International Workshop
“Ultrafast Beams and Applications”

The Resonant Impedance of a Metal-Dielectric Structures in THz Region

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Yerevan, 2019, Jul 03

Abstract

The resonant properties of the impedance of a two-layer circular metal-dielectric waveguide are investigated. Their dependence on the geometric and electromagnetic parameters of the waveguide is considered: The conditions of a single-mode nature of the radiation field of a point particle and a Gaussian bunch are established numerically.

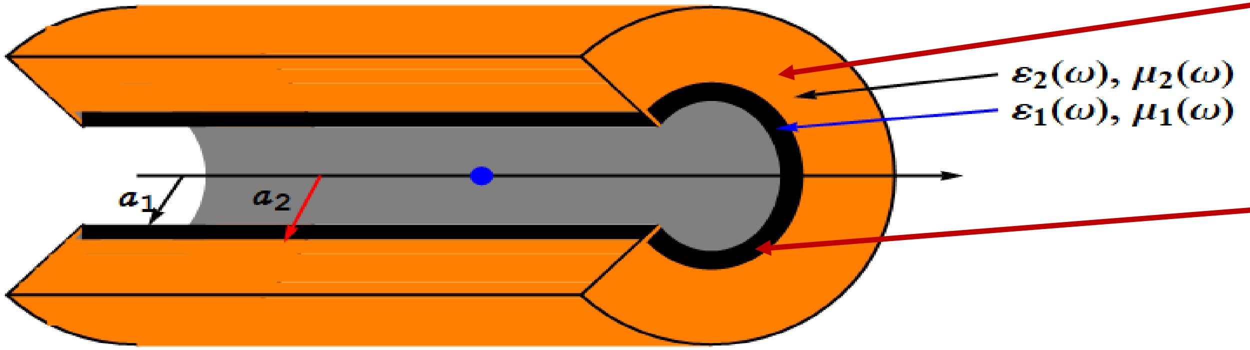
CONTENT

Introduction

- The main source of radiation dissipation in the metal-dielectric waveguide is the finite conductivity of the metal wall and the attenuation due to the losses of dielectric (due to imaginary part of the dielectric constant of the internal dielectric coating). With an idealized consideration of the outer wall as ideally conductive and in the absence of loss in the dielectric, the movement of a particle in a waveguide will not be accompanied by a loss of energy. The impedance of the structure in this case contains only the imaginary component. The equivalent circuit of such a process corresponds to a parallel inductive-capacitive resonant oscillatory circuit without loss.
- The real model of the structure should take into account both dielectric and metal losses.

2. Statement of the Problem

A metal waveguide with an internal dielectric coating is considered.



Metal $\varepsilon_1 = \varepsilon_1' + j\varepsilon_1''$

Dielectric $\varepsilon_2 = 1 + j\dfrac{\sigma_2}{\varepsilon_0 \omega}$

$\varepsilon_1, \varepsilon_2$ - relative dielectric constant of layers

$\sigma_2 = 58 \times 10^6\ \Omega^{-1} m^{-1}$ - conductivity of copper (Cu)

ε_1'=10 ε_1'' = 0, 0.1, 0.5 and 3

$a_1 = 2mm, d = a_2 - a_1, d = 200\mu m\ and\ d = 2\mu m$

The cases of thick $(\frac{d}{a_1} = \mathbf{0.1})$ and thin $(\frac{d}{a_1} = \mathbf{0.001})$ dielectric coatings are considered.

Longitudinal impedance, exact formulae

- $Z_{||} = \{Z_{diel}^{-1} + Z_{rez}^{-1}\}^{-1}$ $\quad \beta_{1,2} = k\sqrt{1 - \varepsilon_{1,2}\mu_{1,2}}$
- $Z_{diel} = j\frac{Z_0}{\pi k a_1^2}\left[1 - \frac{2\varepsilon_1}{a_1\beta_1}\frac{U_3}{U_1}\right]^{-1}$ $\quad \varepsilon_2 = \varepsilon_0 + j\,\sigma_{1,2}/\varepsilon_0\omega$
- $Z_{rez} = -j\frac{Z_0}{2\pi k\varepsilon_1}\beta_1^3 a_2 U_1\left(U_2 + \frac{\beta_1}{\varepsilon_1}\frac{\varepsilon_2}{\beta_2}U_1\alpha\right)$ $\quad \varepsilon_1 = \varepsilon_1' + j\varepsilon_1''$
- $U_1 = I_0(\beta_1 a_2)K_0(\beta_1 a_1) - I_0(\beta_1 a_1)K_0(\beta_1 a_2)$
- $U_2 = I_1(\beta_1 a_2)K_0(\beta_1 a_1) + I_0(\beta_1 a_1)K_1(\beta_1 a_2)$
- $U_3 = -I_0(\beta_1 a_2)K_1(\beta_1 a_1) - I_1(\beta_1 a_1)K_0(\beta_1 a_2)$
- $U_4 = I_1(\beta_1 a_1)K_1(\beta_1 a_2) - I_1(\beta_1 a_2)K_1(\beta_1 a_1)$

Dispersion relations, exact expressions

- $\frac{\varepsilon_1 \nu_0 J_0(\nu_0) W_3 + \nu_1 J_1(\nu_0) W_1}{\varepsilon_1 \nu_0 J_0(\nu_0) W_4 + \nu_1 J_1(\nu_0) W_2} + \frac{\varepsilon_1 \nu_2}{\varepsilon_2 \nu_1} \frac{H_0^{(1)}(\tilde{d}\nu_2)}{H_1^{(1)}(\tilde{d}\nu_2)} = 0$
- Here $\tilde{d} = a_2/a_1$, ν_0 is the desired dimensionless transverse wavenumber in vacuum of the TM mode with zero first index; ν_1 and ν_2 transverse wavenumbers of the same mode in the inner layer and in the outer wall, respectively: $\nu_{1,2} = \sqrt{k^2 a_1^2 (\varepsilon_{1,2}\mu_{1,2} - 1) + \nu_0^2}$
- $W_1 = H_0^{(1)}(\nu_1) J_0(\tilde{d}\nu_1) - J_0(\nu_1) H_0^{(1)}(\tilde{d}\nu_1)$
- $W_2 = -H_0^{(1)}(\nu_1) J_1(\tilde{d}\nu_1) + J_0(\nu_1) H_1^{(1)}(\tilde{d}\nu_1)$
- $W_3 = -H_1^{(1)}(\nu_1) J_0(\tilde{d}\nu_1) + J_1(\nu_1) H_0^{(1)}(\tilde{d}\nu_1)$
- $W_4 = H_1^{(1)}(\nu_1) J_1(\tilde{d}\nu_1) - J_1(\nu_1) H_1^{(1)}(\tilde{d}\nu_1)$

for $\sigma_2 \to \infty$:

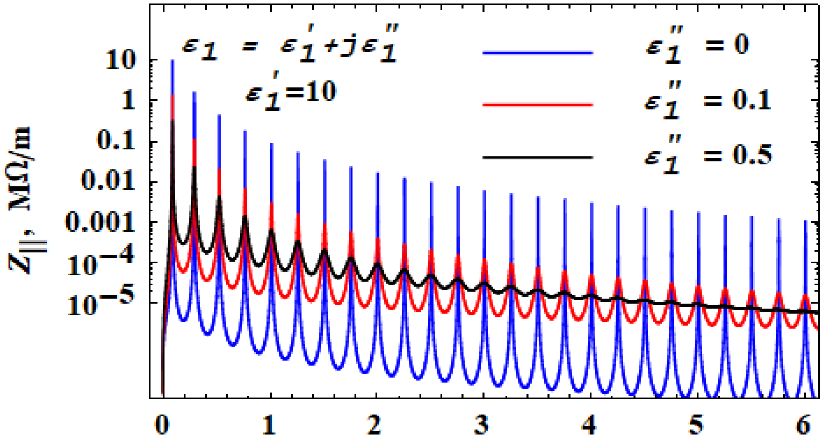
$\varepsilon_1 \nu_0 J_0(\nu_0) W_3 + \nu_1 J_1(\nu_0) W_1 = 0$

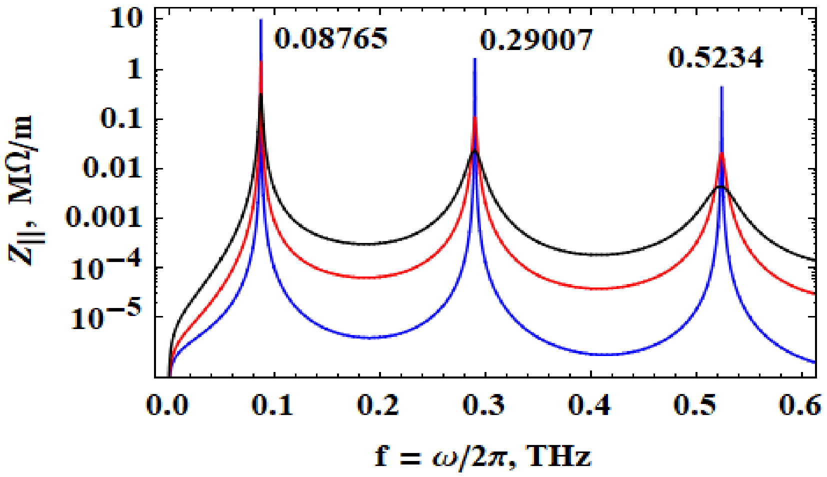
Wake function: $W_{||} = \int_{-\infty}^{\infty} Z_{||} e^{j\frac{\omega}{c}s} d\omega$

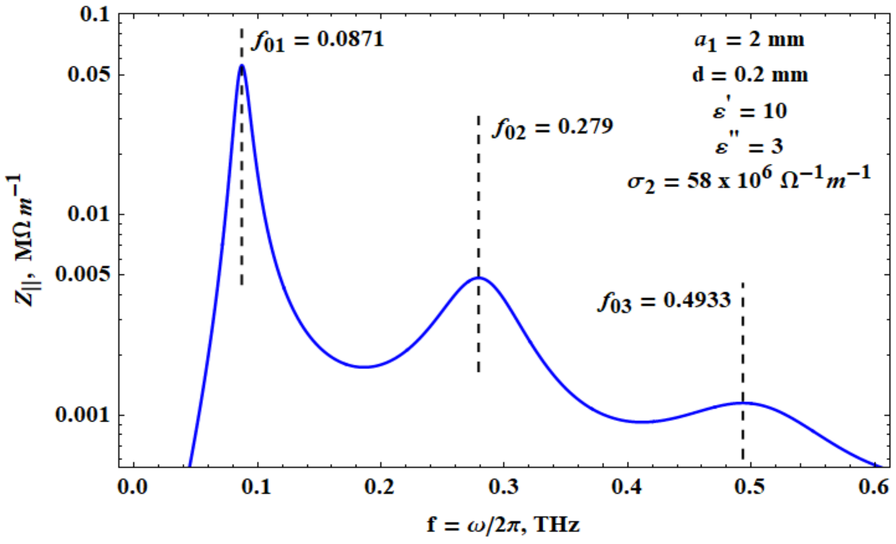
3.Longitudinal Impedance, $\boldsymbol{a_1 = 2mm}$, $\boldsymbol{d = 200\mu m}$

- The next slide presents graphs of the longitudinal impedance of a metal-dielectric tube with a thick $(\boldsymbol{d = 200\mu m})$ internal dielectric coating with relatively small values of the imaginary component of the dielectric constant (ε_1''=0, 0.1, 0.5) of the internal coating.
- A separate graph presents the impedance for the case of a relatively large (ε_1''=3) imaginary component.
- Impedances are characterized by multiple resonant frequencies. The amplitudes of the impedances decrease with an increase of the imaginary component of the dielectric constant.

3. Longitudinal Impedance, $a_1 = 2mm$, $d = 200\mu m$



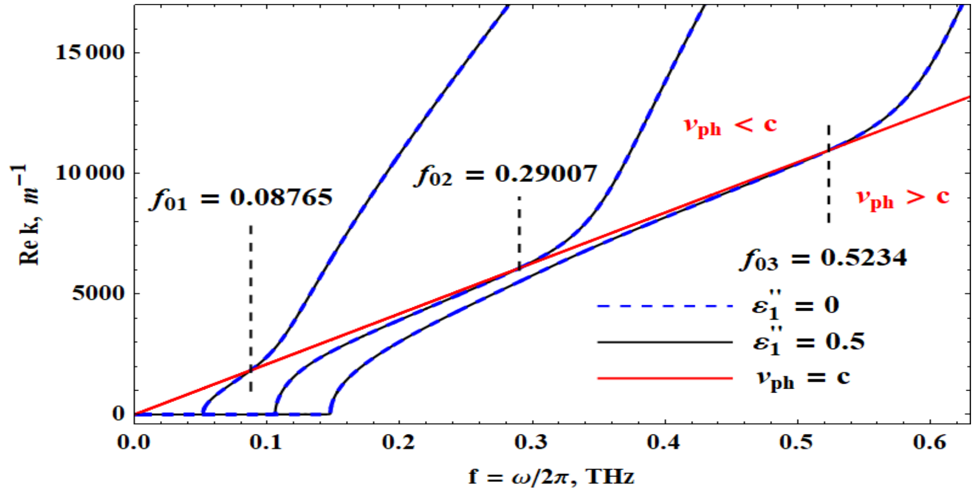


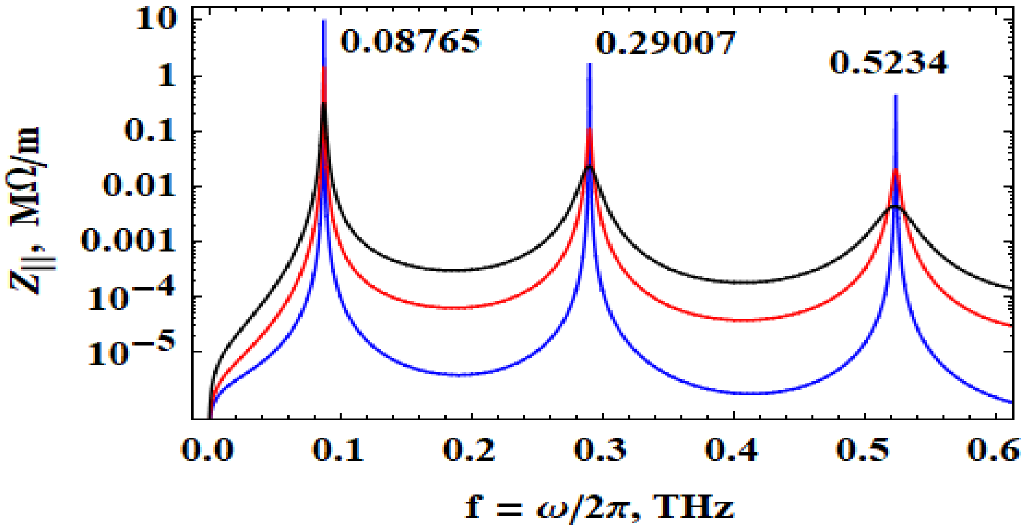


Resonance frequencies

ε_2''	f_1	f_2	f_3
0	0.08765	0.29007	0.5234
0.1	0.08765	0.29005	0.52336
0.5	0.08764	0.28974	0.52257
3	0.0871	0.279	0.4953

4. Dispersion curves, $a_1 = 2mm$, $d = 200\mu m$





In the absence of attenuation ε_1''=0 and at low attenuation ε_1''=0.5, all resonant frequencies correspond to slowly propagating waveguide modes, the phase velocities of which are synchronous with the velocity of propagation of the particle and, therefore, affect to the movement of the test particle.

The effect of higher resonant frequencies on the test particle can be countered by studying the shape of the corresponding wake functions.

5. Wake functions, $\boldsymbol{a_1 = 2mm}$, $\boldsymbol{d = 200\mu m}$

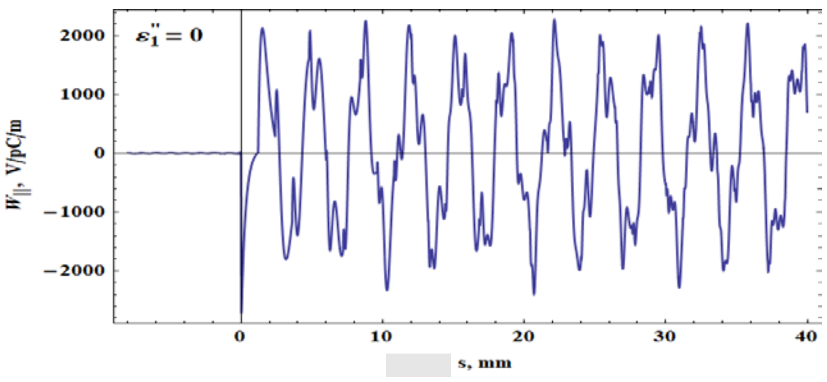
- The next slide shows the distributions of longitudinal wake functions for the following values of the imaginary component of the dielectric constant: ε_1''=0, 0.1 and 0.5.

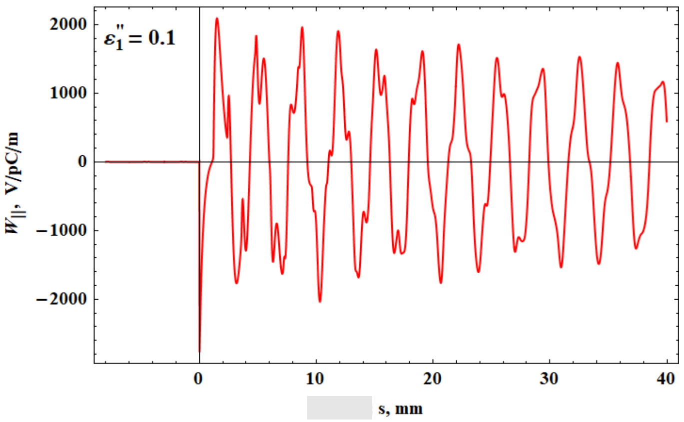
For ε_1''=0 and 0.1 , the effect of higher resonant frequencies is significant throughout the presented maximal distance behind the particle ($s \le 40mm$).

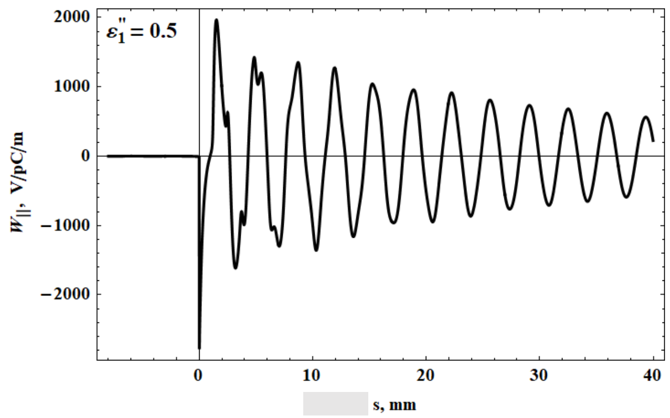
When ε_1''=0.5 their impact becomes insignificant already for s > 10mm.

For these distances, in this case, the wakefield becomes monochromatic.

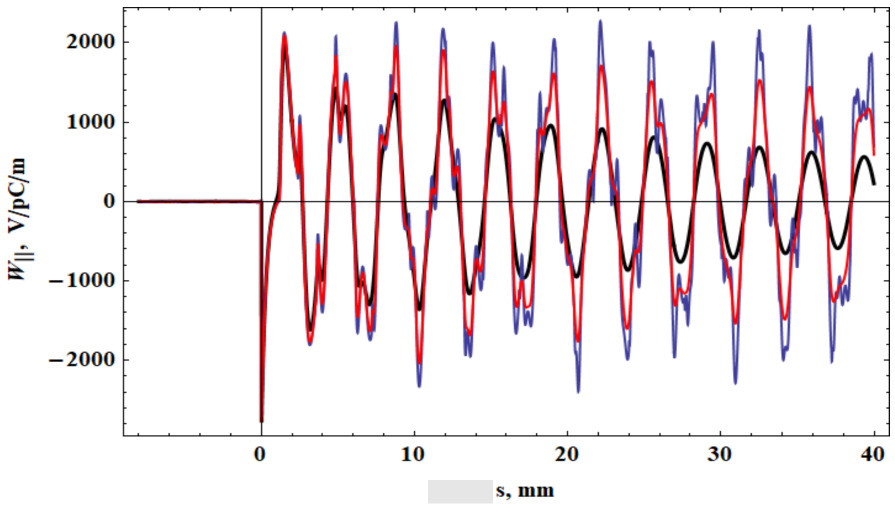
5. Wake functions, $\boldsymbol{a_1 = 2mm}$, $\boldsymbol{d = 200\mu m}$







With increasing attenuation in the dielectric while maintaining the fundamental resonant frequency, the contributions of higher resonant frequencies are noticeable. With increasing attenuation, their influence decreases.

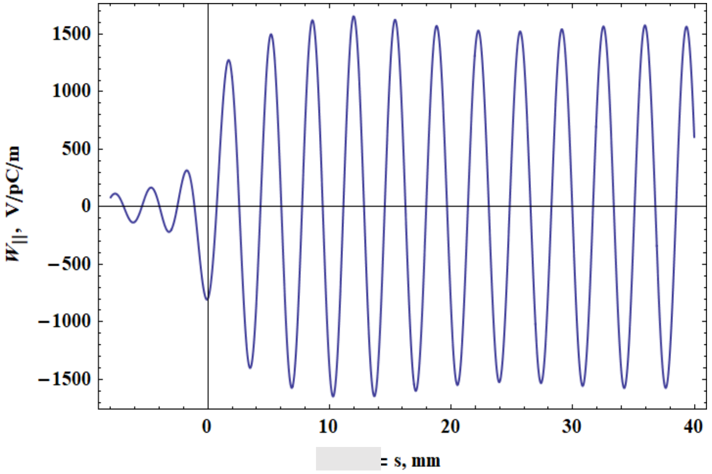


6. Impacts of higher resonance frequencies

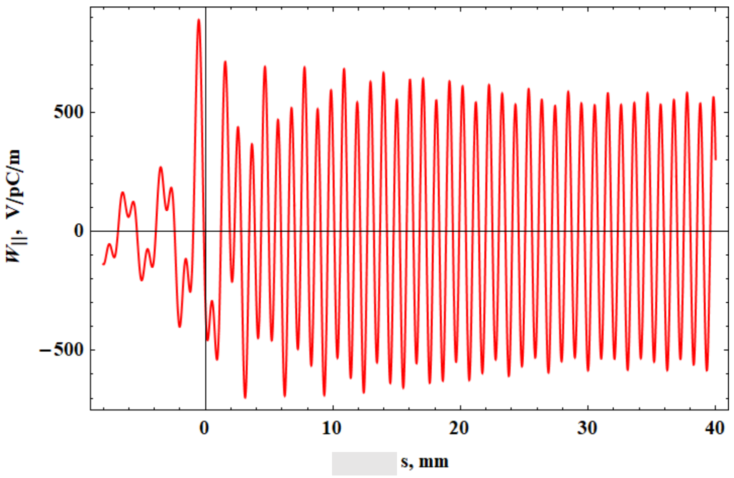
- The next two slides (9 and 10) show separately the contributions of the first three resonant frequencies to the wake function for cases ε_1''=0 and ε_1''=0.5.
- In the first case (no attenuation, ε_1''=0), all three harmonics make a significant contribution to the total wakefield function. The sum of the three harmonics basically repeats the form of the full wake function, except for the immediate vicinity of the particle.
- In the second case, the convergence of harmonics to the full wake function is faster. Mainly the first two harmonics participate in the formation of the wake function.

6. Impacts of higher resonance frequencies, $\varepsilon_1''=0$

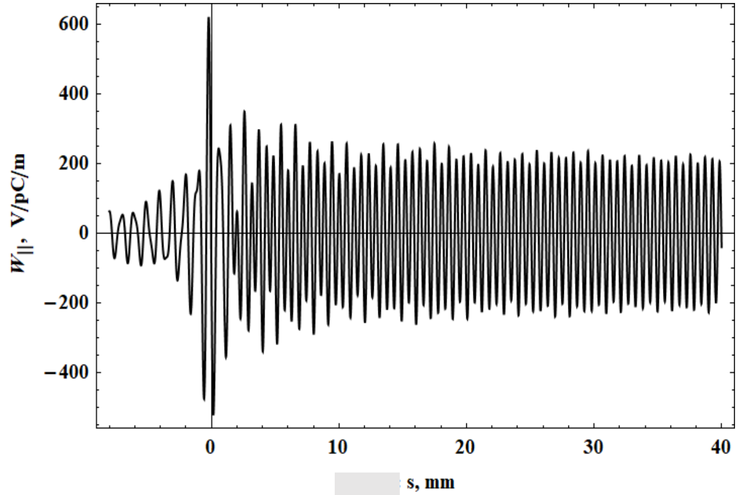
f_{01}



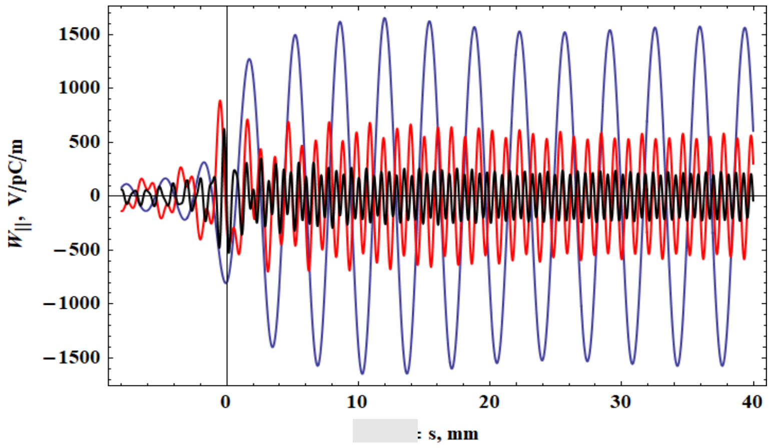
f_{02}



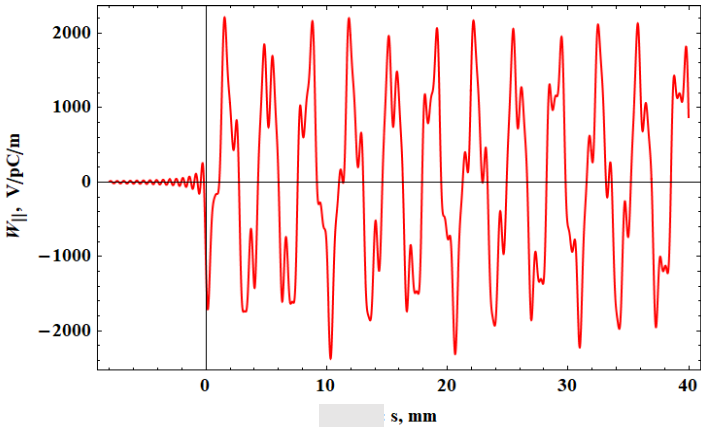
f_{03}



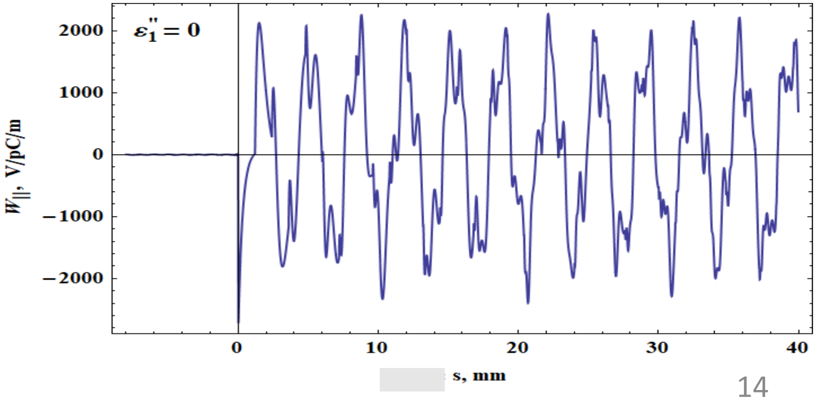
f_{01}, f_{02}, f_{03}



$f_{01}+f_{02}+f_{03}$

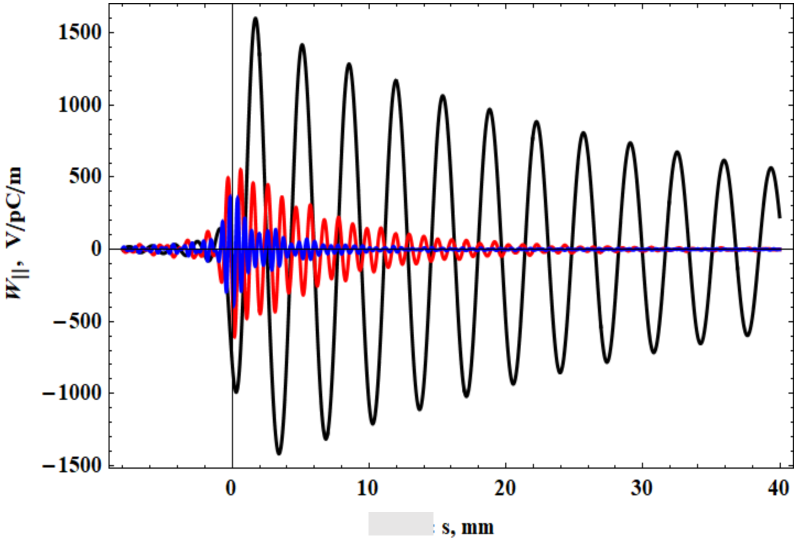


Full wake function

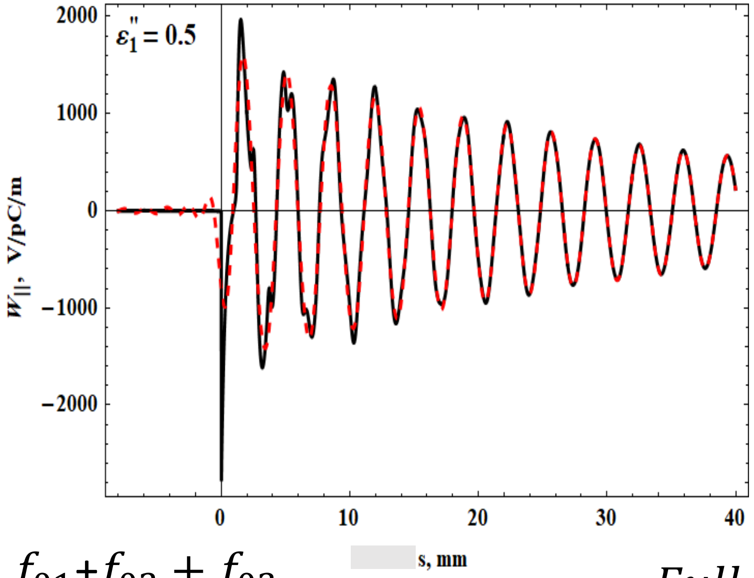


6. Impacts of higher resonance frequencies, $\varepsilon_1''=0.5$

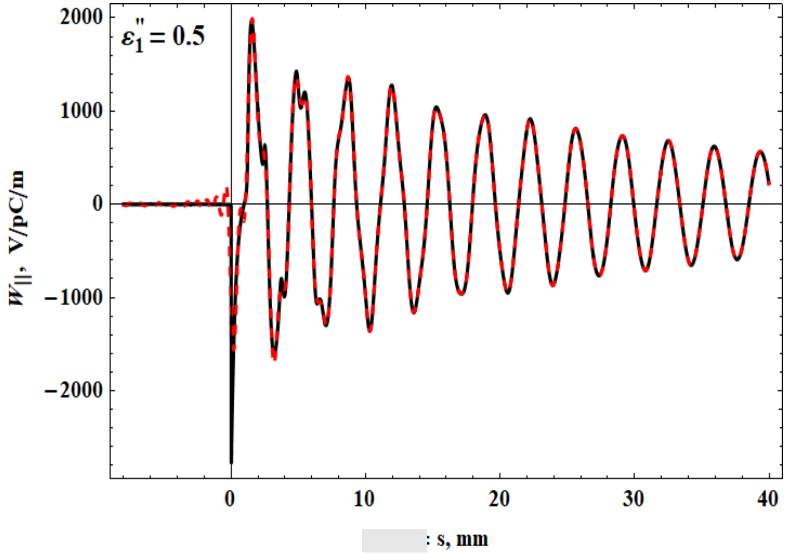
f_{01}, f_{02}, f_{03}



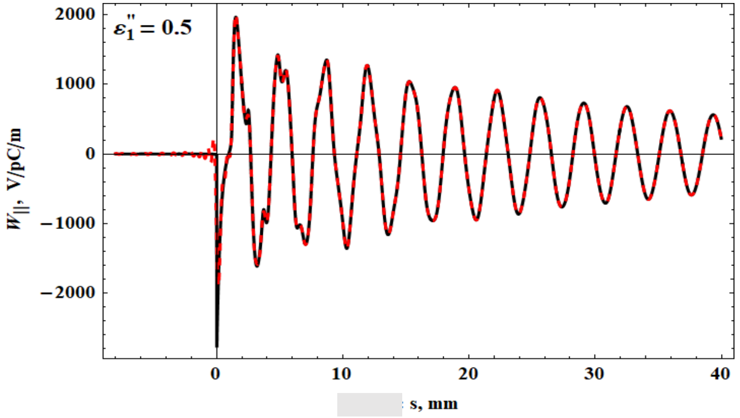
$Full\ wake\ function, f_{01}$



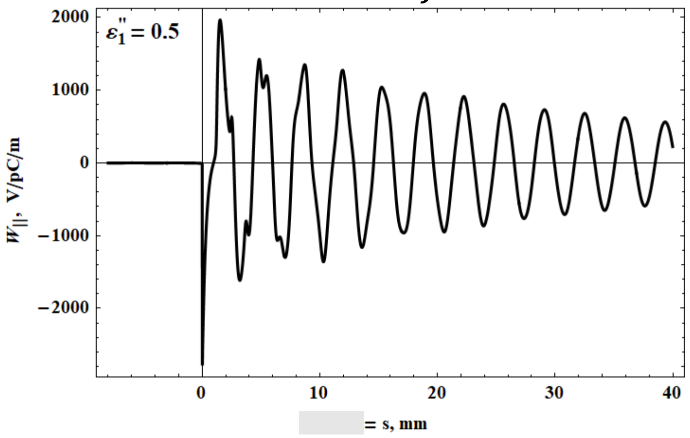
$Full\ wake\ function, f_{01}+f_{02}$



$Full\ wake\ function, f_{01}+f_{02} + f_{03}$

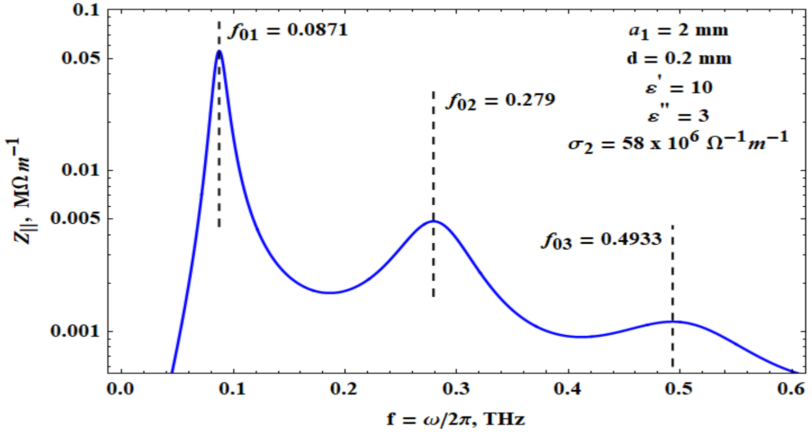


$Full\ wake\ function$

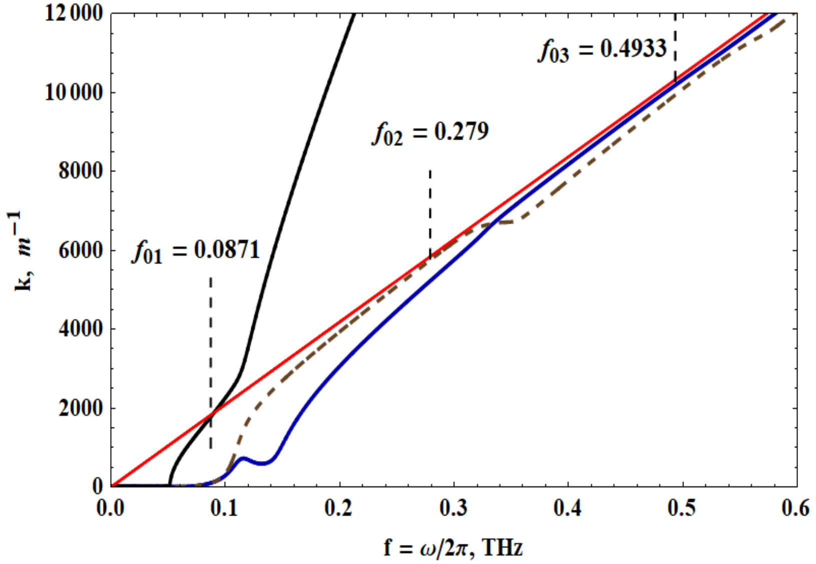


7. Impedance, wake function and dispersion curves for the case of high attenuation $\varepsilon_1''=3$

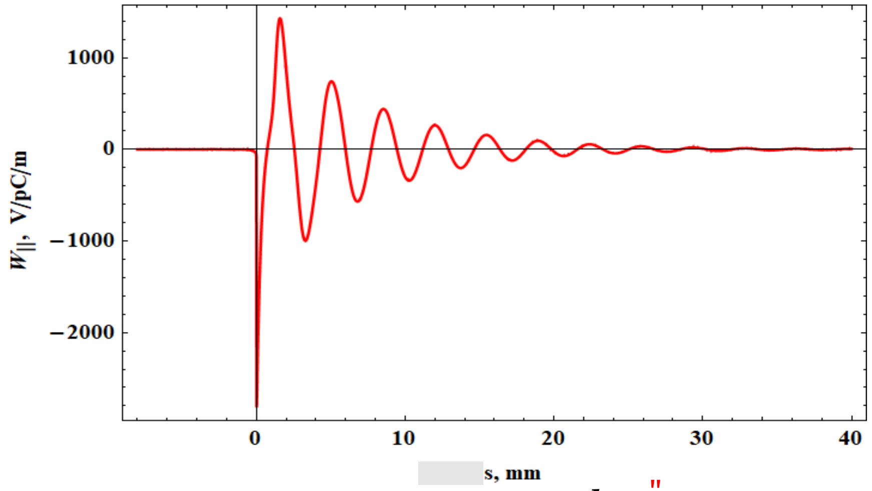
Impedance



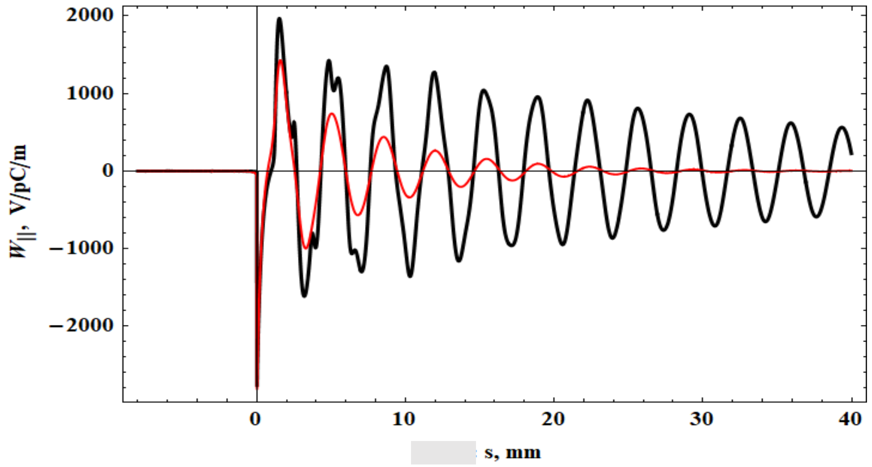
dispersion curves



wake function



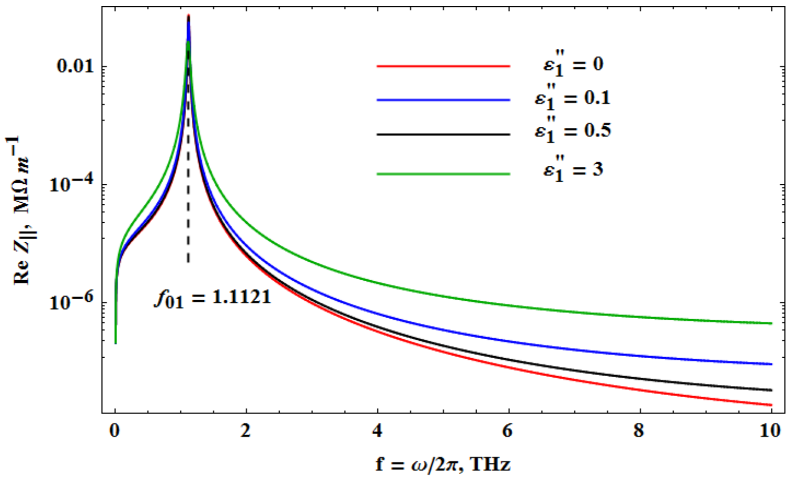
comparison with $\varepsilon_1''=0.5$

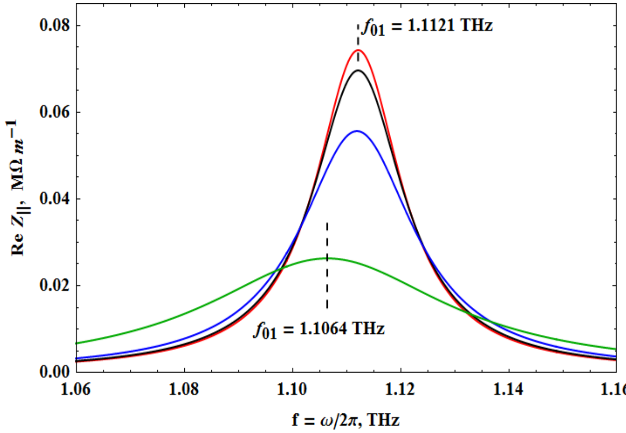


For relatively large attenuation values in the presence of multiple resonances, only the main resonance frequency turns out to be in phase with the particle velocity. Phase velocities corresponding to higher harmonics turn out to be greater than the speed of light, i.e. rapidly propagating. However, in this case, the amplitude of the wake function decreases rapidly.

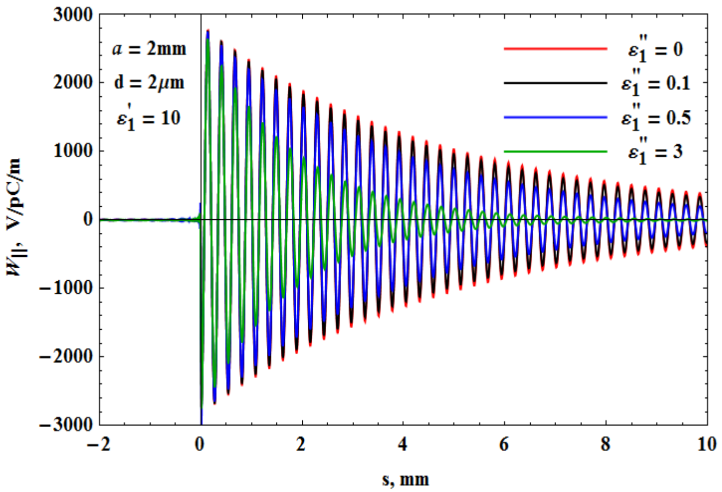
8. Case of thin dielectric cover, $d = 2\mu m$

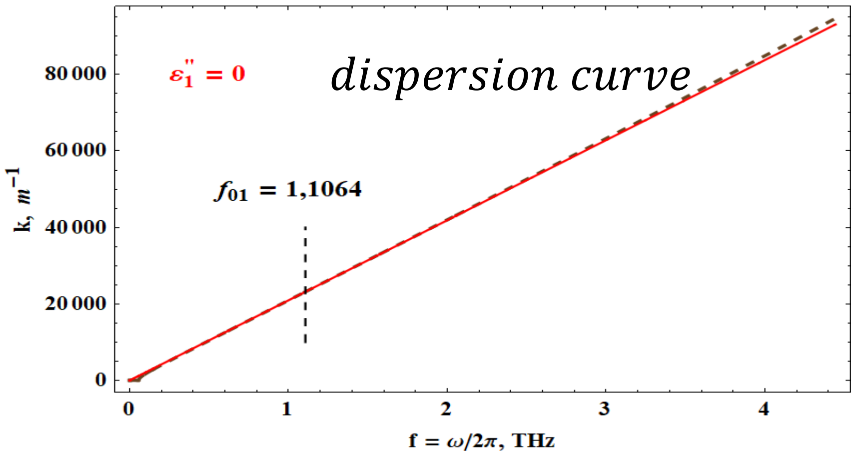
Impedance





wake function





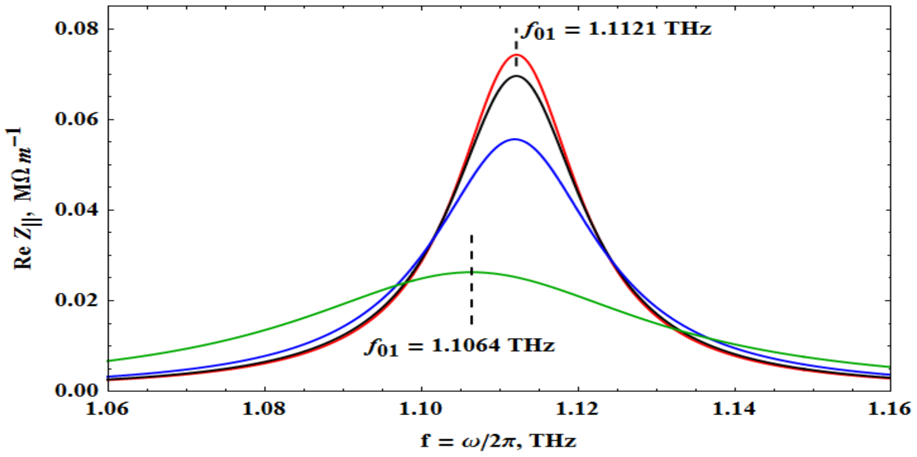
In this case, there is a single resonant frequency. The phase velocity of the harmonic at this frequency coincides with the velocity of the particle. As a result, the wakefield turns out to be monochromatic even in the immediate vicinity of the particle.
The advantage of a small thickness of the dielectric layer is also in the weak dependence of the wake field on the attenuation in the dielectric. The decrease in the amplitude of the wake function is mainly due to the finite conductivity of the outer wall.

8. Case of thin dielectric cover, $d = 2\mu m$

- For a thin lossless $(\varepsilon_1'' = 0)$ inner dielectric layer with an ideally conducting outer wall, the resonant frequency is determined by the expression

$$f_{rez} = \frac{c}{2\pi}\sqrt{\frac{\varepsilon_1'}{\varepsilon_1' - 1}\frac{2}{a_1 d}}$$

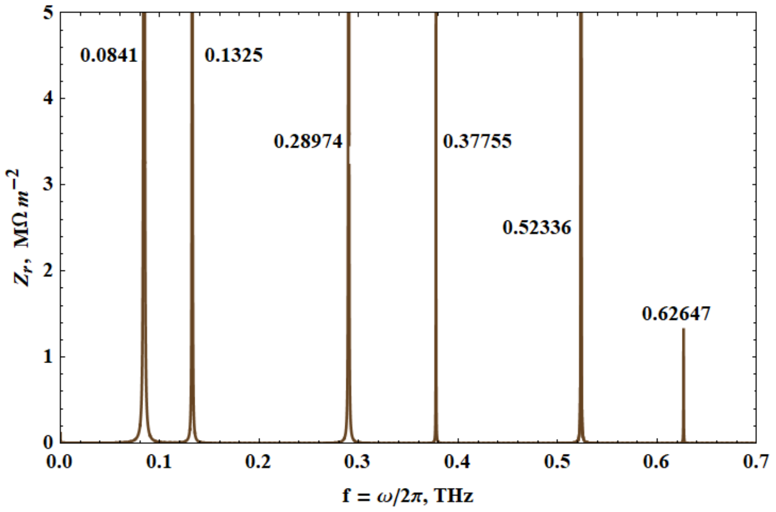
- The resonant frequency calculated using this formula for selected parameters gives $\boldsymbol{f_{rez} = 1.1254\ THz}$, whereas the same frequency calculated for the copper wall using exact formulas gives a slightly lower value $\boldsymbol{f_{rez} = 1.1121\ THz}$. The difference $\boldsymbol{\Delta f_{rez} = 13.2\ GHz}$ is due to the finite conductivity of the outer copper wall



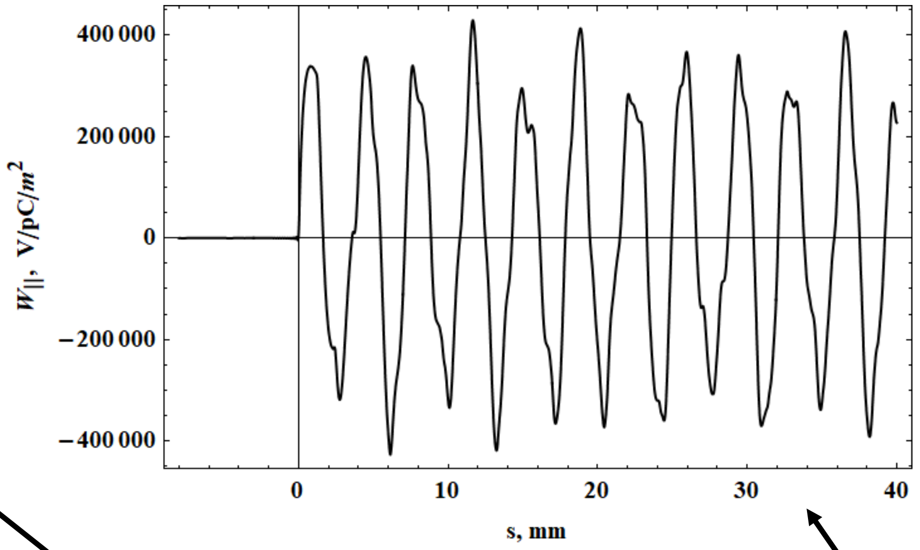
9. Transverse Impedance and Wake function. Thick inner cover

$$a = 2mm, d = 200\mu m, \varepsilon_1' = 10, \varepsilon_1'' = 0$$

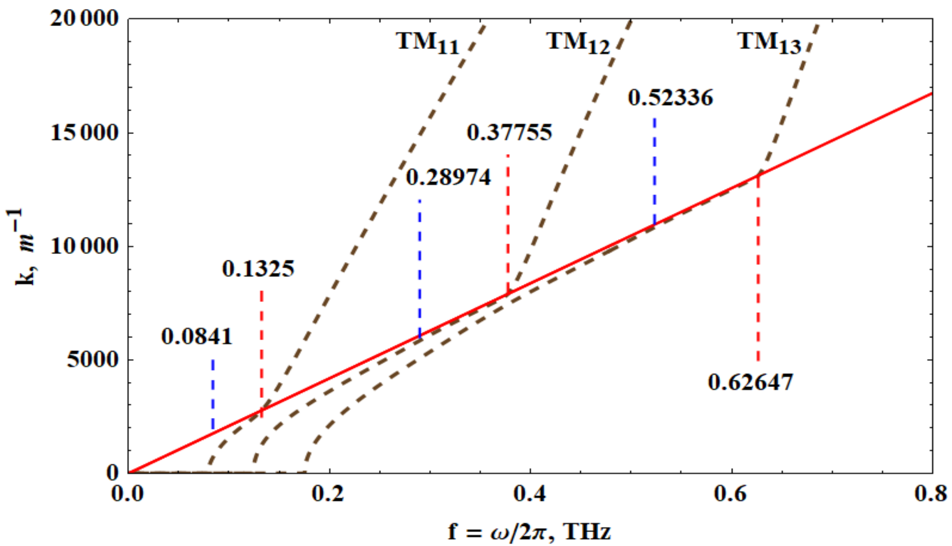
Transverse dipole Impedance



Transverse dipole Wake function



Dispersion curves

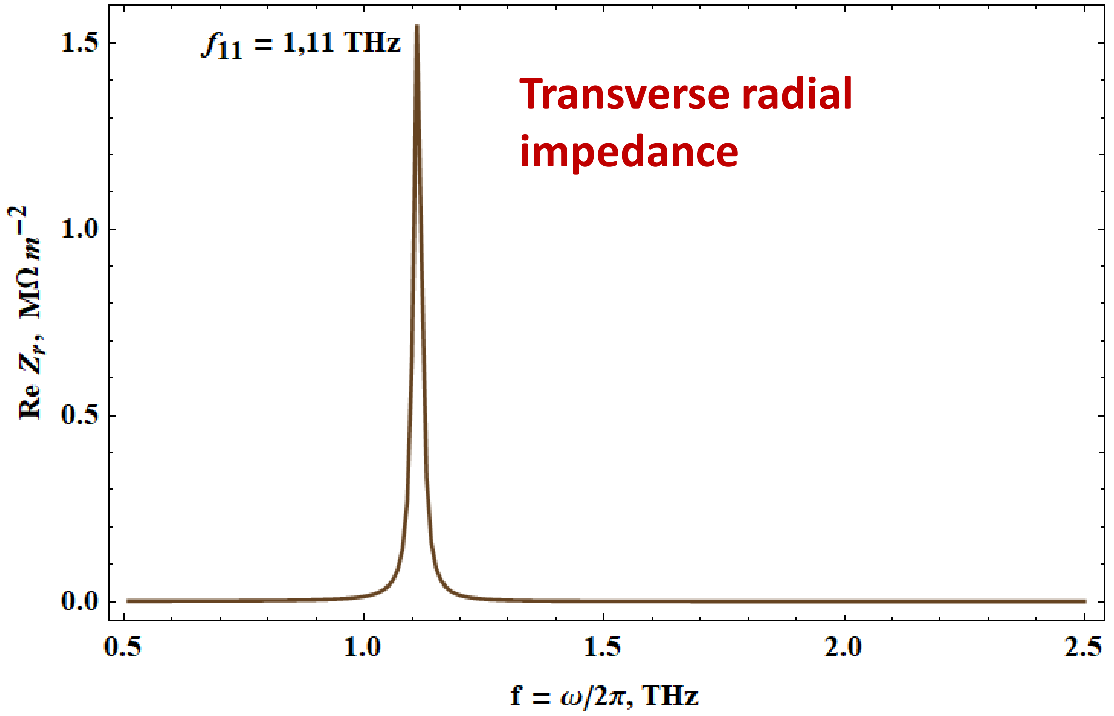


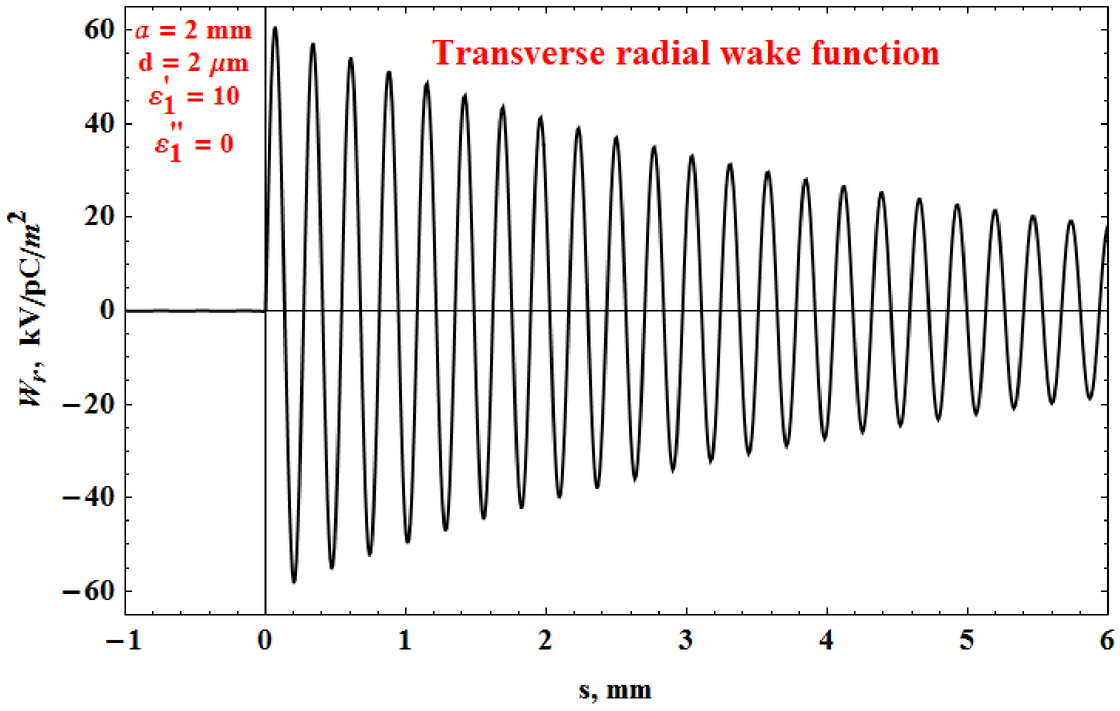
There are many resonances.
The wake function is not harmonic and is composed of contributions from several resonances.
Dispersion curves through one intersect with the line of inphase.

Coupling is strong

10. Transverse Impedance and Wake function. Thin inner cover

$$a = 2mm, d = 2\mu m, \varepsilon_1' = 10, \varepsilon_1'' = 0$$



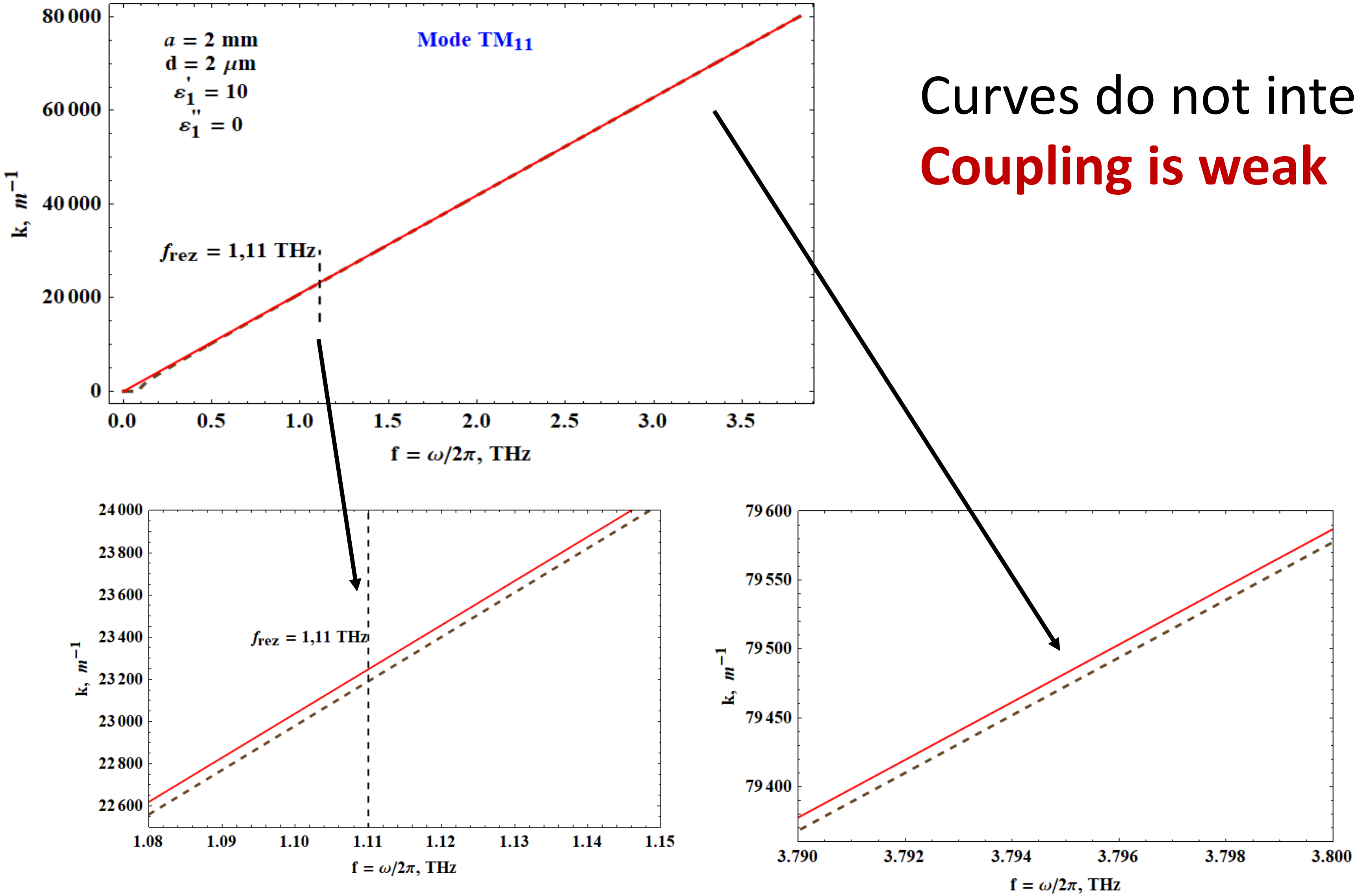


There is only one resonance.

The wake function is harmonious

Dispersion curve. Thick inner cover

$a = 2mm, d = 2\mu m, \varepsilon_1' = 10, \varepsilon_1'' = 0;\ Mode\ TM_{11}$

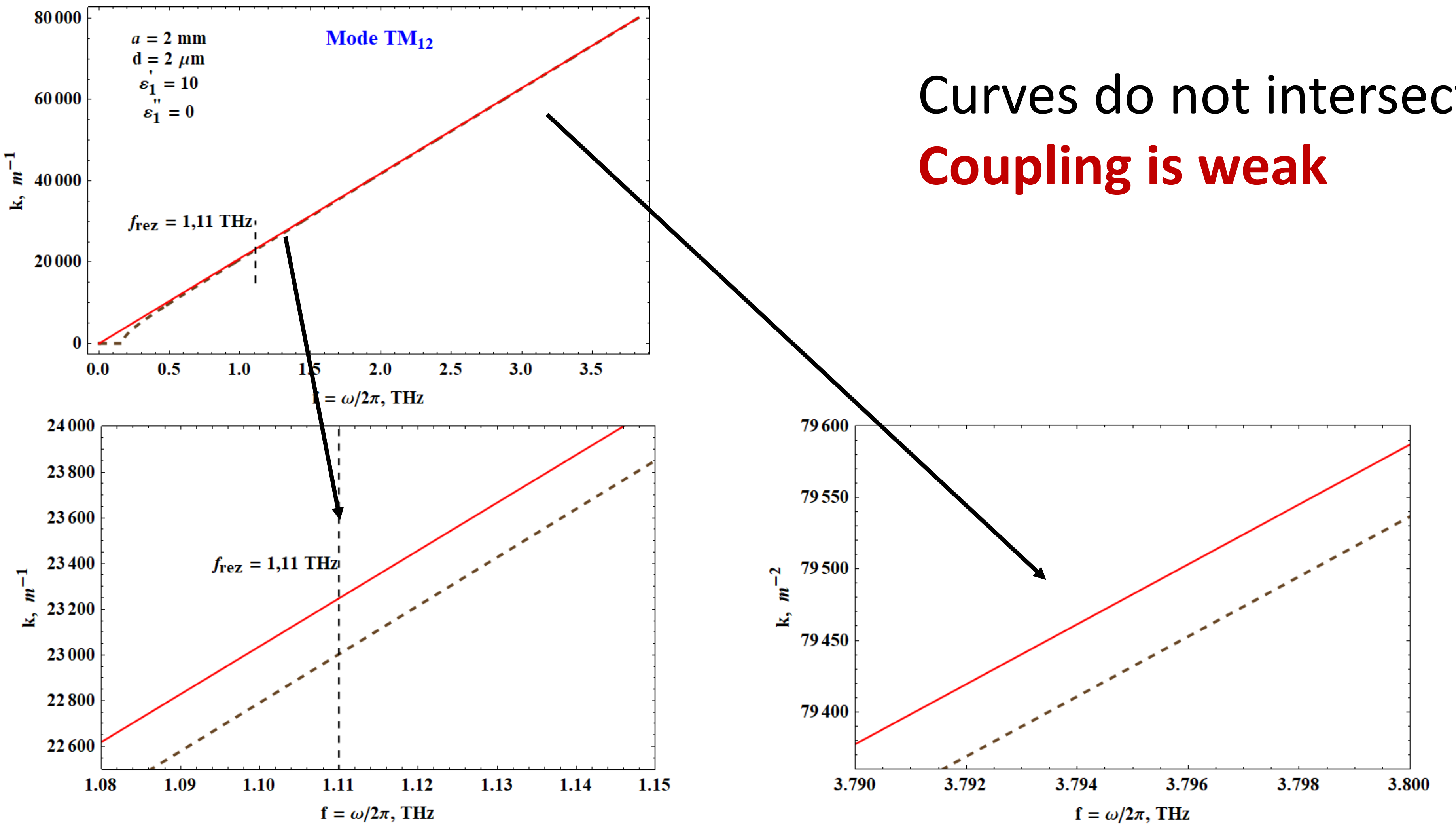


Curves do not intersect

Coupling is weak

Dispersion curve. Thick inner cover

$a = 2mm, d = 2\mu m, \varepsilon_1' = 10, \varepsilon_1'' = 0;\ Mode\ TM_{12}$

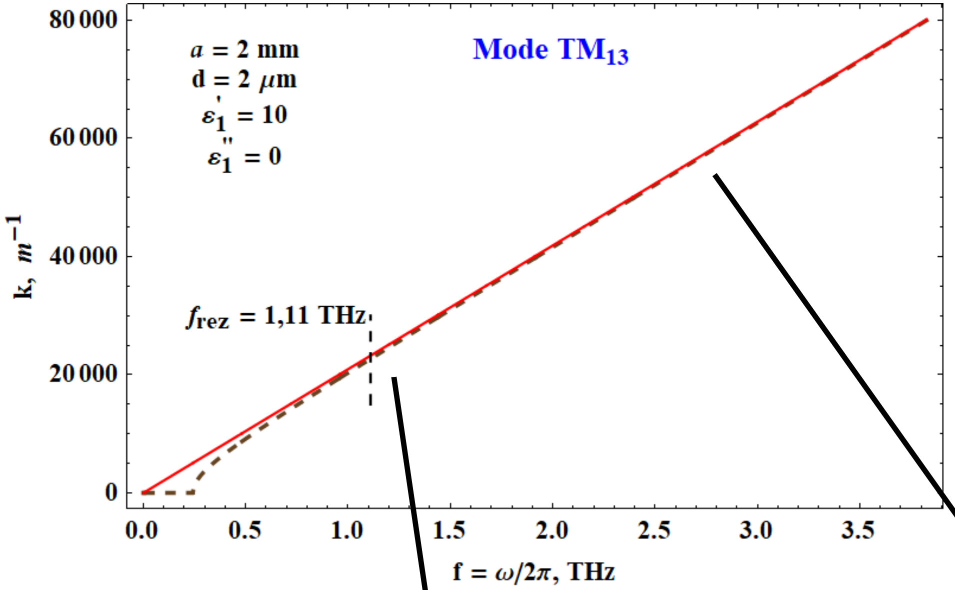


Curves do not intersect

Coupling is weak

Dispersion curve. <u>Thin inner cover</u>

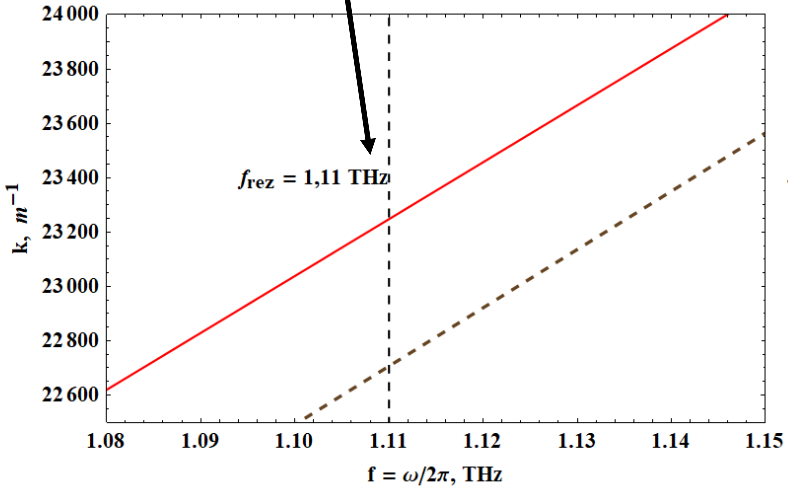
$a = 2mm, d = 2\mu m, \varepsilon_1' = 10, \varepsilon_1'' = 0;\ Mode\ TM_{13}$

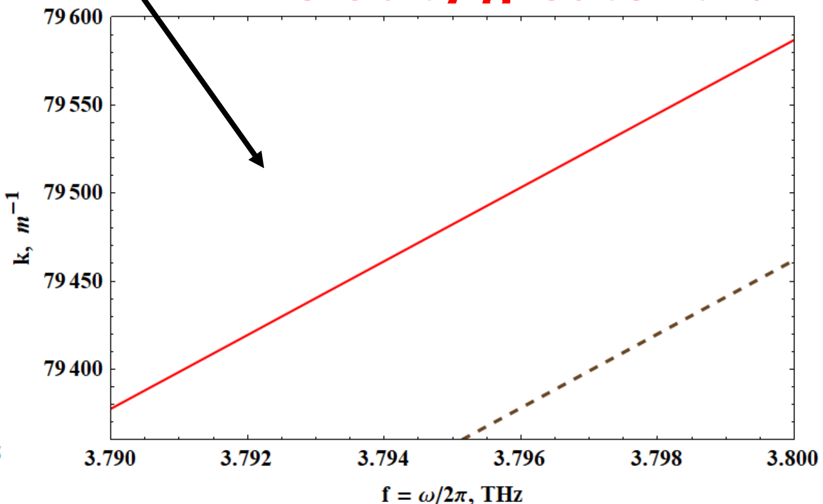


Curves do not intersect

Coupling is weak

Conclusion: In the case of a thin coating, slowly propagating dipole modes are not generated. Generated modes have a phase velocity greater than the speed of light.

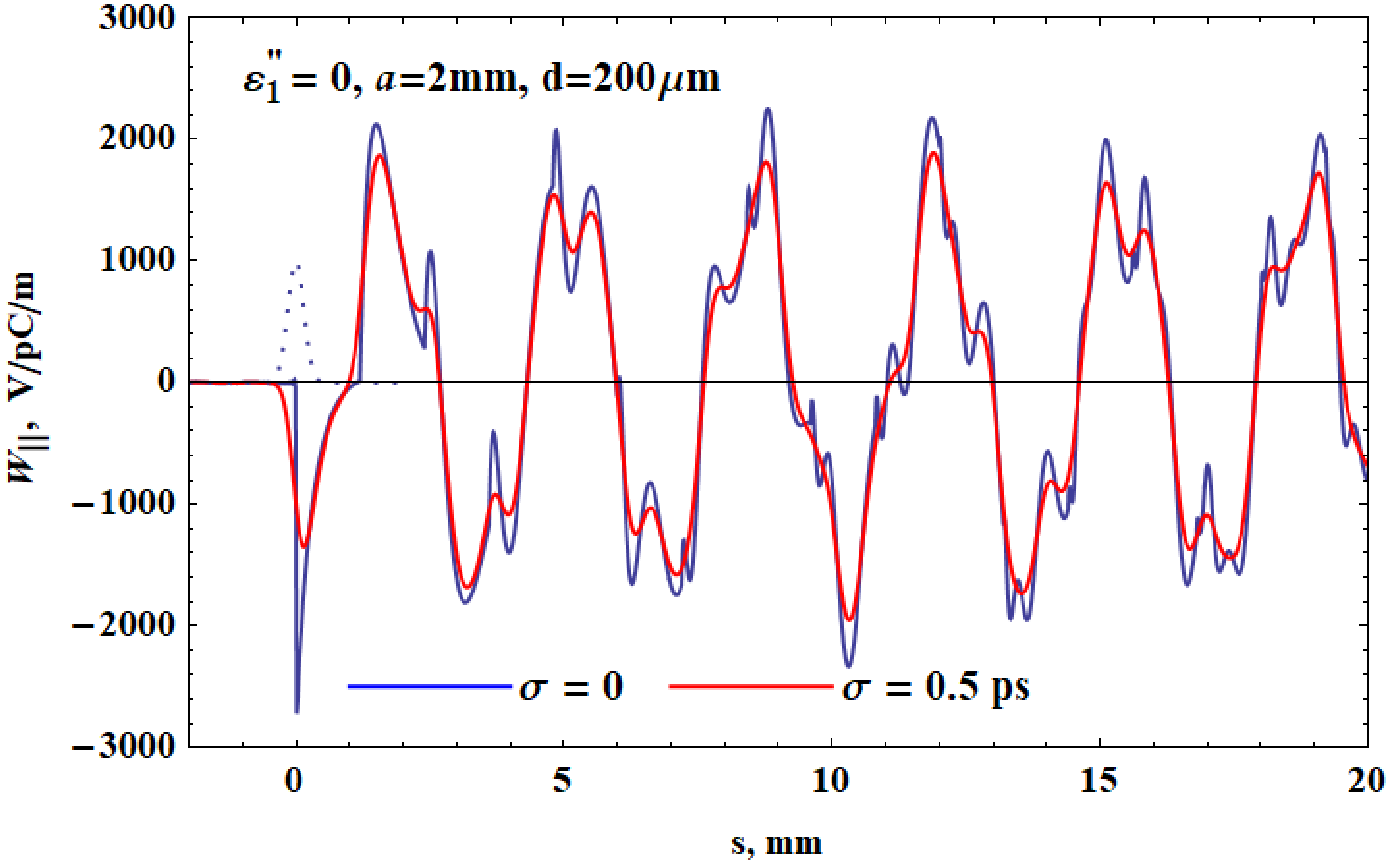




Thus, no effect of dipole modes on the transverse beam dynamics in the case of a thin internal dielectric coating

11. Case of Gaussian Bunch, rms length $\sigma = 0.5ps\ (150\mu m)$

Longitudinal wake potential



The presence of a bunch partially suppresses the contributions from higher modes, which in practice leads to the smoothing of additional bursts on the wake potential graph

12. CONCLUSION

- Studies have shown the possibility of optimal selection of the thickness and electromagnetic characteristics of the internal dielectric coating. For monochromatization of particle radiation, an important role is played the selection of the attenuation degree (selection of the optimal imaginary component of the dielectric constant of the dielectric). In particular, at ε_1'=10 and when the thickness of the inner layer is about 200 μm (that is, if the thickness is sufficiently large), the dielectric attenuation index should exceed 0.5 $(tg\delta = 0.05)$. On the other hand, the complete elimination of the influence of higher resonant frequencies can be avoided with attenuation greater than 3 $(tg\delta = 0.3)$. However, in this case, the amplitude of the wake function decreases rapidly.
- The preferred alternative is to use a thin dielectric coating with small attenuation. In this case, the equivalent circuit of the impedance of the structure is a parallel resonant oscillating circuit with the frequency-dependent complex shunt impedance and the emission of a particle is monochromatic, even in the immediate vicinity of it.
- The advantage of a small thickness of the dielectric layer is also in the weak dependence of the wake field on the attenuation in the dielectric. The decrease in the amplitude of the wake function is mainly due to the finite conductivity of the outer wall.
- Another advantage is the possibility of achieving high frequencies, since the resonant frequency is inversely proportional to the square of the layer thickness.
- No effect of dipole modes on the transverse beam dynamics in the case of a thin internal dielectric coating

Thank you!