GEOMETRICAL INTERPRETATION OF TRANSITION RADIATION IN A WAVEGUIDE

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03. 07. 2019

Ultrafast Beams and Applications 2019

2-5 July 2019, Yerevan, Armenia

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ABSTRACT

The presentation demonstrates the geometric optic properties of the transition radiation (TR) of an ultrarelativistic point-like charged particle crossing the transverse wall of a circular semi-infinite waveguide with ideally conducting walls. It is shown that the rays forming the TR field emitted from the point of entry are exposed the laws of geometric optics. The TR field at the observation point is formed from geometric-optical rays which are multi re-reflected from the waveguide wall.

CONTENT

- ***INTRODUCTION***
- ***ANALYTICAL INVESTIGATION OF LONGITUDINAL ELECTRIC COMPONENT OF TR***
- ***FORMATION OF TRANSITION RADIATION BY POINT CHARGE***
- ***SOLUTION OF THE PROBLEM***
- ***TIME DEPENDENCE OF THE LONGITUDINAL ELECTRIC COMPONENT OF TR FIELD AT A FIXED POINT OF OBSERVATION***
- ***CONCLUSION***

INTRODUCTION

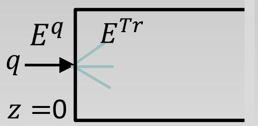
- As it is known, Transition Radiation (TR) in unlimited space, occurs when a charged particle crosses the boundary between two media with different dielectric constants and obeys the geometric-optic laws of refraction and reflection of light.

- The purpose of this work is to demonstrates the geometric properties of *TR* and to show the connection between the propagation of radiation and the laws of geometrical optics in a waveguide.

- In this research work, we use numerical examples to show that the TR field in a waveguide strictly follows the geometric-optical reflection laws.

ANALYTICAL INVESTIGATION OF LONGITUDINAL ELECTRIC COMPONENT OF TR

There is two field in waveguide Coulomb field and TR field emitted by charge



$$\left. \begin{aligned} E_r^q &= E_r^{Tr} \\ E_\varphi^q &= E_\varphi^{Tr} \end{aligned} \right|_{z=0}$$

The frequency distribution of the longitudinal electric component E_z^{Tr}

$$E_z^{Tr} = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} jq \frac{Z_0}{\pi a} \frac{J_m(\chi_{mn}\tilde{r}_0)}{\sqrt{\tilde{k}^2 - \chi_{mn}^2}} \frac{J_m(\chi_{mn}\tilde{r})}{J_{m+1}^2(\chi_{mn})} exp(jm\varphi) exp\left\{ j\left(\tilde{k}\tilde{t} - \sqrt{\tilde{k}_c^2 - \chi_{mn}^2}\, \tilde{z} \right) \right\}$$

Making a transition from ω to t

The time distribution of the longitudinal electric component E_z^{Tr}

$$E_z^{Tr}(r,\varphi,z,t) = \int_{-\infty}^{\infty} E_z^{Tr} d\omega = jq \frac{Z_0 c}{\pi a^2} \sum_{m=0}^{\infty} \delta_m \sum_{n=1}^{\infty} J_m(\chi_{mn}\tilde{r}_0) \frac{J_m(\chi_{mn}\tilde{r})}{J_{m+1}^2(\chi_{mn})} Cos\{m\varphi\} G(\tilde{z},\tilde{t})$$

$$G(\tilde{z},\tilde{t}) = \begin{cases} 2K_0\left[\chi_{mn}\sqrt{\tilde{z}^2 - \tilde{t}^2}\right], & 0 < \tilde{t} < \tilde{z} \\ j\pi H_0^{(1)}\left[\chi_{mn}\sqrt{\tilde{t}^2 - \tilde{z}^2}\right], & \tilde{z} < \tilde{t} < \infty \end{cases}$$

where

$\tilde{k} = \omega a/c$, *dimensionless wavenumber*
$\tilde{z} = z/a$, $\tilde{r} = r/a$, *normed longitudinal and radial coordinates*
$\tilde{t} = ct/a$, *dimensionless time*

The remaining components of the TR fields expressed in terms of derivatives of the longitudinal electrical components

$$E_\phi^{Tr} = \frac{a^2}{\chi_{mn}^2 r} \frac{\partial^2}{\partial\varphi\partial z} E_z^{Tr}, \quad E_r^{Tr} = \frac{a}{\chi_{mn}} \frac{\partial^2}{\partial r\partial z} E_z^{Tr},$$

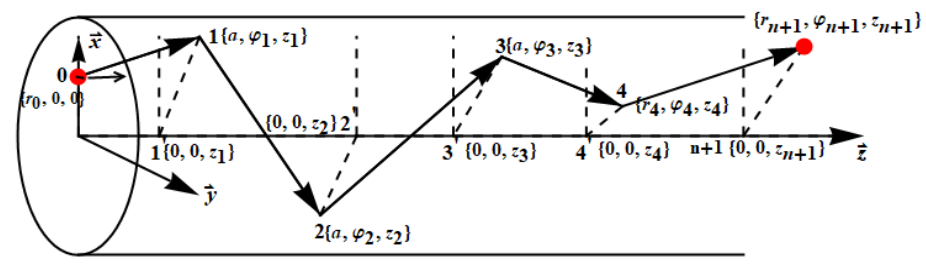
$$H_\varphi^{Tr} = j\frac{a}{\chi_{mn}} \frac{\partial^2}{\partial t\partial r} E_z^{Tr}, \quad H_r^{Tr} = -j\frac{a}{\chi_{mn}^2 r} \frac{\partial^2}{\partial t\partial\varphi} E_z^{Tr}, \quad H_z^{Tr} = 0$$

FORMATION OF TRANSITION RADIATION BY POINT CHARGE

In this chapter it is more detailed described the formation of TR and distribution of emitted rays, also a method is developed for determining the longitudinal and polar coordinates of the points of reflection from the surface of the waveguide and the selection of various options for finding the trajectories which would satisfy the stated requirements.

FORMATION OF TRANSITION RADIATION BY POINT CHARGE

Figure schematically shows the course of the rays



where

$\tilde{r}_0, 0,0$ particle entry point (0< $\tilde{r}_0 < a$)

$r_{n+1}, \varphi_{n+1}, z_{n+1}$ observation point

When $\varphi \neq 0$ the rays have complicated trajectory

Requirements

- To match the path of the rays to geometrical optics is to find in the same plane four points :- the refracted and reflected rays and the normal, dropped from the reflection point to the axis of the waveguide, (determinant of system must be equal to 0)
- equality of the angles of refracted and reflected rays $\angle\{i-1, i, i'\} = \angle\{i', i, i+1\}, i = 1,2, \ldots n$

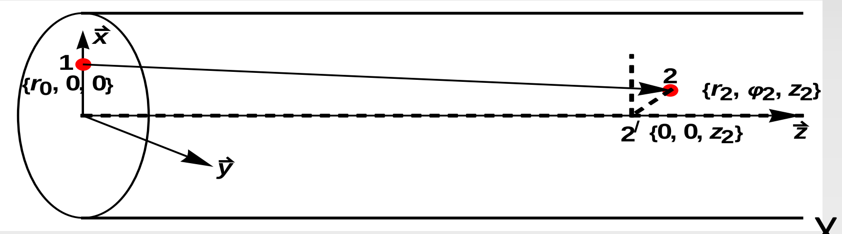
$$Det \begin{Bmatrix} x_{i-1} & y_{i-1} & z_{i-1} & 1 \\ x_i & y_i & z_i & 1 \\ x_{i'} & y_{i'} & z_{i'} & 1 \\ x_{i+1} & y_{i+1} & z_{i+1} & 1 \end{Bmatrix} = 0, \qquad i = 1,2, \ldots n$$

Here $\{x_0, y_0, z_0\}$ entry point,
$\{x_{n+1}, y_{n+1}, z_{n+1}\}$ observation point.
This system defines the relationship between the longitudinal z_i and angular φ_i coordinates of the reflection points

SOLUTION OF THE PROBLEM

$i-1, i, i'$ and $i+1$, $i = 1,2,..n$, where n is the number of reflections

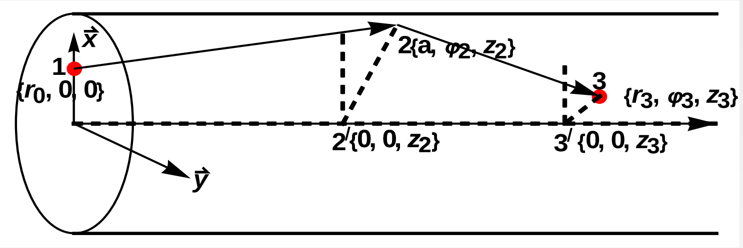
For $n = 0$ (direct hit of the ray):

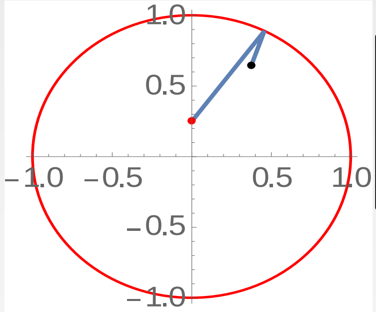


$$L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2 + (z_i - z_{i+1})^2}$$

X,Y plane

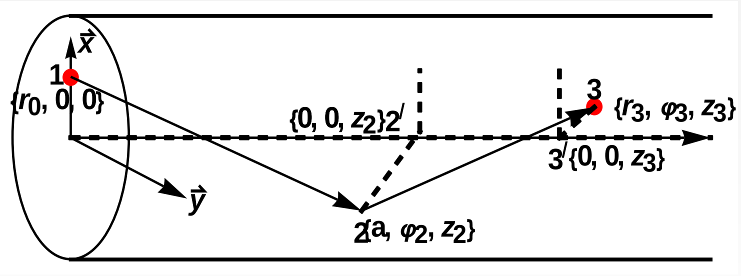
For $n = 1$

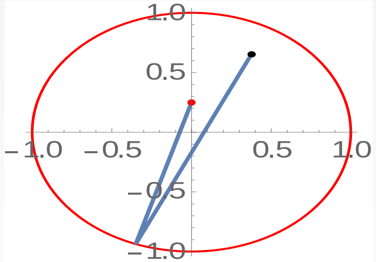




$$A_0 + \sum_1^3\{A_1 cos(\varphi_1) + B_1 sin(\varphi_1)\} = 0 \text{ with}$$
$$A_0 = a^2(r_0^2 - r_1^2)$$

where



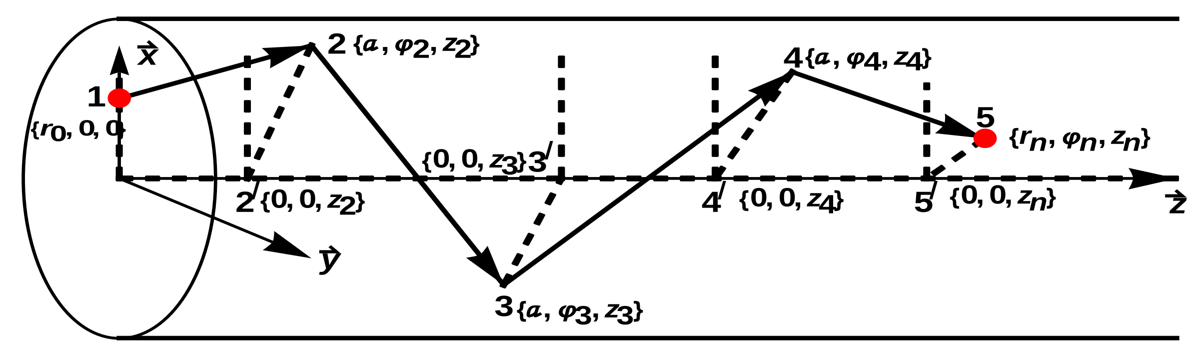


$$A_1 = -ar_0 r_{n+1}(r_0 cos\varphi_{n+1} - r_{n+1}(2 - cos2\varphi_{n+1}))$$
$$B_1 = -ar_0 r_{n+1}(3r_0 + 2r_{n+1} cos\varphi_{n+1})sin\varphi_{n+1}$$

SOLUTION OF THE PROBLEM

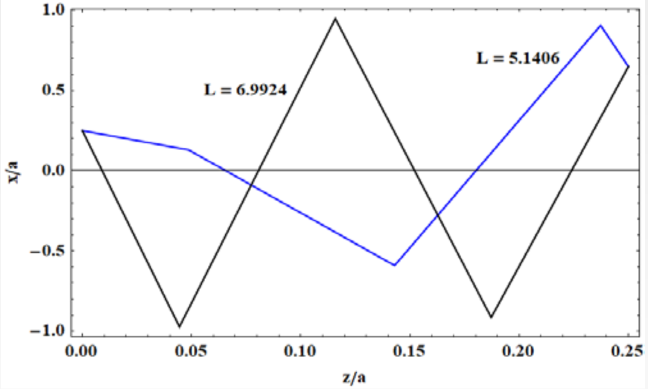
For $n > 1$(considered a case when n=3 ,)

Geometrical interpretation for n=3 case

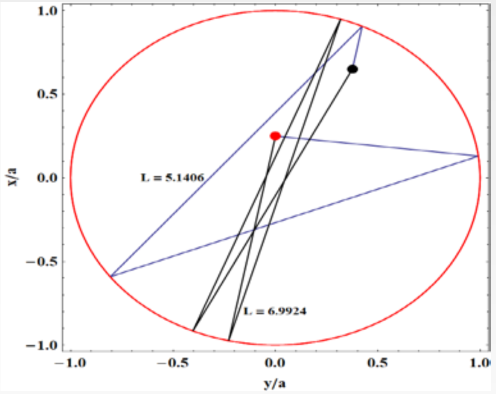


$1 \rightarrow \{\tilde{r}_0 = 0.75, \varphi = 0, \tilde{z} = 0\}$- point of entry

$5 \rightarrow \{\tilde{r} = 0.25, \varphi = \pi/6, \tilde{z} = 0.25\}$- observation point



X,Y plane



The ray's trajectories projection on the plane x, z (left) and x, y (right); enter point (red), observation point (black).

The relationship between the longitudinal z_i and angular φ_i coordinates of the reflection points:

$$a^2 + (a^2 + r_n^2)cos\beta - 4ar_n cos^2\frac{\beta}{2} cos\alpha + r_n^2 cos2\alpha = 0$$

$$\beta = (-1)^j Arccos\left(-\frac{a^2-2ar_0+r_0^2 cos2\varphi_1}{a^2+r_0^2-2ar_0 cos\varphi_1}\right) j = 0,1;$$

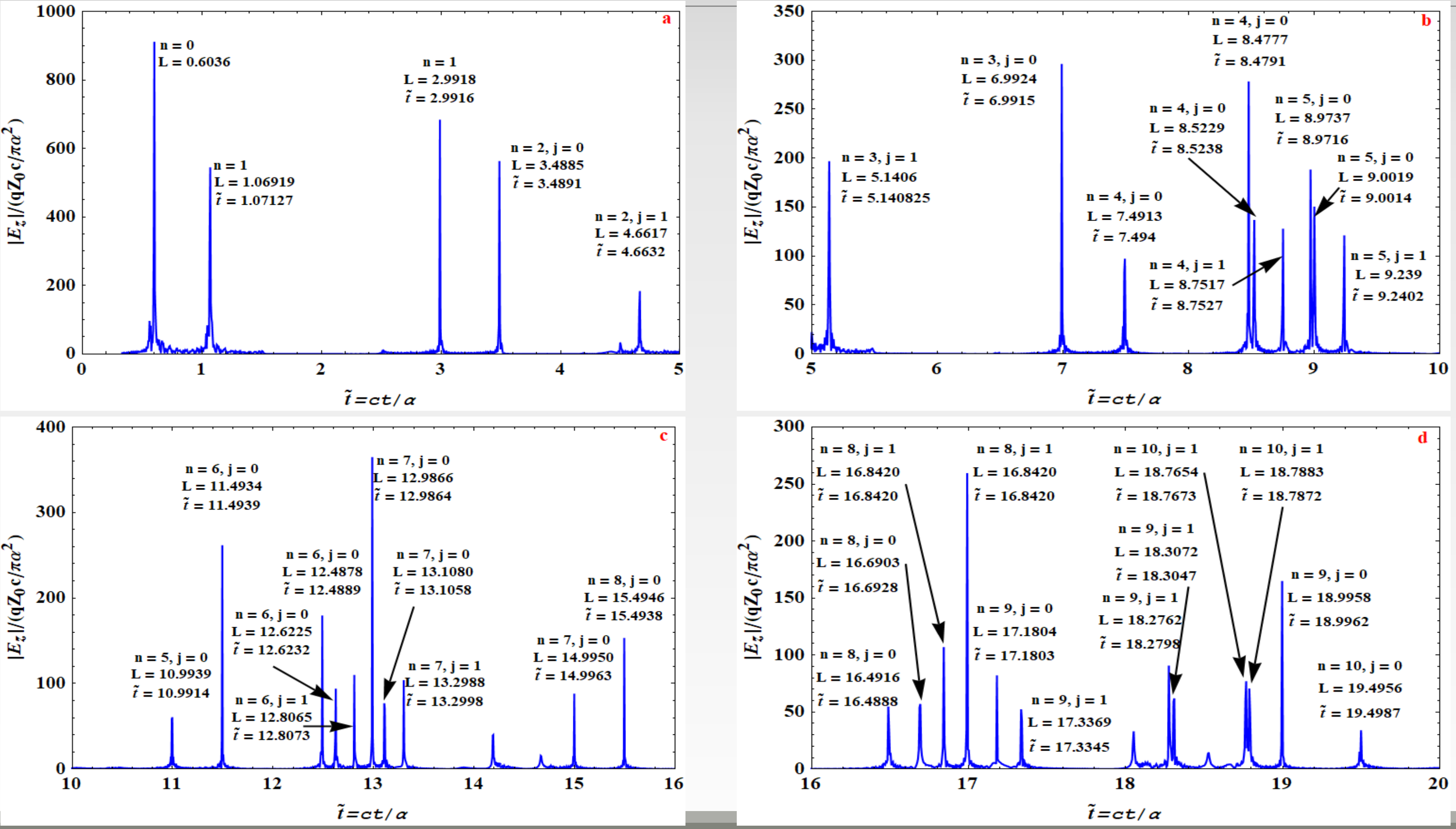
$$\alpha = (n-1)\beta - \varphi_1 + \tilde{\varphi}_{n+1}; \varphi_2 = \varphi_1 - \beta$$

TIME DEPENDENCE OF THE LONGITUDINAL ELECTRIC COMPONENT OF TR FIELD AT A FIXED POINT OF OBSERVATION

- In this chapter was calculated the time dependence of TR field on a fixed point of observation $\underline{\{\tilde{r}=0.25,\ \varphi=\pi/6,\ \tilde{z}=0.25\}}$ with the point of particle entry $\underline{\{\tilde{r}_0=0.75,\ \varphi=0,\ \tilde{z}=0\}}$ for corresponding numbers up to ***n=10*** of reflections .

- Considered a case for a multiple numbers of waveguide modes and reflections.(*m=500 and n=1000*)

TIME DEPENDENCE OF THE LONGITUDINAL ELECTRIC COMPONENT OF TR FIELD AT A FIXED POINT OF OBSERVATION

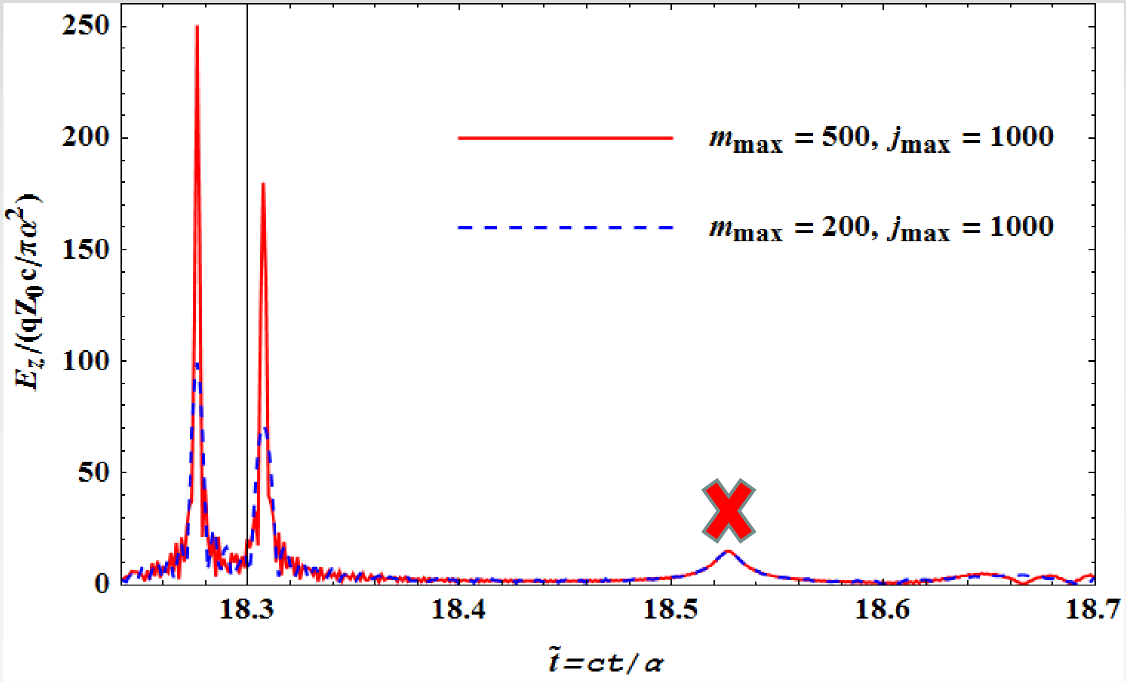


- The occurrence of these peaks can be explained if we assume that the point of entry of particle is an instant source of radiation, rays are emitting inside the waveguide in all directions, but only rays which optical path corresponds to the lows of geometric optics reach the observation point

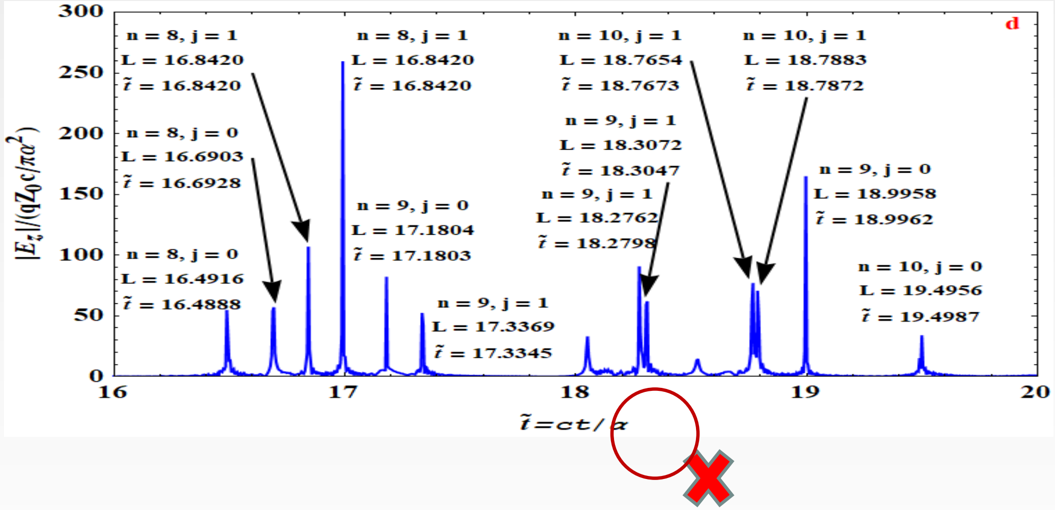
- Each of the peaks corresponds to a ray emitting from the point of entry of the particle and reaching the observation point after the corresponding number (n) of reflections. The optical path (L) of the corresponding ray is equal to the normed time moment $(\tilde{t})$ of formation of the peak.

- Thus, our statement about the formation of a TR field in accordance with the laws of geometric optics can be considered as proven.

TIME DEPENDENCE OF THE LONGITUDINAL ELECTRIC COMPONENT OF TR FIELD ON A FIXED POINT OF OBSERVATION



- In this figure taken into an account increasing number of waveguide modes and reflections $m = 0,1,2,..200; j = 1,2,...1000$. This can explain the presence in previous figure d of several unidentified peaks. When using more harmonics , the peaks due to the geometrical optics increase, and the height of the unidentified peaks remains unchanged (on figure it is marked with a cross).



CONCLUSION

- *The regularities demonstrated for an ideal waveguide can be extended to both resistive waveguides and waveguides partially filled with the medium. The applied technique can be extended to closed resonators.*

- *It has been shown that the rays of transition radiation spread in all directions but only rays which are exposed to geometrical optics laws can rich the arbitrary viewpoint.*

- *Due to this new approach, with the obtained results, it was possible to give a more figurative interpretation and possiblility to predict the distribution of Transition Radiation at a fixed point of observation.*

Acknowledgement

I would like to express my very great appreciation to Dr M. Ivanyan, my research supervisor for his valuable and constructive suggestions and for his help during the development of this research work.

Note

PIERS 2019 in Rome

PhotonIcs & Electromagnetics Research Symposium

17.06.2019

THANK YOU FOR YOUR ATTENTION!