Synchronous acceleration with tapered dielectric-lined waveguides

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Acceleration of non-relativistic particles in high frequency structures

What is the challenge?

Energy gain per rf period for a constant phase

$$\Delta\gamma = \int_0^{kz=2\pi} \frac{eE(z,t)}{mc^2} dz = \int_0^{kz=2\pi} \frac{e}{mc^2} E_0 \left(\sin(\phi) + \underbrace{\sin(\phi + 2kz)}_{\text{only in standing wave structures, does not contribute}} \right) dz$$

$$\Delta\gamma = 2\pi \frac{eE_0}{mc^2 k} \sin(\phi) = 2\pi\alpha \sin(\phi)$$

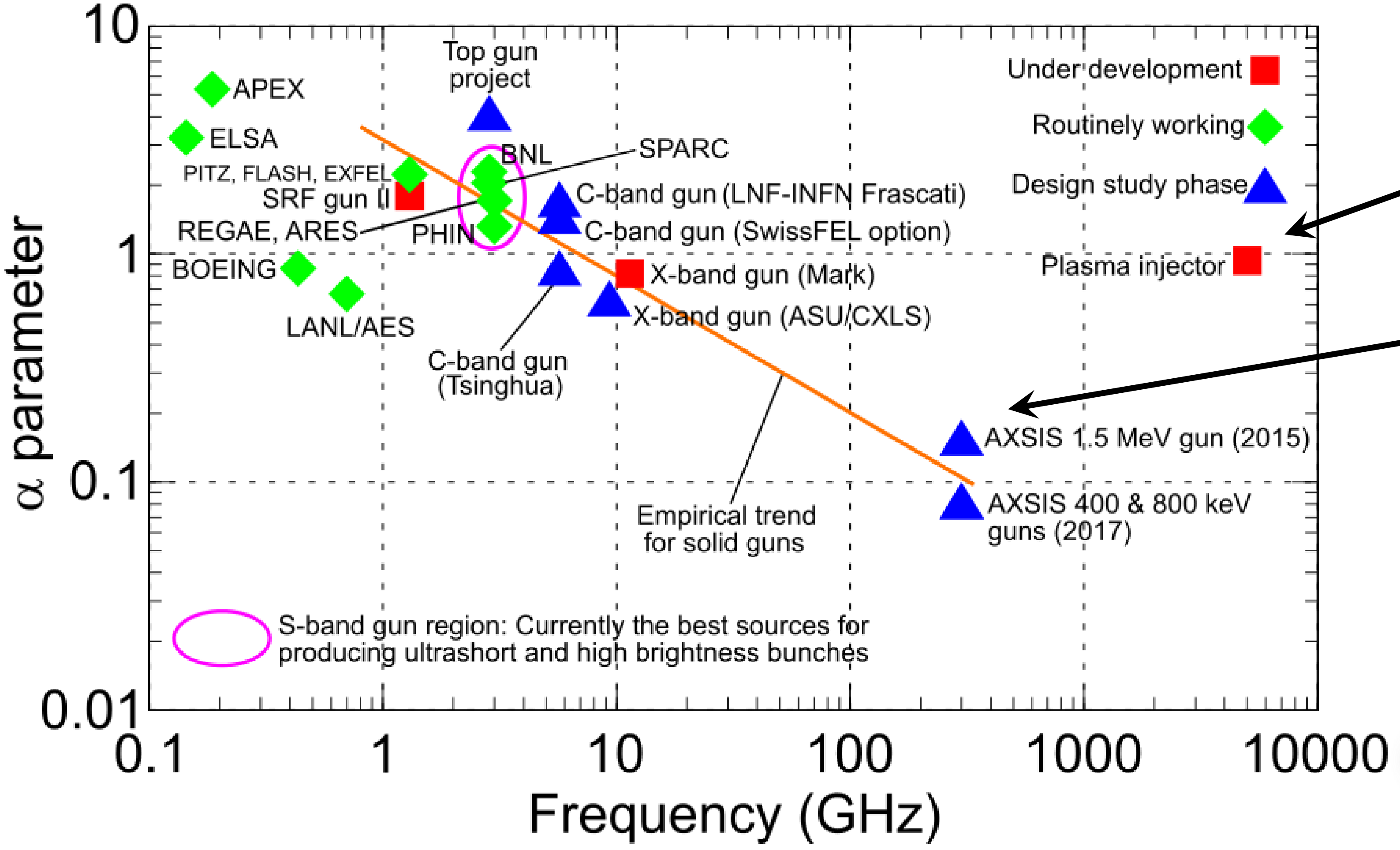
The normalized vector potential $\alpha = \frac{eE_0}{mc^2 k}$ describes the longitudinal beam dynamics independent of the rf wavelength.

For $\alpha > 1$ a particles starting at rest reaches relativistic energies within one rf period.

For $\alpha < 1$ phase slippage is significant. Efficient acceleration requires matching of the phase velocity of the wave to the (increasing) velocity of the particle.

The normalized vector potential shrinks with increasing frequency

…because the gradient is not increased as much as the wavelength decrease



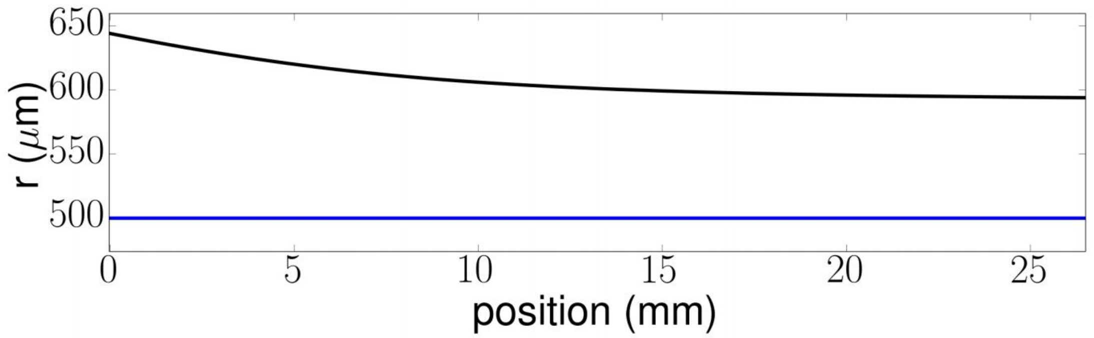
The exception: Plasma accelerators break the trend

THz-Projects

optical wavelength (ACHIP)

Th. Vinatier et al. EAAC 2017, NIM A 909 (2018) 185–192

Synchronous acceleration in a tapered dielectric waveguide

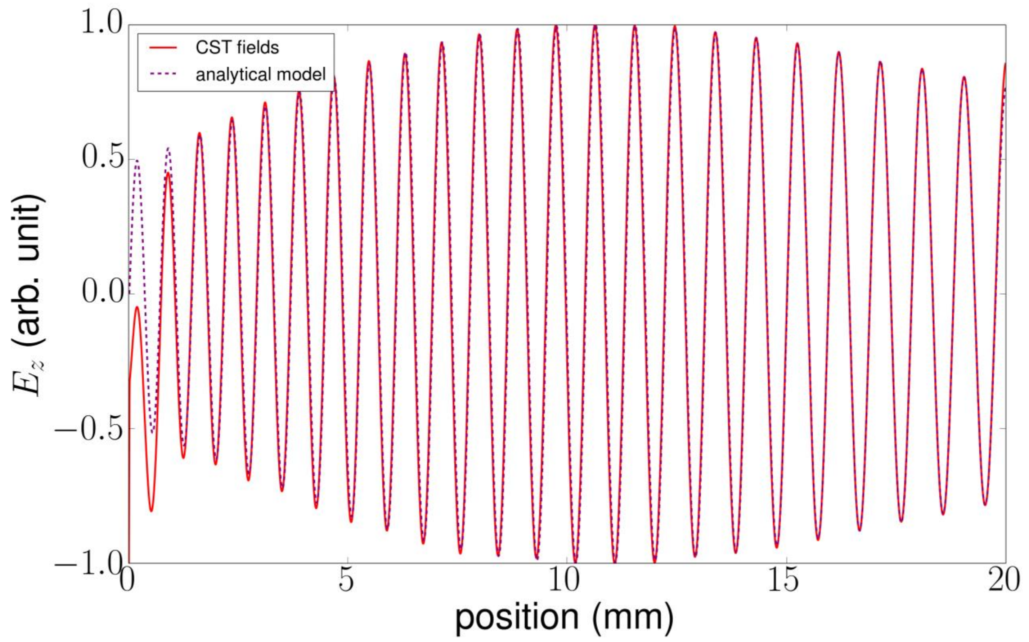


outer radius

quartz, $\varepsilon_r = 4.41$

inner radius

vacuum

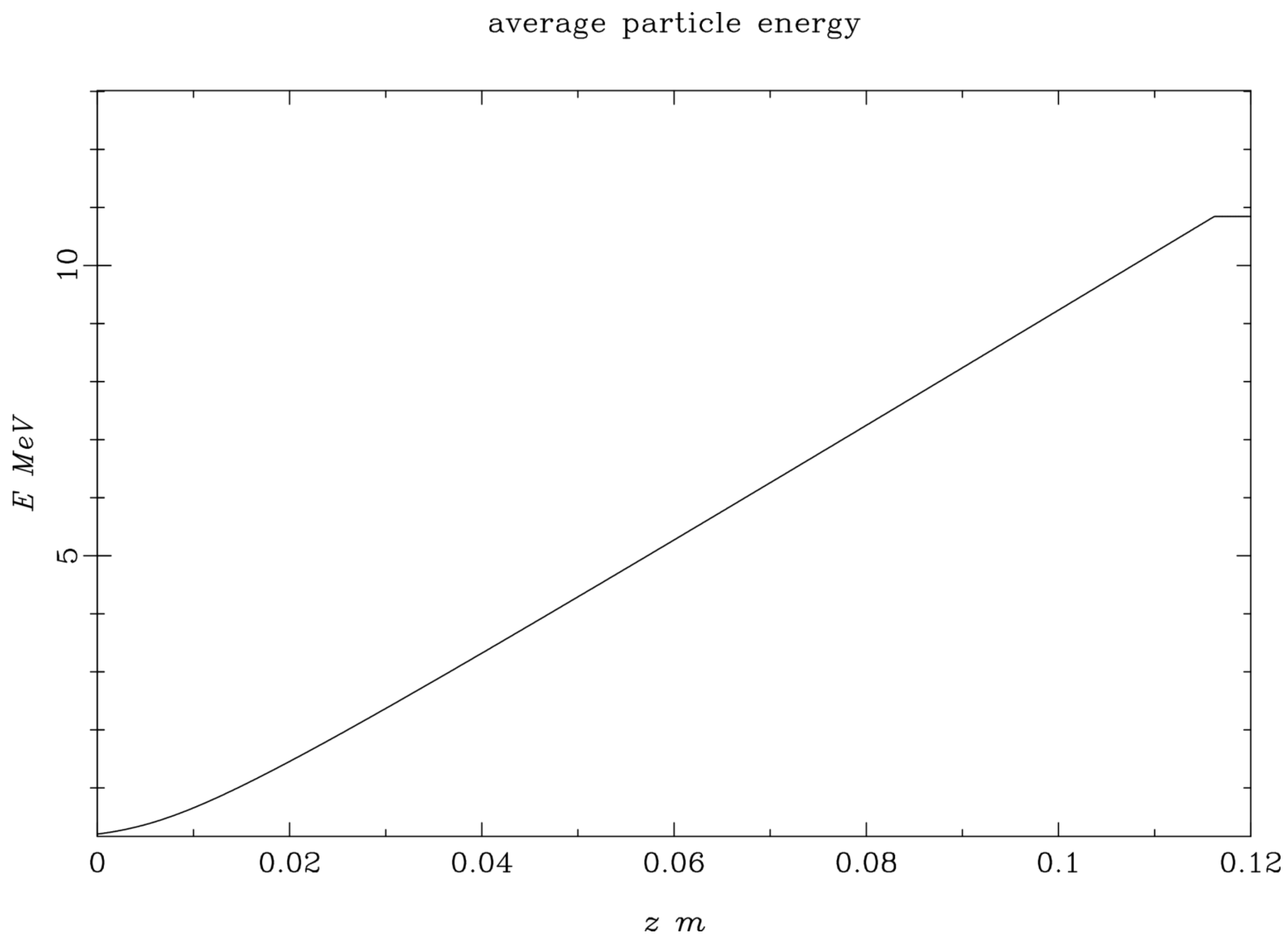


Generalized TM_{01} mode description:

$$E_z = E_0 I_0 \left(r k_r \right) \sin\left(\omega t - \int_0^z k_z dz + \varphi \right)$$

E_0, k_r and k_z are functions of the longitudinal coordinate

Synchronous acceleration in a tapered dielectric waveguide



Example: Energy gain in a tapered dielectric structure

IT WORKS:

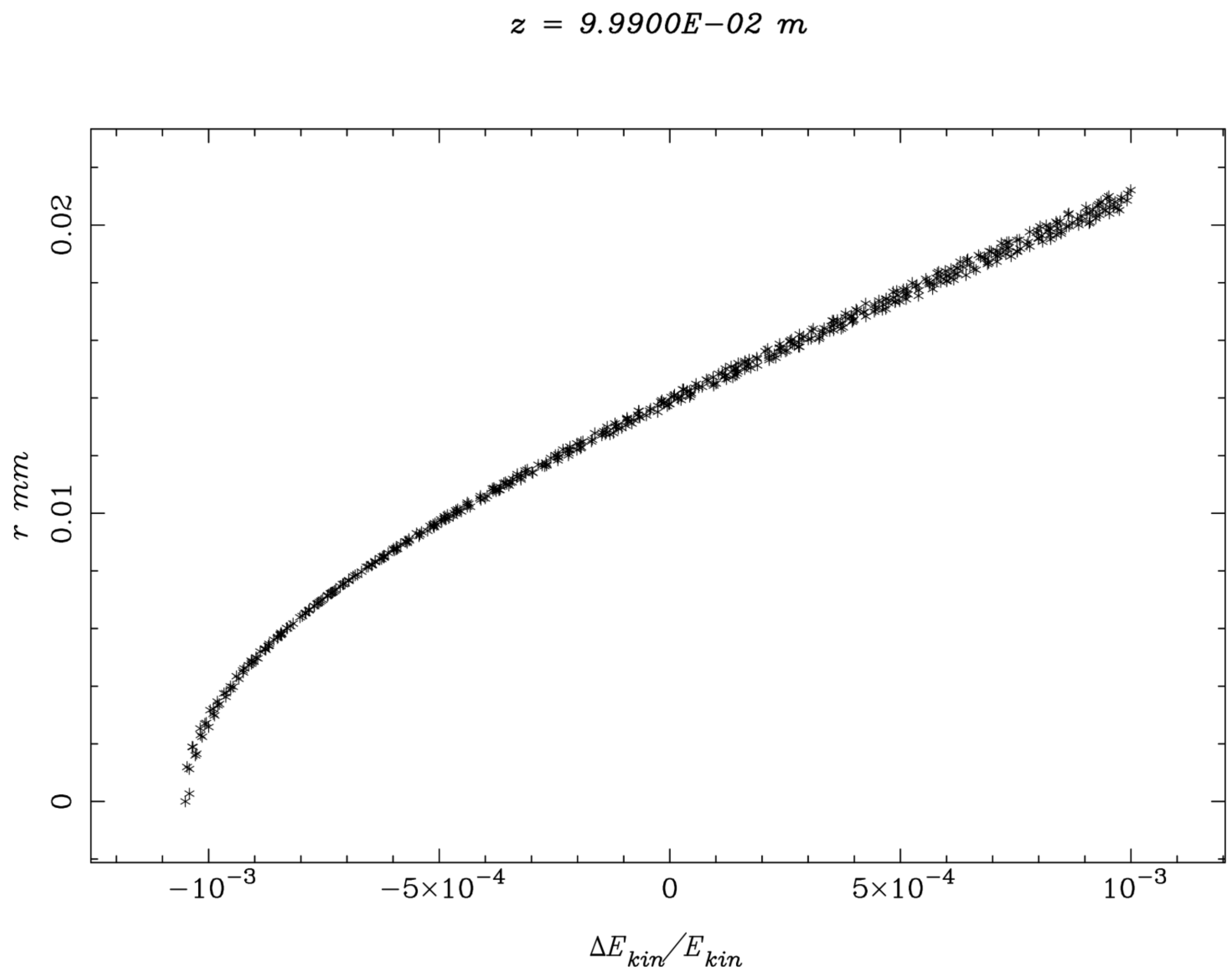
The electron stays on a constant phase and thus gains continuously energy.

BUT:

The fields are nonlinear; not due to the tapering but due to the matching to velocities below speed of light:

THIS IS FUNDAMENTAL!

Synchronous acceleration in a tapered dielectric waveguide



Particles at large radii see higher fields…

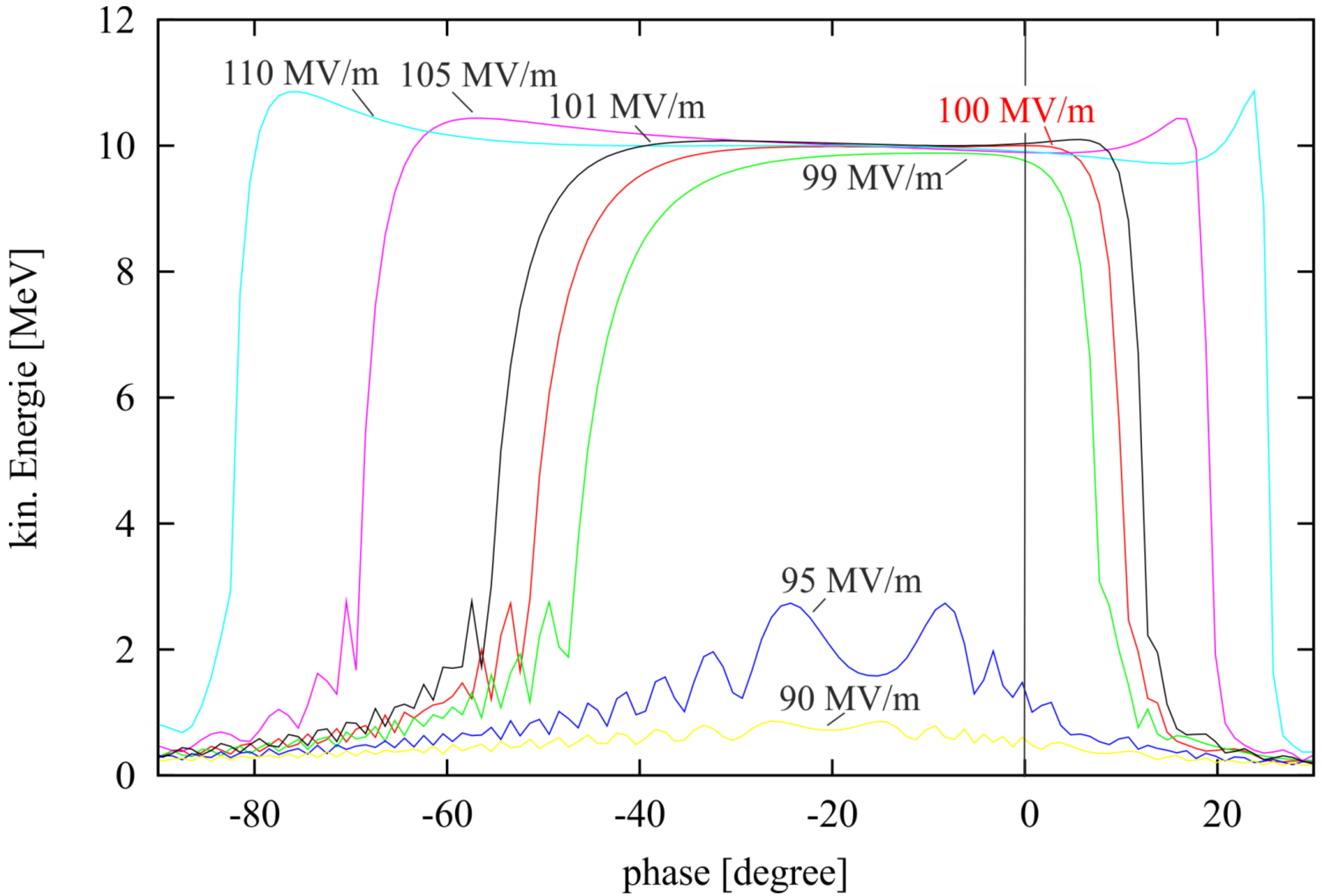
First order approximations:

$$F_z = eE_0 \sin(\varphi)\left(1 + \frac{{k_r}^2 r^2}{4}\right)$$

$$F_r = eE_0 \cos(\varphi)\frac{k_0}{\gamma^2 \beta}\left(\frac{r}{2} + \frac{{k_r}^2 r^3}{16}\right)$$

- the beam size has to be small in comparison to the wavelength
- higher initial energies are beneficial

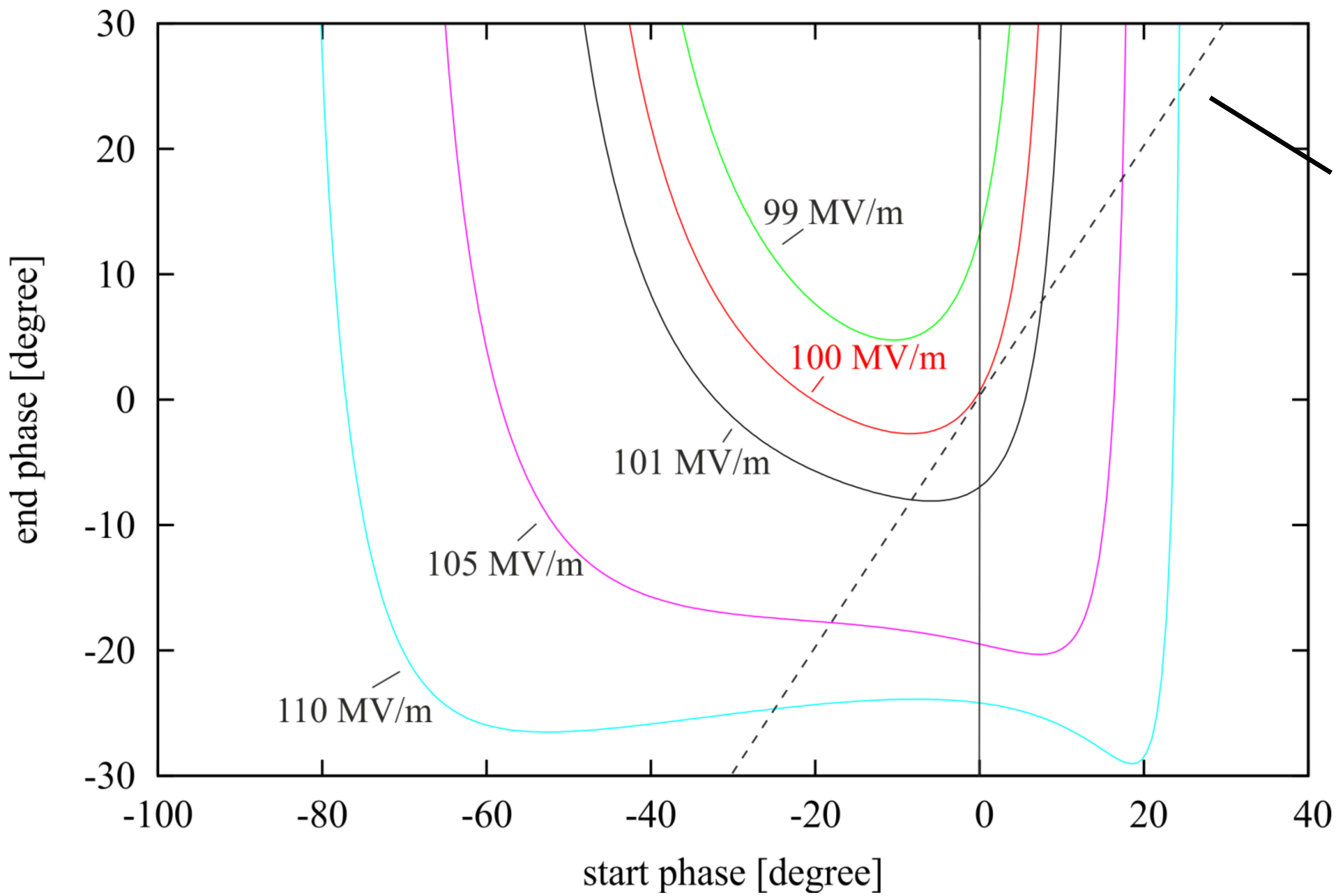
Stable operation when slightly over-powered…



Energy gain vs. start phase for different operation gradients

- structure is matched for a gradient of 100 MV/m
- initial energy 100 keV, 10 cm DLW
- broad phase acceptance when the gradient is somewhat higher than the design gradient

…. strong (on-axis) bunch compression when slightly over-powered



End phase vs. start phase for different operation gradients

broken line: end phase = start phase, i.e. no phase slippage

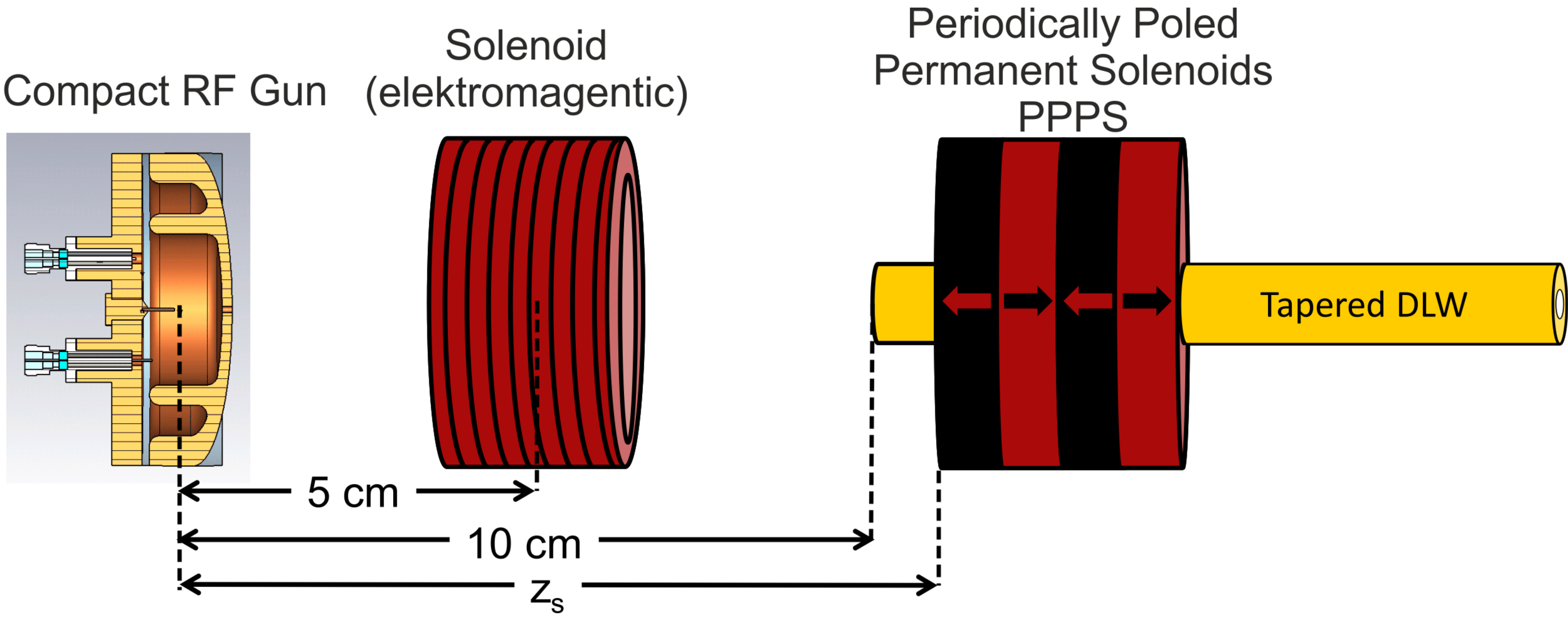
The line is crossed at

$$\varphi_0 = \pm \operatorname{acos} {}^{1}\!/_{\eta}$$

with

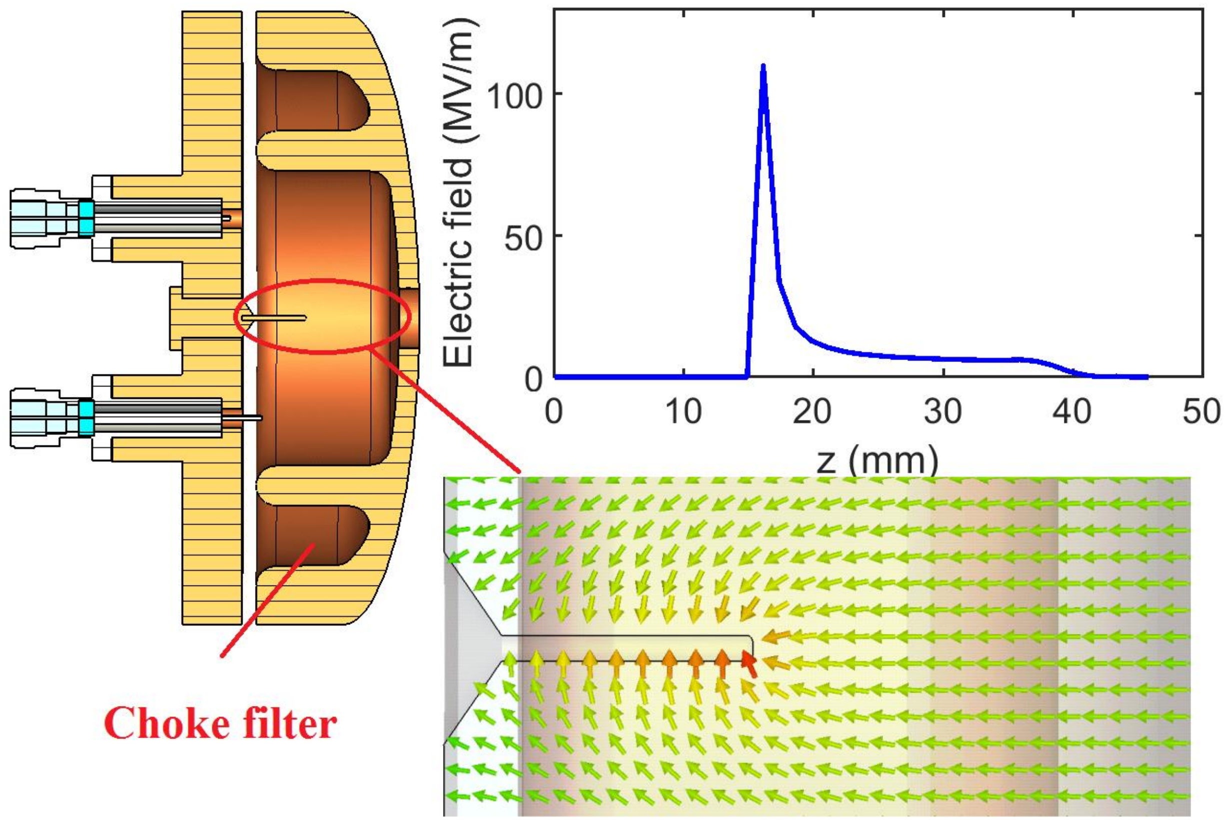
$$\eta = {}^{E}\!/_{E_m}$$

Schematic layout of compact THz accelerator



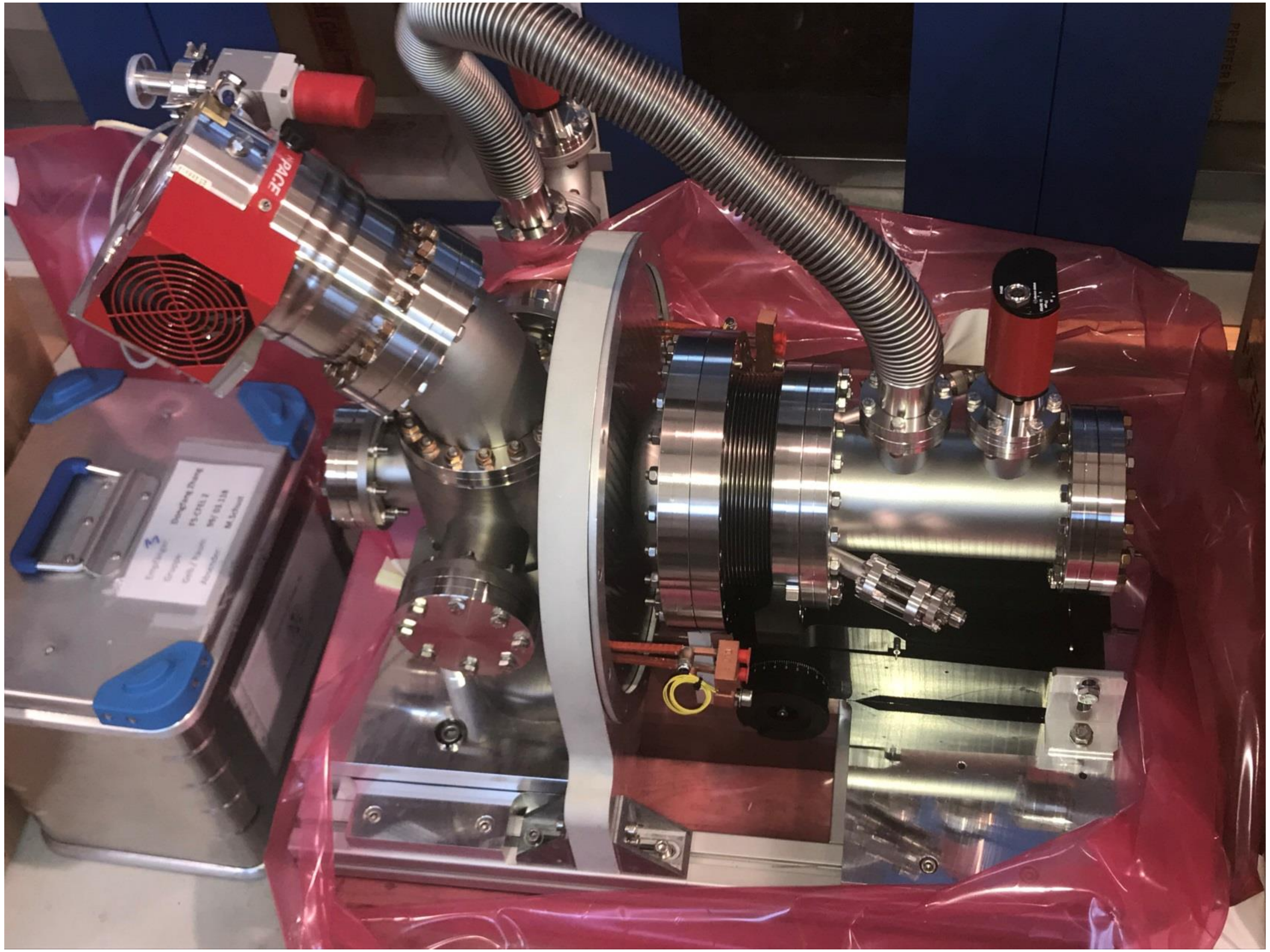
Compact RF Gun

design and construction by group of Franz Kaertner

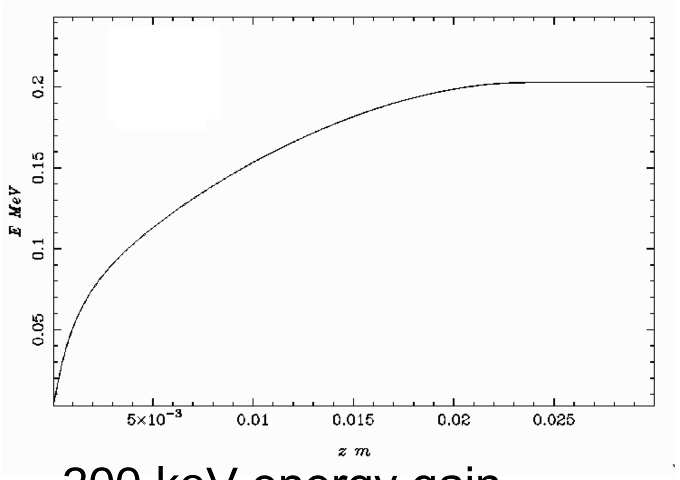


- 3 GHz single cell cavity with cathode tip and choke mode filter
- driven by a compact solid state amplifier (10 kW)
- high field (100 MV/m) on the cathode but low end energy (150 – 200 keV)

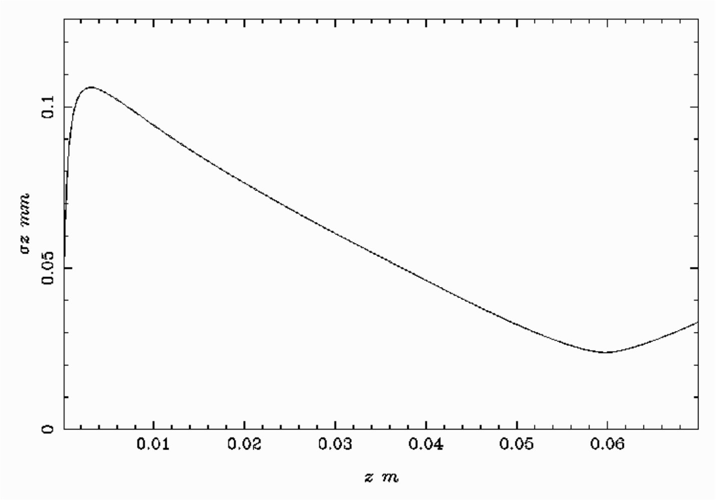
Prototype of the Compact RF Gun



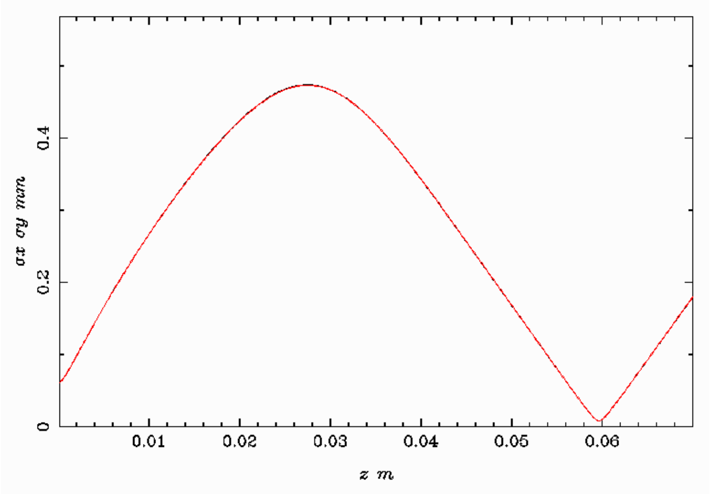
Simulation results for the Compact Gun



200 keV energy gain…



…25 μm bunch length….



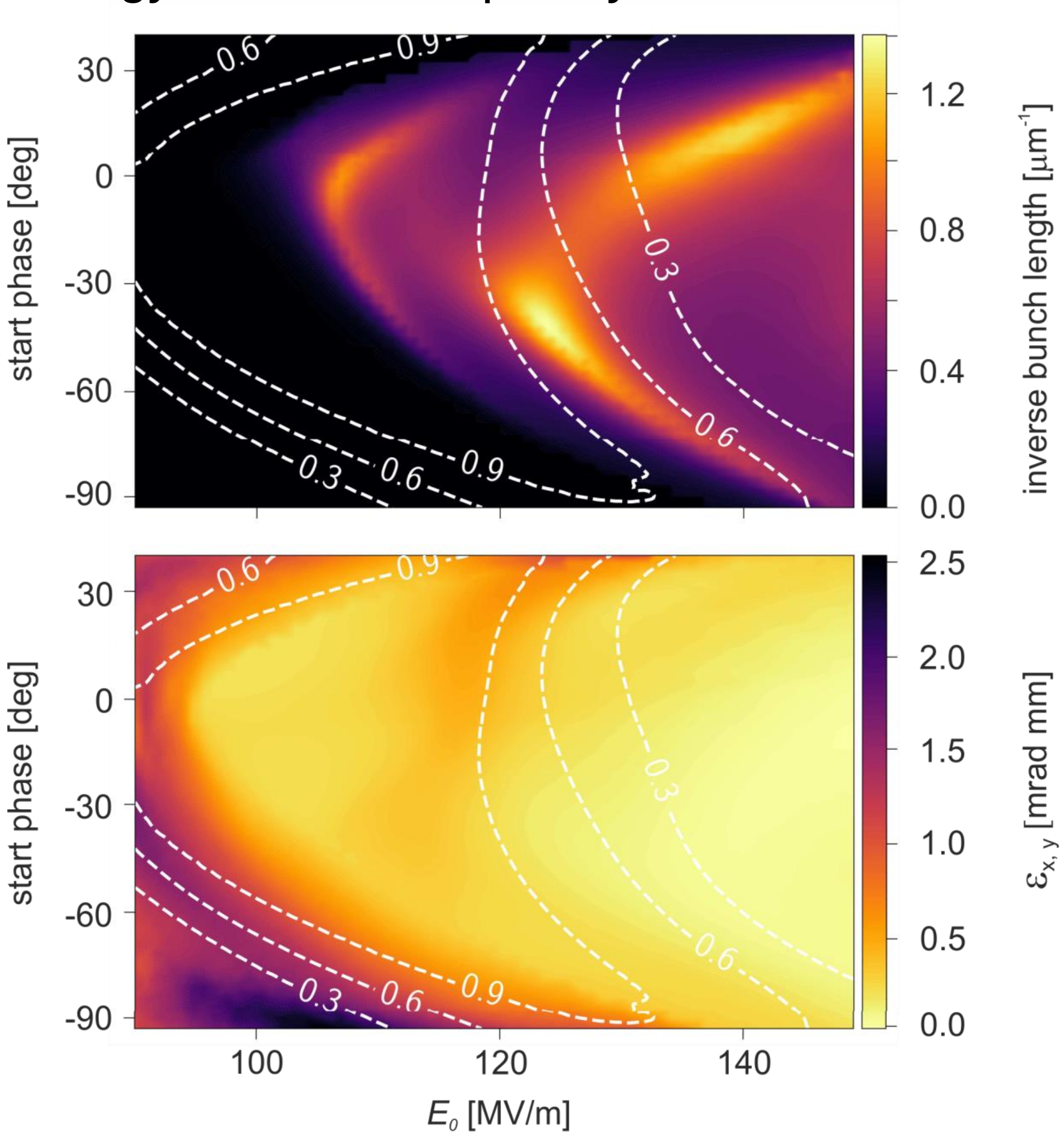
…less than 10 μm beam size

- 150 fC bunch
- 200 keV energy gain
- Negative phase injection → bunch compression
- Transverse emittance: 0.1 mm mrad

courtesy Moien Fakhari

Simulation results Gun & DLW & Focusing

initial energy 200 keV, frequency of the DLW 300 GHz, matched gradient 100 MV/m, α = 0.03



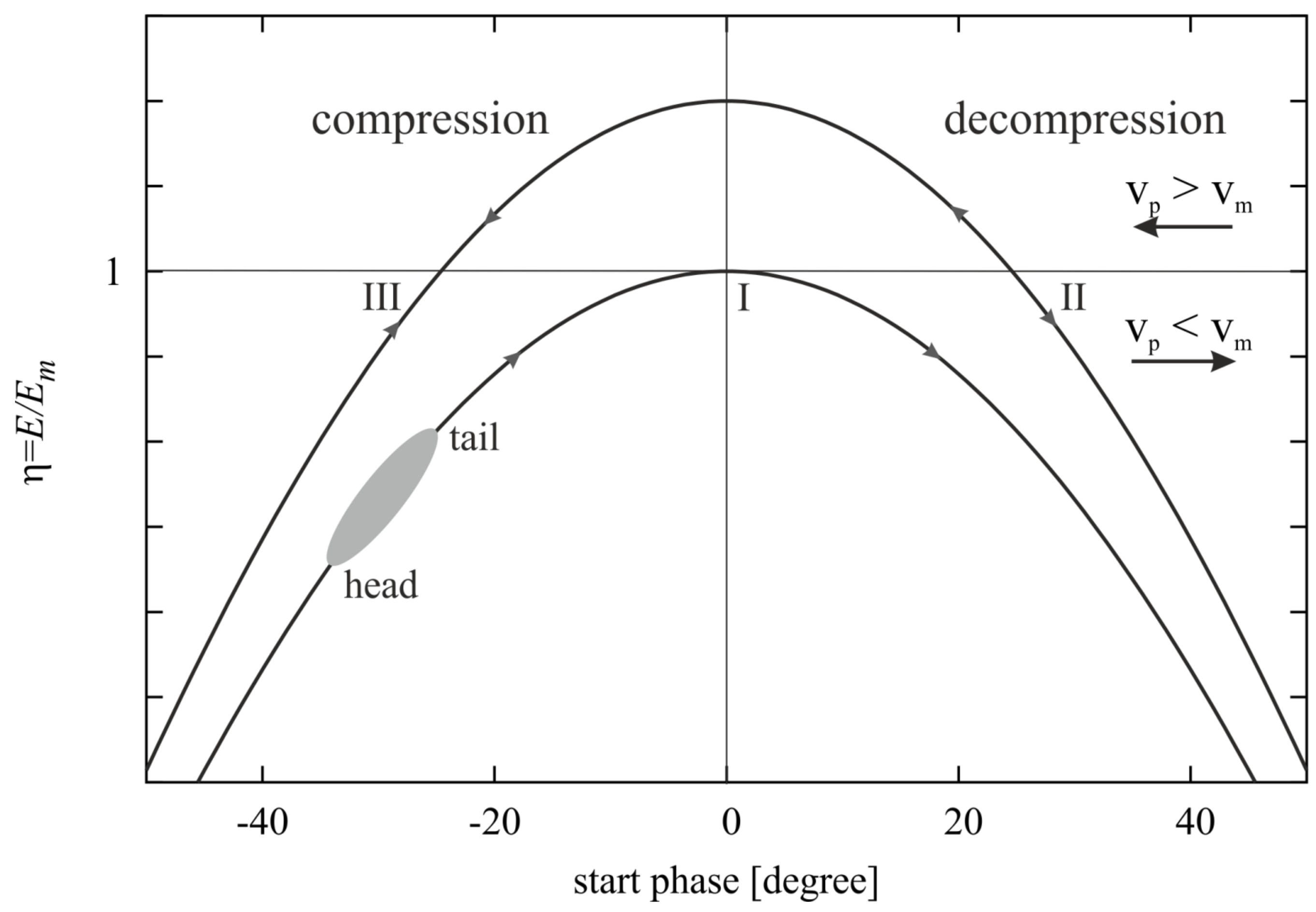
Inverse bunch length (top) and transverse emittance (bottom) as function of gradient and start phase.

White contour lines show the charge transmission.

Parameters for shortest bunch length:

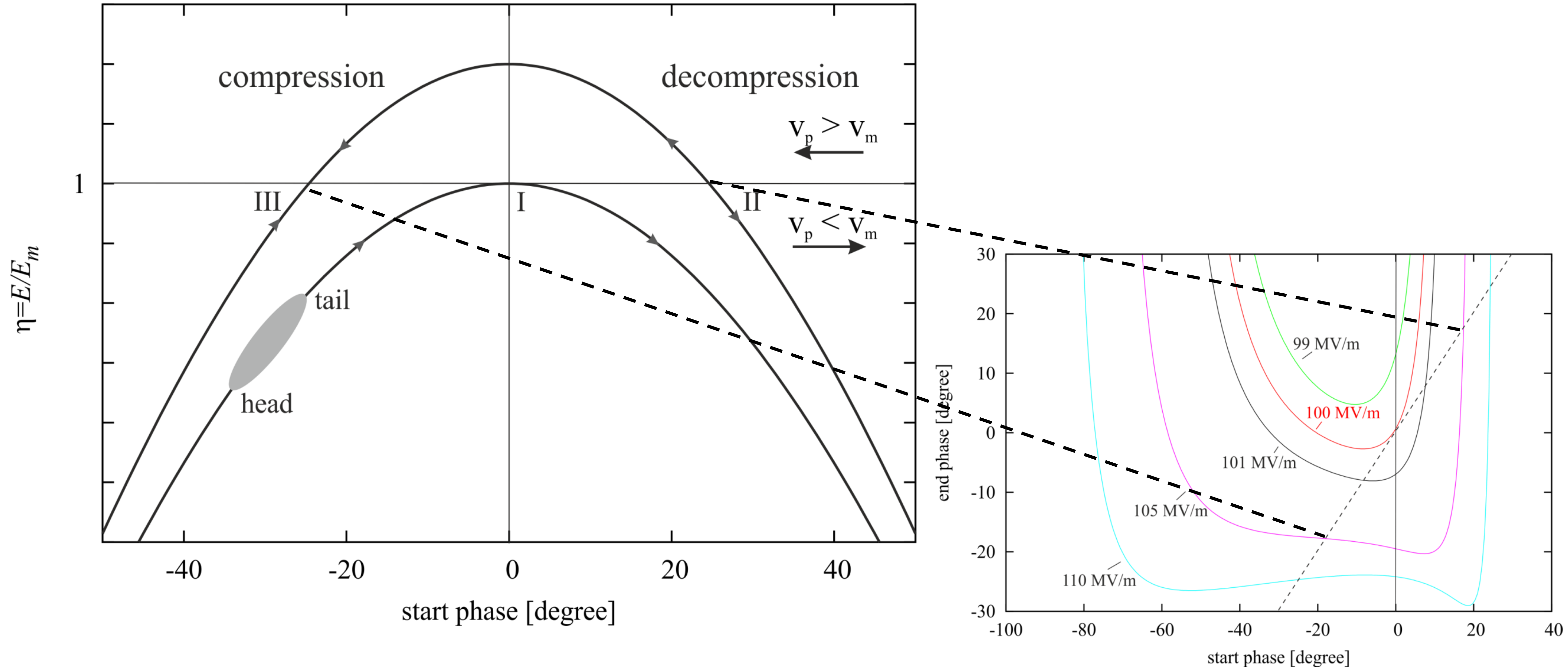
- charge: 80 fC
- bunch length: 730 nm
- emittance: 158 nm
- energy spread: 83 keV

Bunching mechanism in standard and tapered structures



- negative starting phase: tail gains more energy than head the bunch is thus compressed; for positive starting phases the bunch is decompressed
- particles starting below the $\eta = 1$ line move toward higher phase (from left to right); particles starting above the $\eta = 1$ move toward lower phase (from right to left)
- fix points I and II are not stable, while fix point III is stable

Bunching mechanism in standard and tapered structures



Beam dynamics near fix point III

A first order Taylor expansion of the phase near the fix point yields the differential equation

$$\Delta\varphi'' + \frac{3\gamma'}{\gamma}\Delta\varphi' + \frac{k_0\eta\gamma'|\sin\varphi_0|}{\gamma^3}\Delta\varphi = 0$$

(simplified form for $\beta = 1$), with the solution

$$\Delta\varphi(\gamma) = \Delta\varphi_0\frac{\gamma_0}{\kappa_0\gamma}[-C_1 J_2(\kappa) + C_2 Y(\kappa)]$$

C_1, C_2 = constants

J_2, Y_2 = Bessel functions

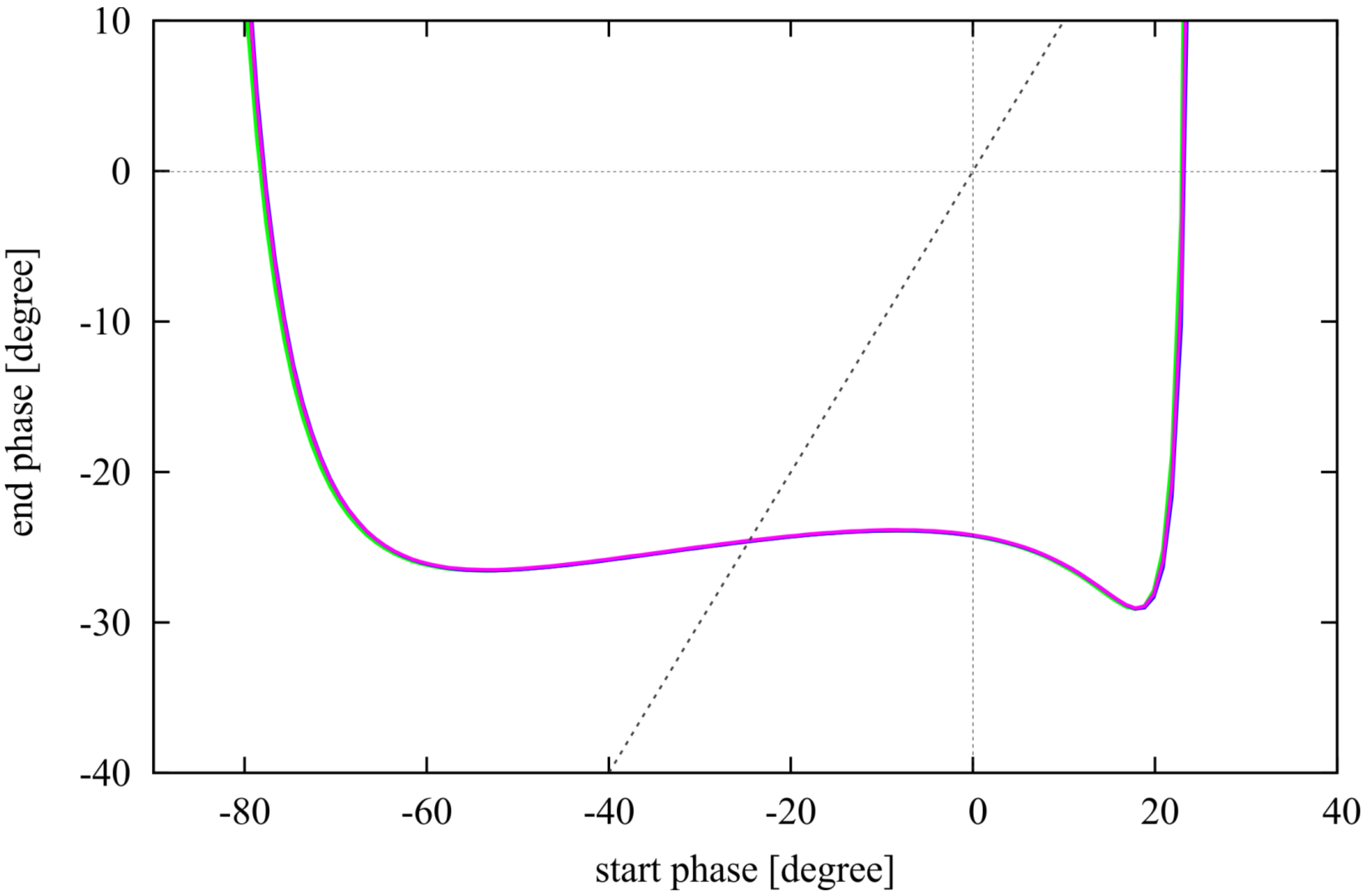
$\Delta\varphi_0$ = initial phase relative to fix point phase

γ_0 = initial energy

$$\kappa = 2\sqrt{\frac{\sqrt{\eta^2-1}}{\alpha\gamma}}$$

The relevant parameters to describe the beam dynamics are: the initial energy γ_0, the normalized vector potential α and the ratio of gradient to matched gradient η.

Proof of relevant parameters



End phase vs. start phase for $\eta = 1.1$, $\alpha = 0.03$, and $E_0 = 100$ keV.

Overlapping curves for:

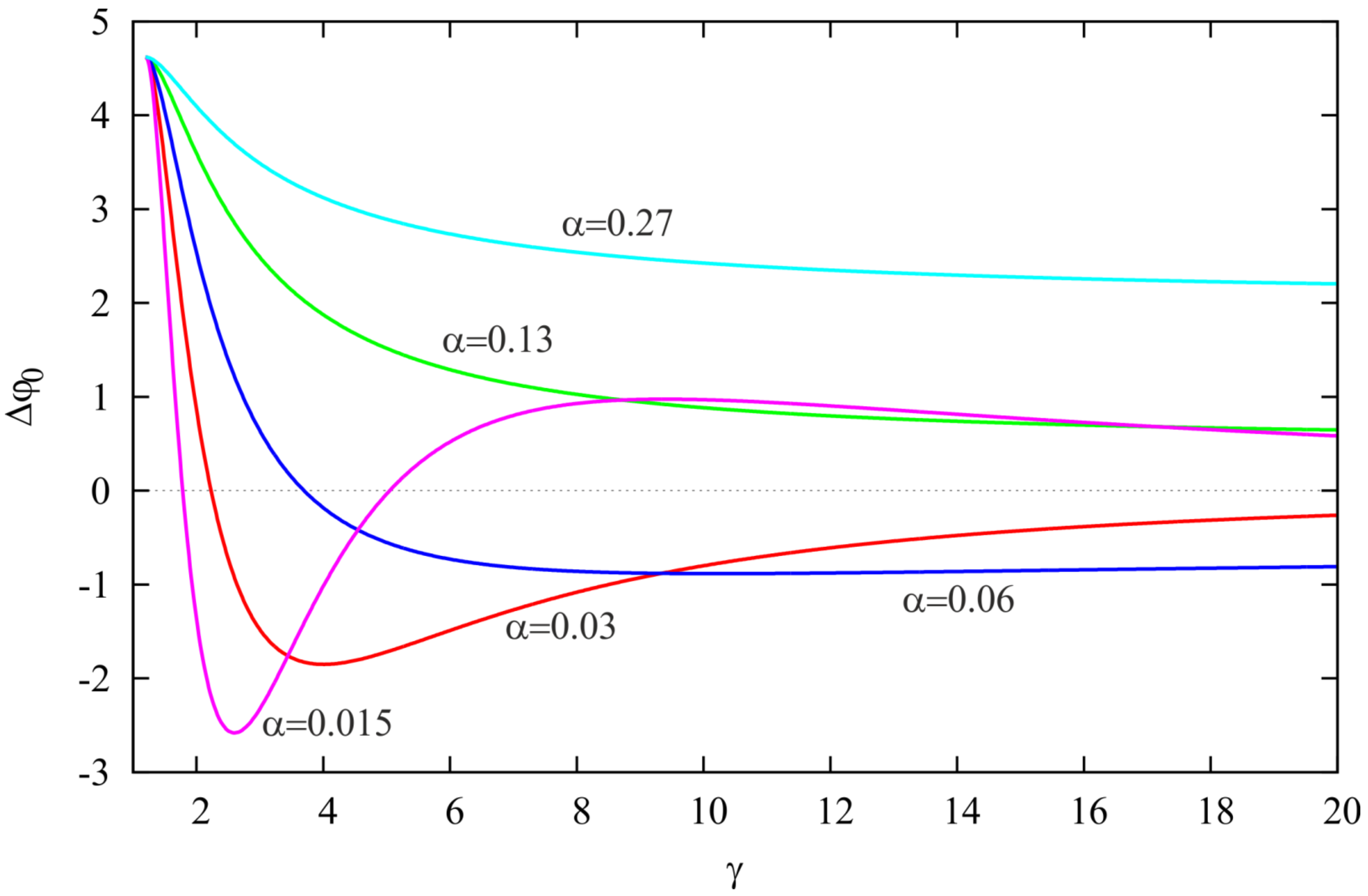
600 GHz, 200 MV/m

300 GHz, 100 MV/m

150 GHz, 50 MV/m

75 GHz, 25 MV/m

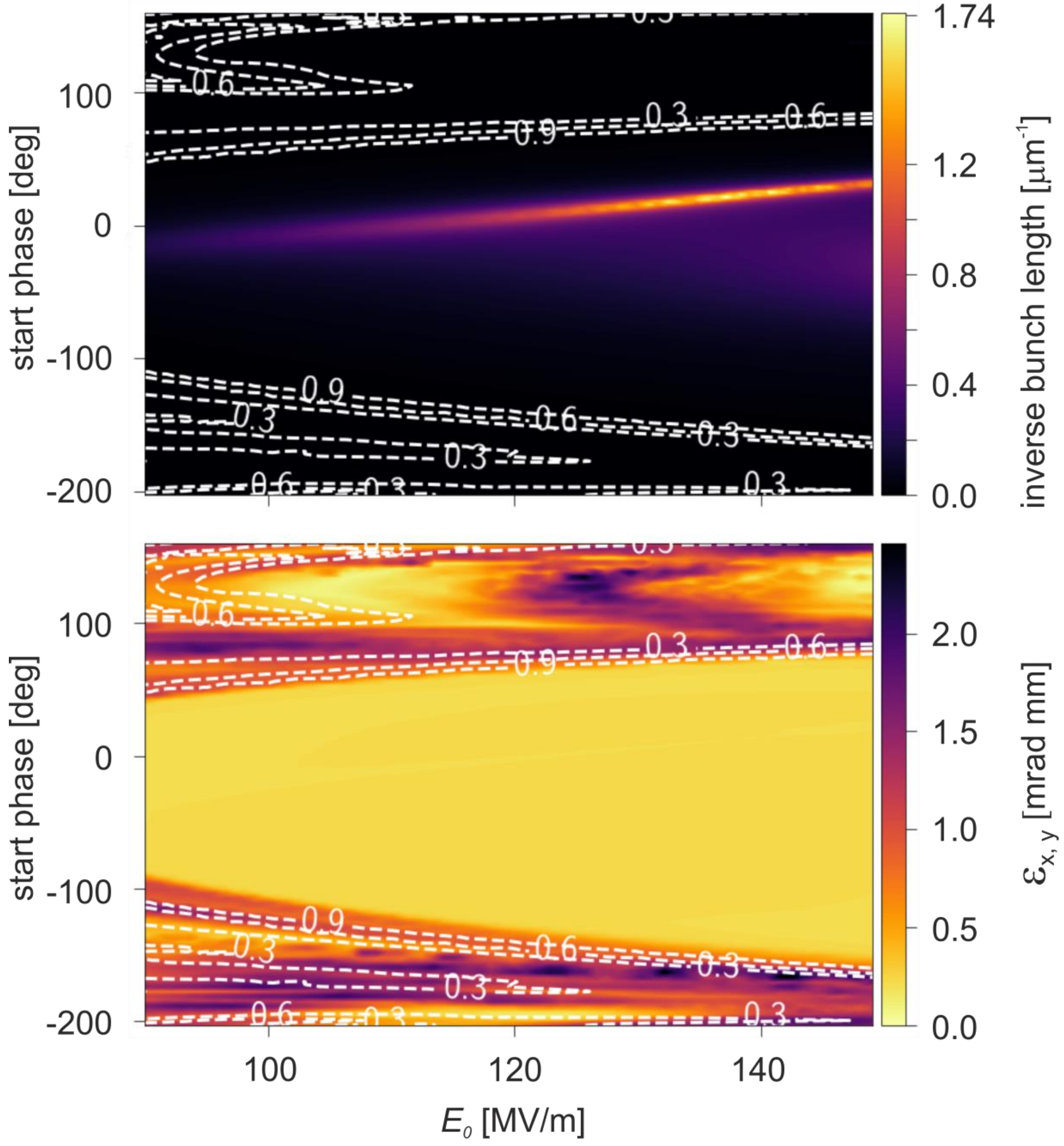
Phase offset vs. energy for different vector potentials



The phase swings over for too low alpha ($\alpha \leq 0.03$), i.e. a bunch is compressed, decompressed and compressed again.
For large alpha ($\alpha = 0.27$), the damping due to the increased gamma is too strong, the compression is stopped too early
$\alpha = 0.13$ is close to the aperiodic limit.
η = const=1.1

Simulation results Gun & DLW & Focusing

initial energy 200 keV, frequency of the DLW 75 GHz, matched gradient 100 MV/m

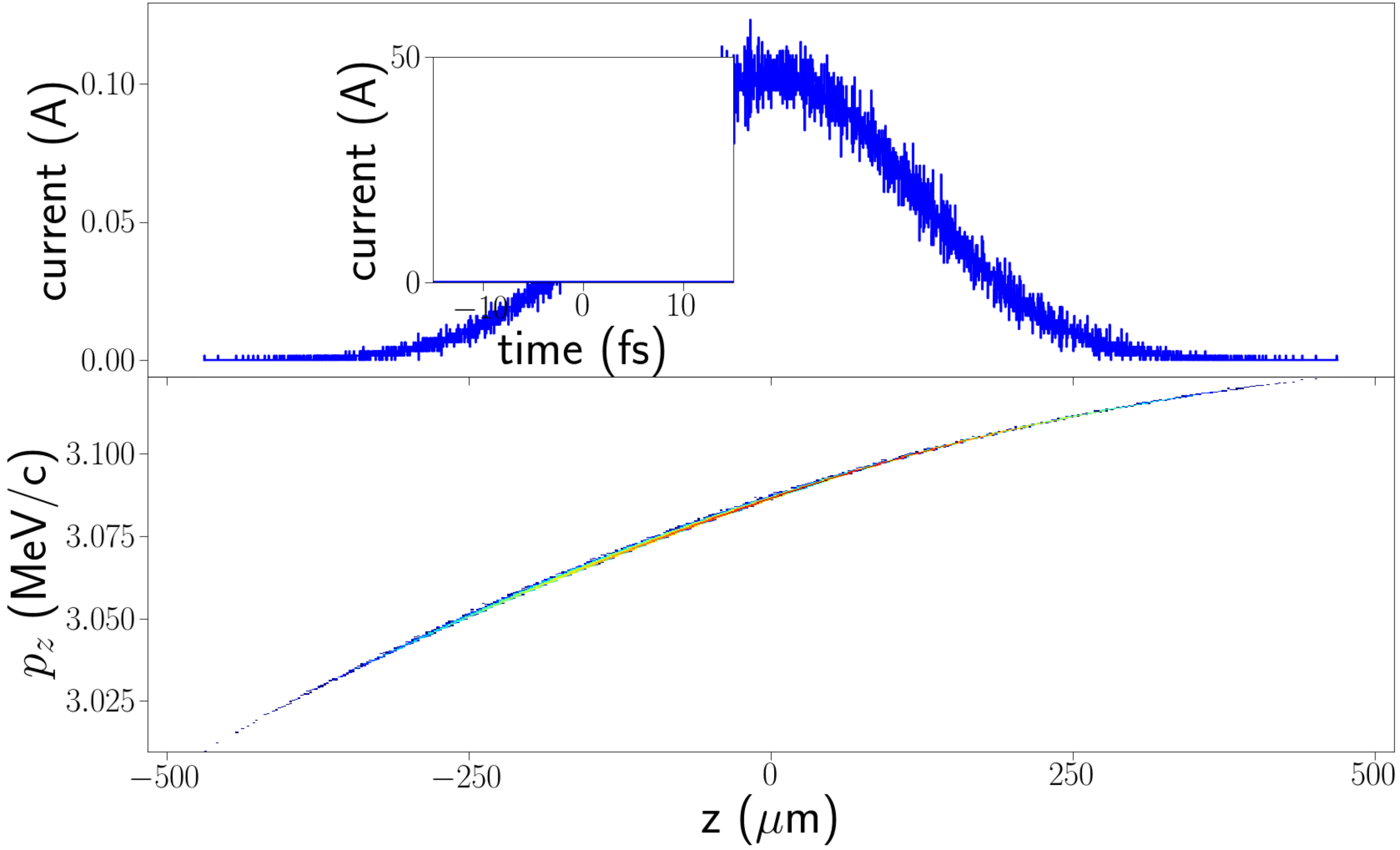


75 GHZ, 100 MV/m: α = 0.12

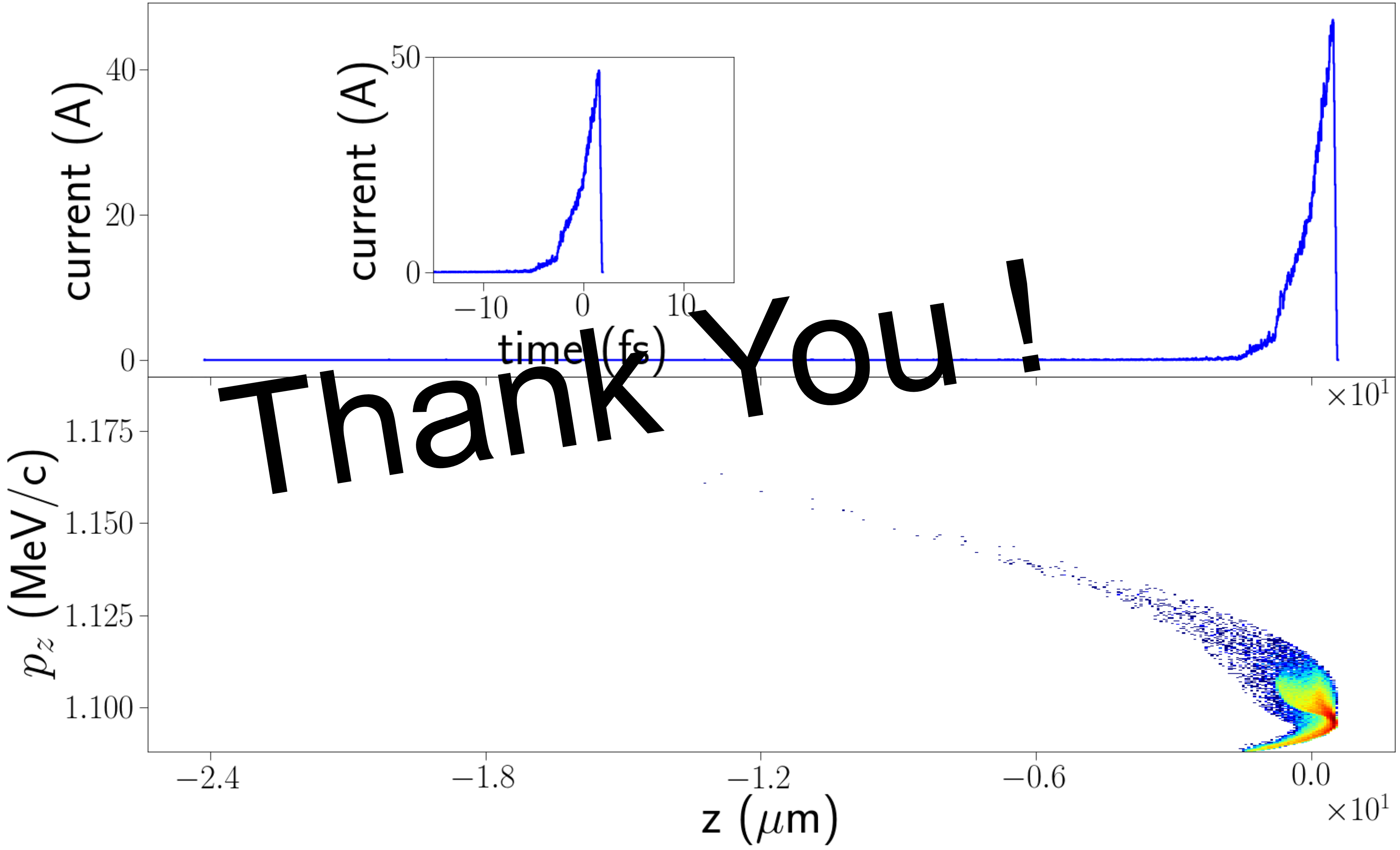
Parameters for shortest bunch length:

- charge: 100 fC
- bunch length: 473 nm
- emittance: 227 nm
- energy spread: 59 keV

Bunch length during the acceleration process



Final bunch length



1.5 fs
bunch length
rms = fwhm