+j}{2\pi R}\sqrt{\frac{\omega Z_0}{2c\sigma(\omega)}}$$



$\mathbf{TiAl - \sigma = 0.58MS/m}$

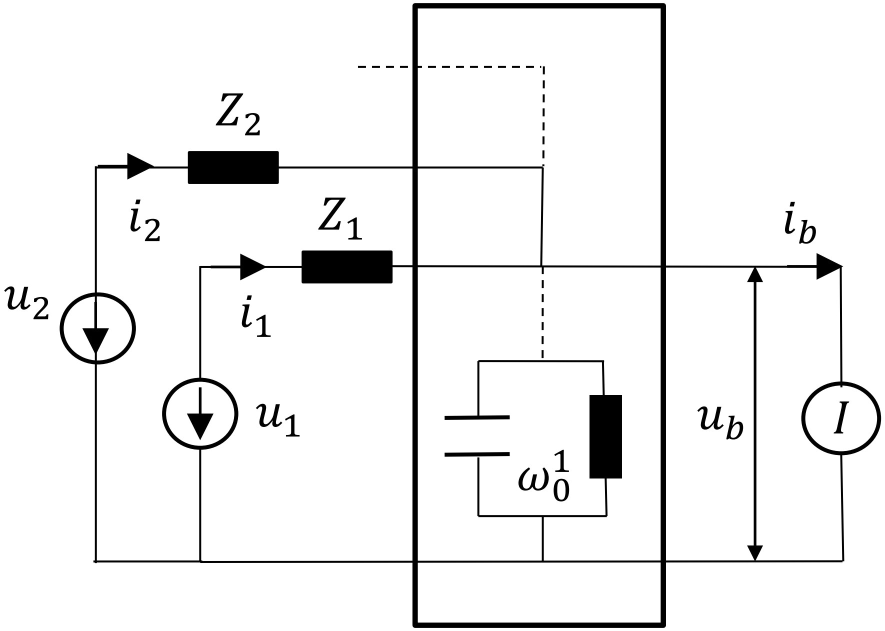


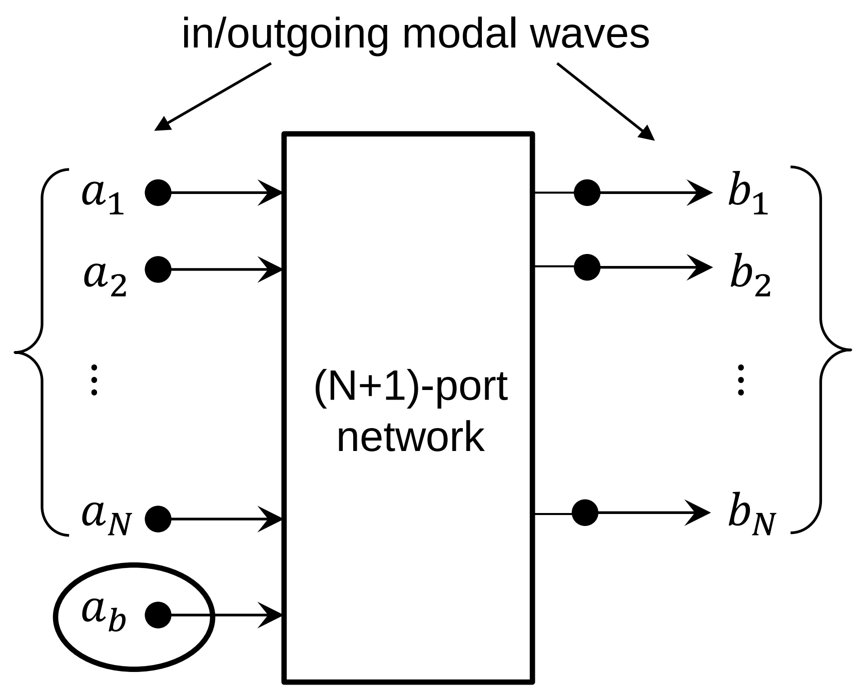


- Large mesh errors in time domain codes
- Sparse unstructured mesh
- Few evaluation points in the frequency domain

Generalized S-Matrix

- Equivalent circuit representation





$$a_b = \frac{u_b + Z_b i_b}{2\sqrt{Z_b}}, \quad b_b = 0$$

- Only synchronous field prop. in the +z direction contributes to beam voltage
- Beam as a one-way transmission line of infinite length

Generalized S-Matrix

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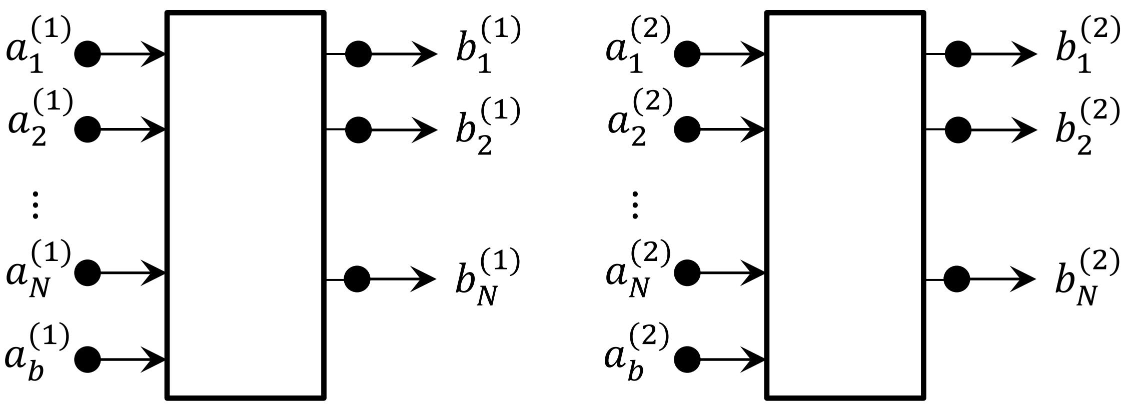
- S-Matrix / hybrid matrix representation

$$\begin{pmatrix} S & k \\ h & r \end{pmatrix}\begin{pmatrix} a_m \\ a_b \end{pmatrix} = \begin{pmatrix} b_m \\ 0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} S & \tilde{k} \\ \tilde{h} & Z_b \end{pmatrix}\begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ -u_b \end{pmatrix}$$

  - Concatenation for cascaded structures:



Matching conditions:

$$b_i^{(n)} = a_i^{(n+1)}$$

$$i_b^{(n)} = i_b^{(n-1)} e^{ik_0 L_{n-1}}$$

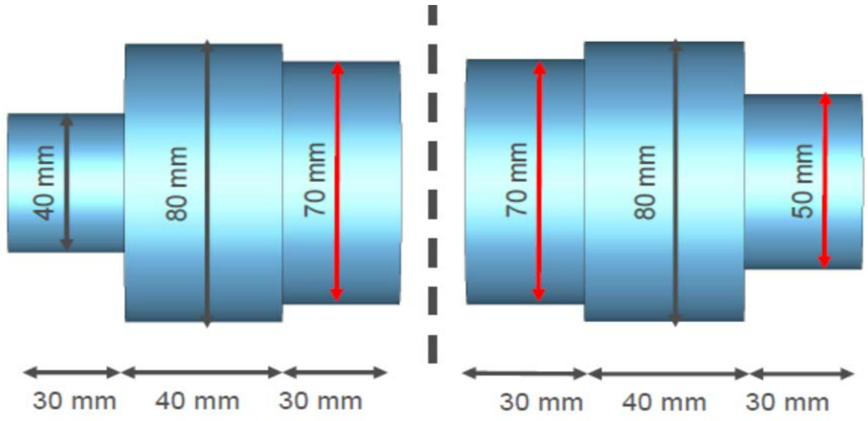
$$\sum_n u_b^{(n)} = u_b^{tot}$$

  - Reduces to total scattering matrix:

$$\boxed{\begin{pmatrix} S^{tot} & \tilde{k} \\ \tilde{h} & Z_b^{tot} \end{pmatrix}\begin{pmatrix} a_m \\ i_b \end{pmatrix} = \begin{pmatrix} b_m \\ -u_b^{tot} \end{pmatrix}}$$

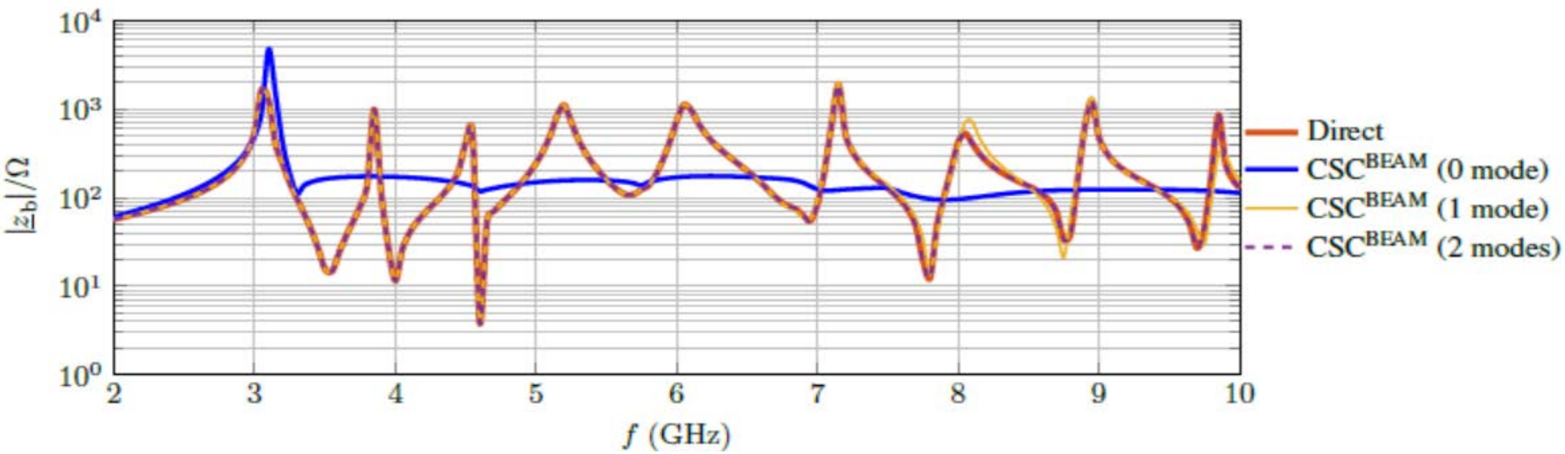
Generalized S-Matrix

- Two cavity / two mode coupling example



Cavity 1: $\tilde{S}\begin{pmatrix} a_{TM_{01}}^{(1)} \\ i_b^{(1)} \end{pmatrix} = \begin{pmatrix} b_{TM_{01}}^{(1)} \\ b_{TM_{02}}^{(1)} \\ -u_b^{(1)} \end{pmatrix}$

Cavity 2: $\tilde{S}\begin{pmatrix} a_{TM_{01}}^{(2)} \\ a_{TM_{02}}^{(2)} \\ -i_b^{(2)} \end{pmatrix} = \begin{pmatrix} b_{TM_{01}}^{(2)} \\ -u_b^{(2)} \end{pmatrix}$



Generalized S-Matrix

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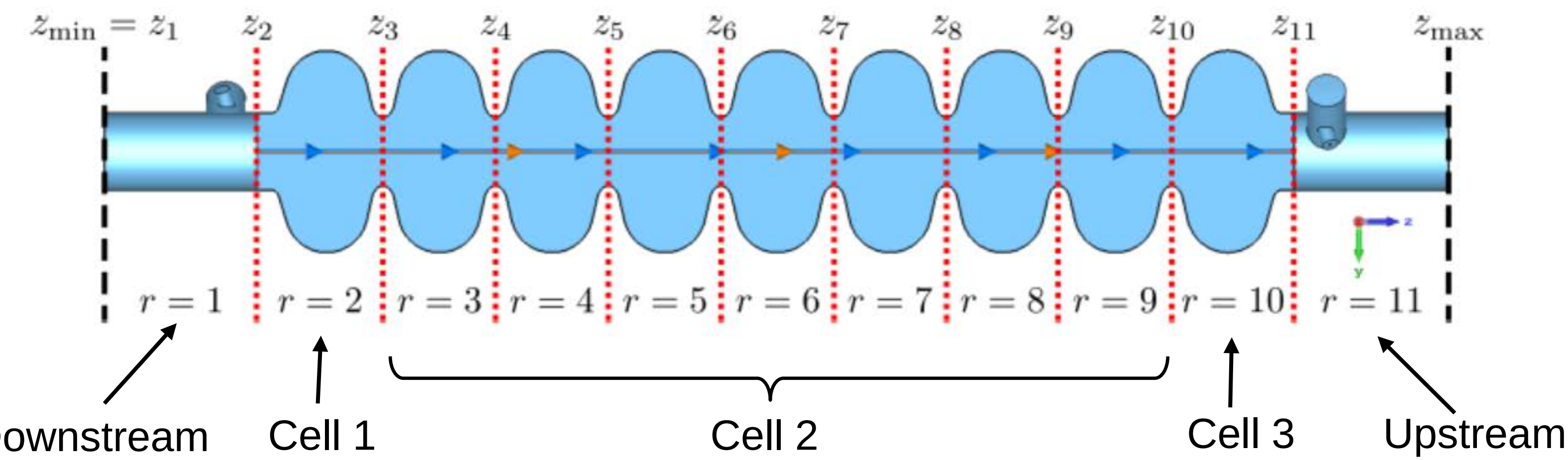
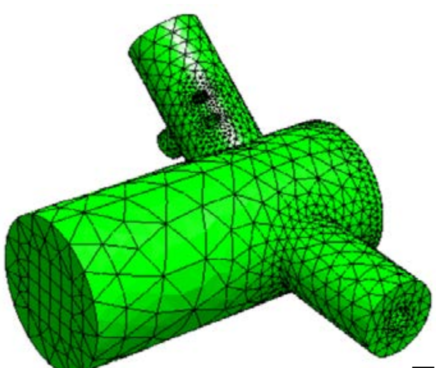
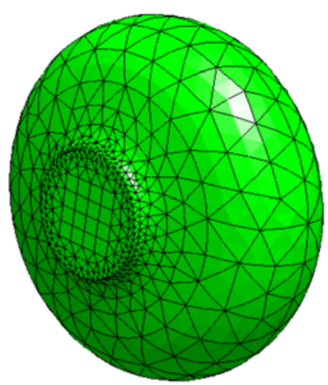
- Tesla 1.3GHz cavity



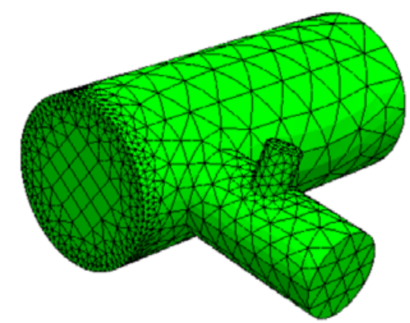


15 TE-Modes
$(f_{max} = 8.2\text{GHz})$
15 TM-Modes
$(f_{max} = 10.6\text{GHz})$

TEM, …

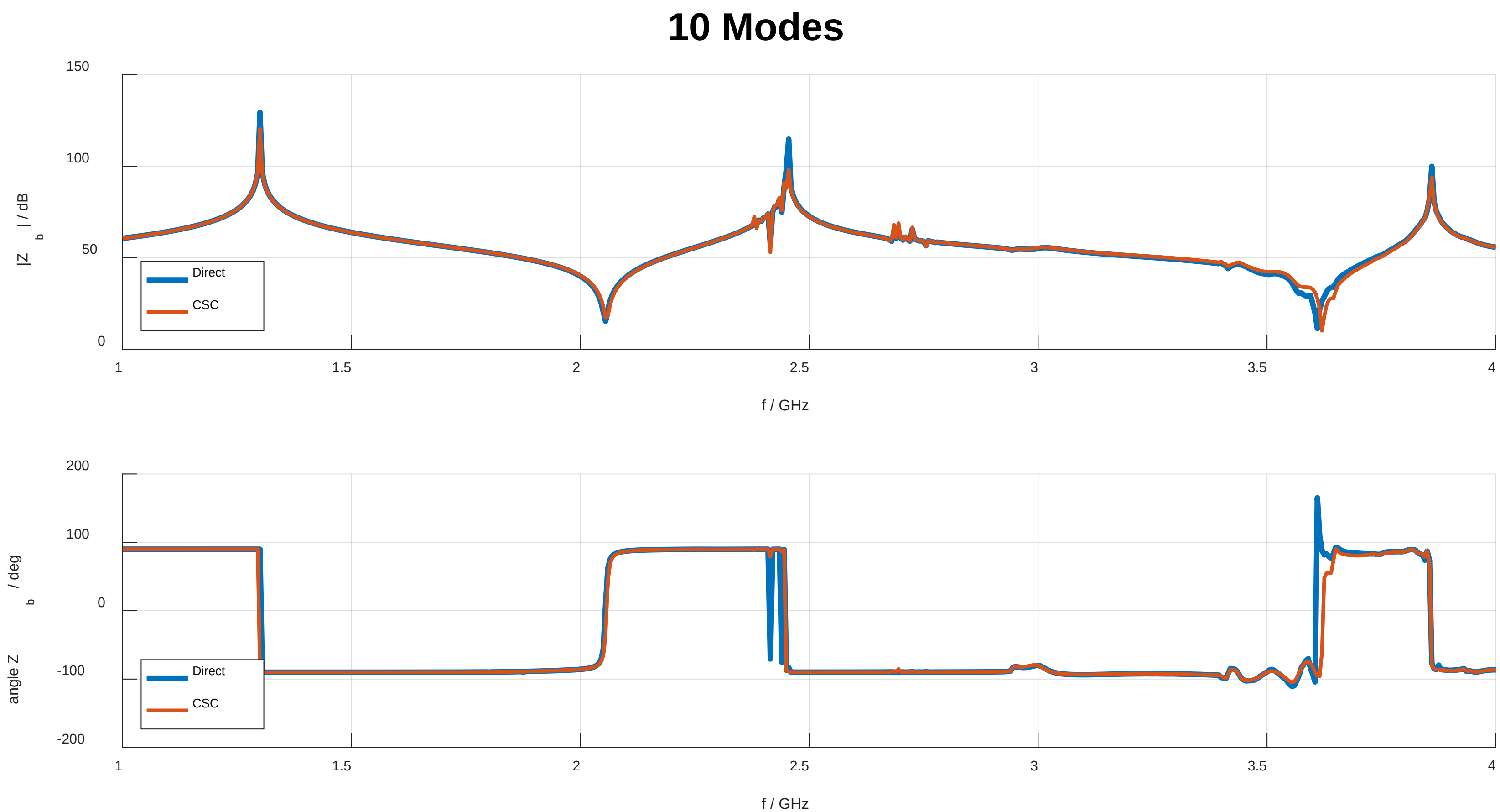


15 TE-Modes
$(f_{max} = 8.2\text{GHz})$
15 TM-Modes
$(f_{max} = 10.6\text{GHz})$



Generalized S-Matrix

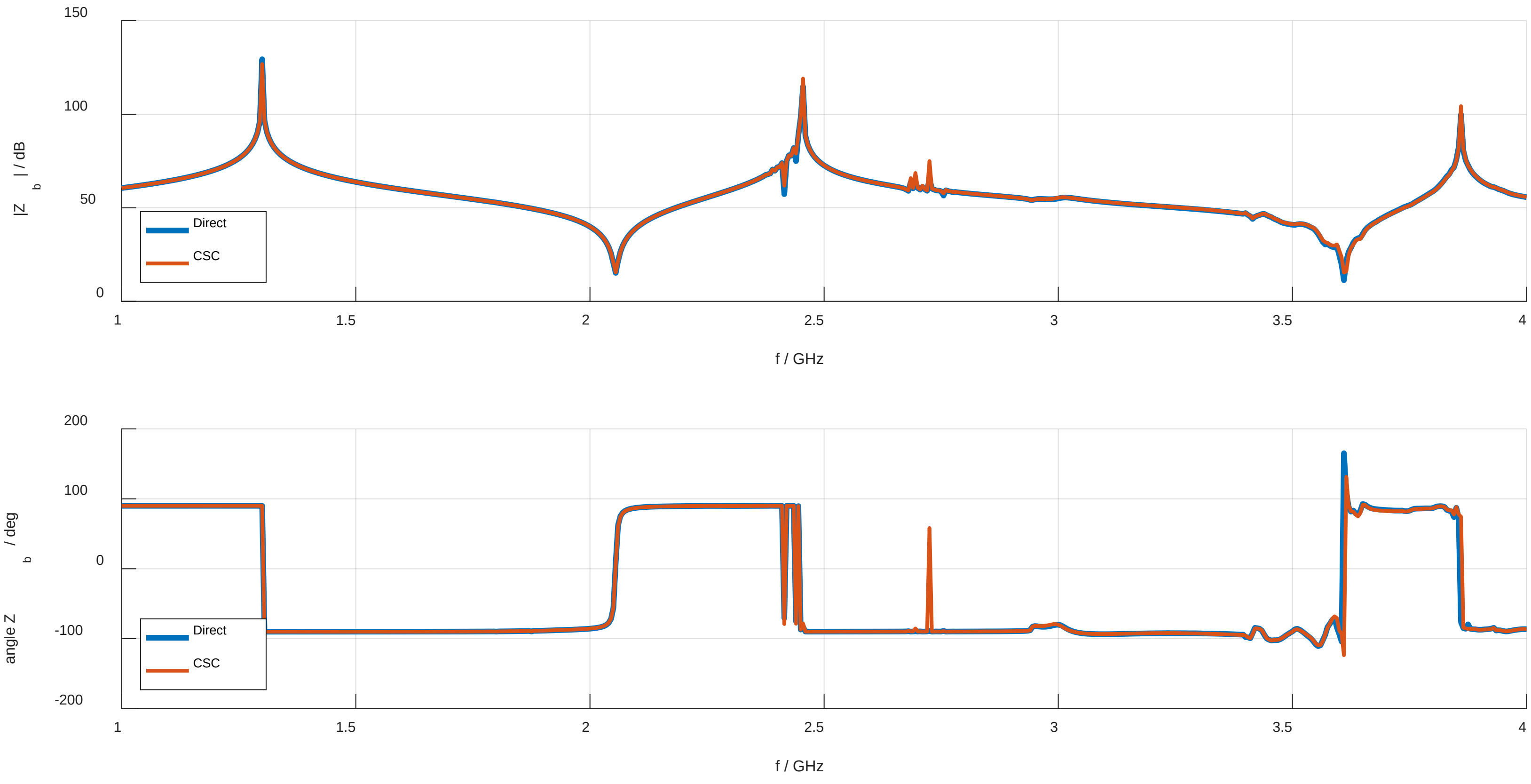
- Tesla 1.3GHz cavity



Generalized S-Matrix

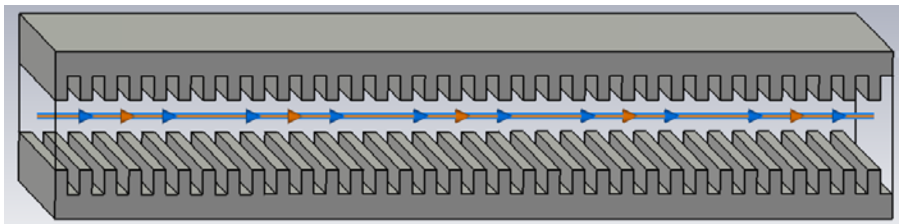
- Tesla 1.3GHz cavity

30 Modes



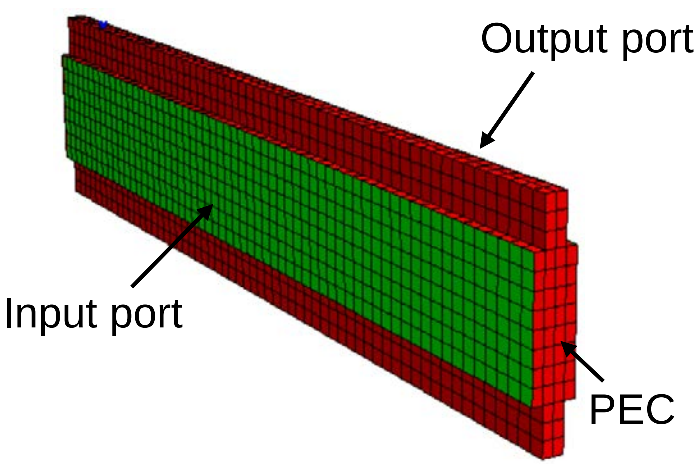
Generalized S-Matrix

- Dechirper (LCLS, E-XFEL)



  - Single period computation layout

Parameter	Value
Depth, *h*	0.5mm
Gap, *t*	0.25mm
Period, *p*	0.5mm
Half aperture, *a*	0.7mm
Half width, *w*	6mm
Length, *L*	2m



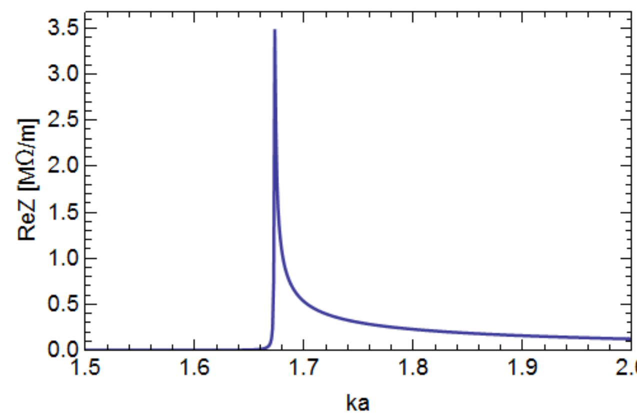
No. elements: 2640

On each port:

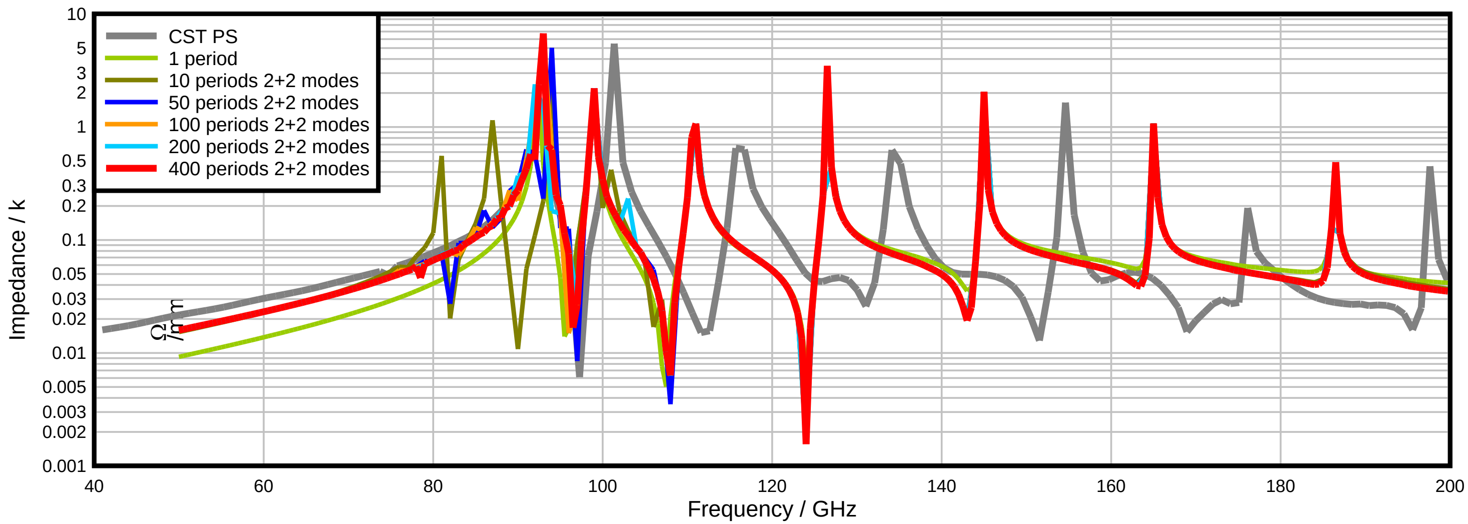
100 TE-Modes $(f_{max} = 375\text{Ghz})$

100 TM-Modes $(f_{max} = 448\text{Ghz})$

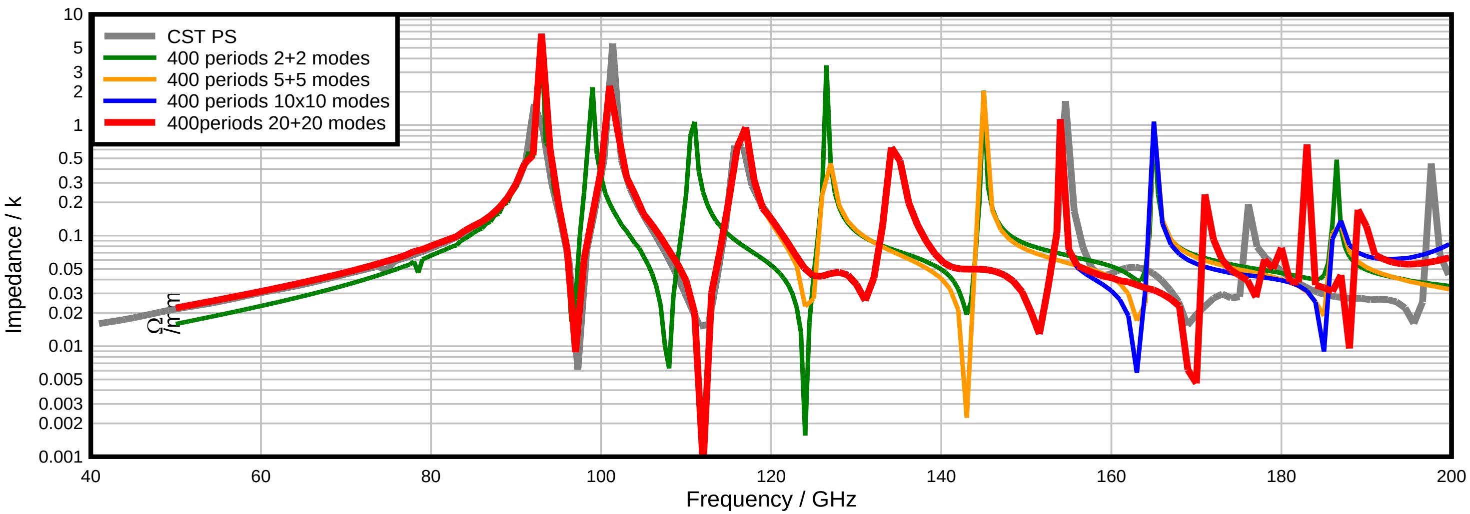
Analytical (Bane et al.)



Generalized S-Matrix



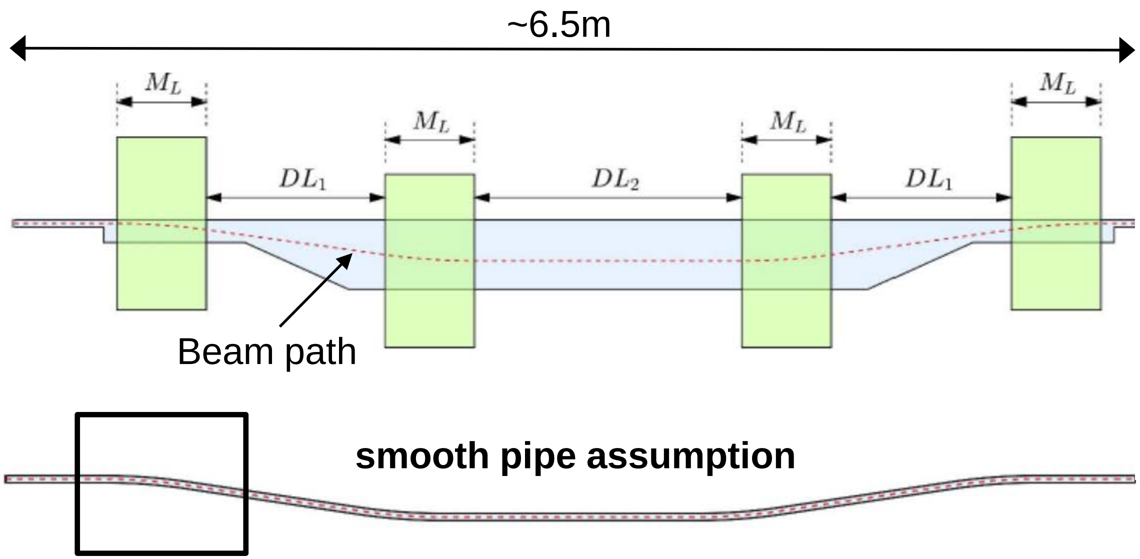
Impedance vs. no. periods

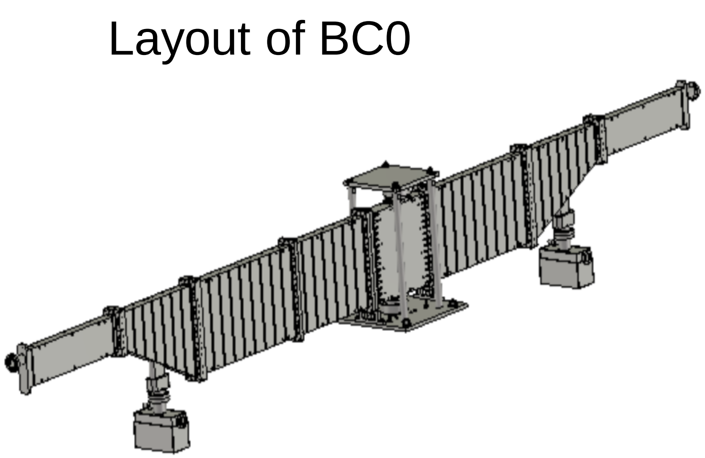


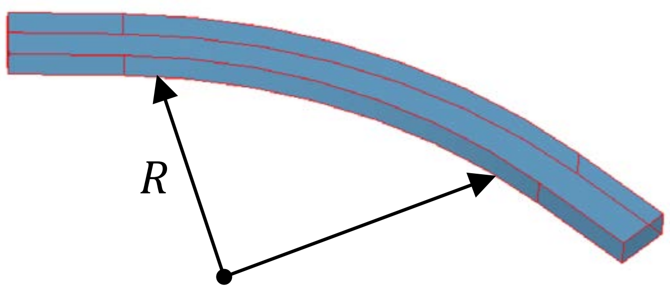
Impedance vs. coupling modes

Curved beam trajectories

- E-XFEL BC0





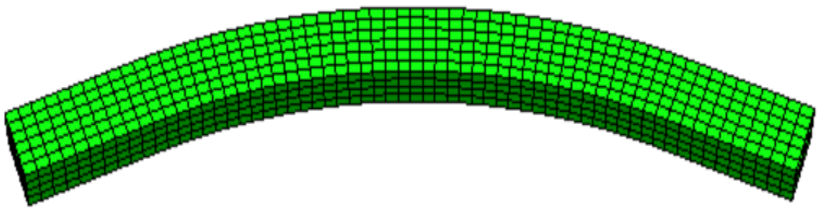


1st BC0 chicane:

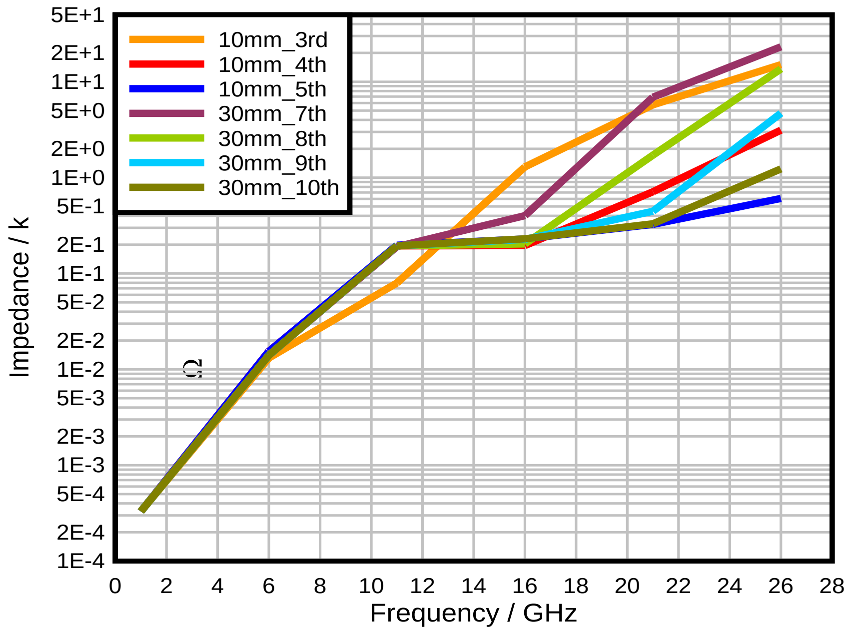
- Bending radius: 1m
- Bending angle: 45°
- Length: ~1.2m
- Pipe dimensions: 100x50mm

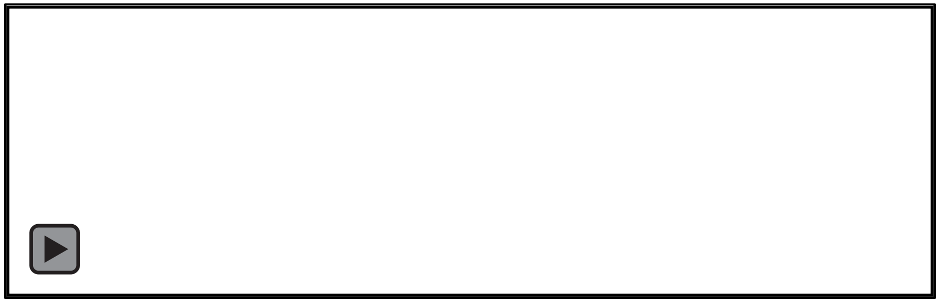
Curved beam trajectories

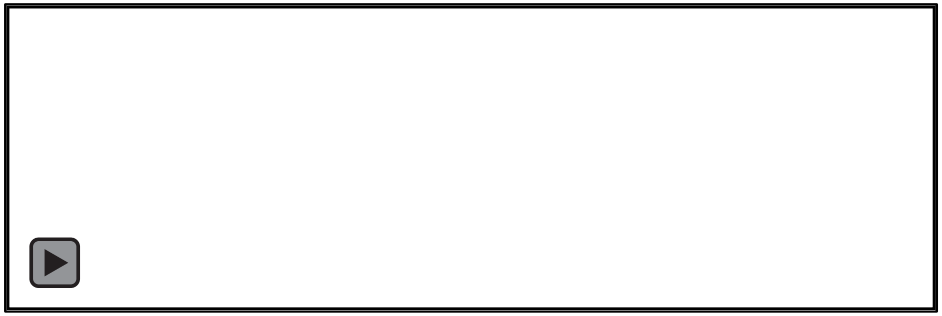
extruded hex-mesh



num. convergence







Summary & Conclusions

- The frequency domain approach
  - **Fills the gap for some important wakefield/impedance problems**
    - Complicated chamber geometry, long range / low frequency fields, resistive, rough surfaces, dispersive materials, beam signals on waveguide openings
  - **FEM Frequency domain formulation**
    - Beam port boundary conditions
    - Mixed mesh discretization
  - **Concatenation using generalized S-Matrix formulation**
    - Efficient /accurate impedance computation of large cavity chains
    - Periodic structures (dechirper)
  - **Impedance (+wall losses/radiation) computation for curved beam trajectories**
  - **Limitation: huge size of discrete problem for ultra-high frequencies**
    - Domain decomposition
    - Parallel multigrid solvers
    - Fast frequency sweeps and spectral evaluation by model order reduction,…

Thank You for your attention