Kharazmi University

Stability of Shear Rotational Viscose Dusty Plasma Systems Merged in the Helical Background Magnetic Field by Multi-Fluid Model

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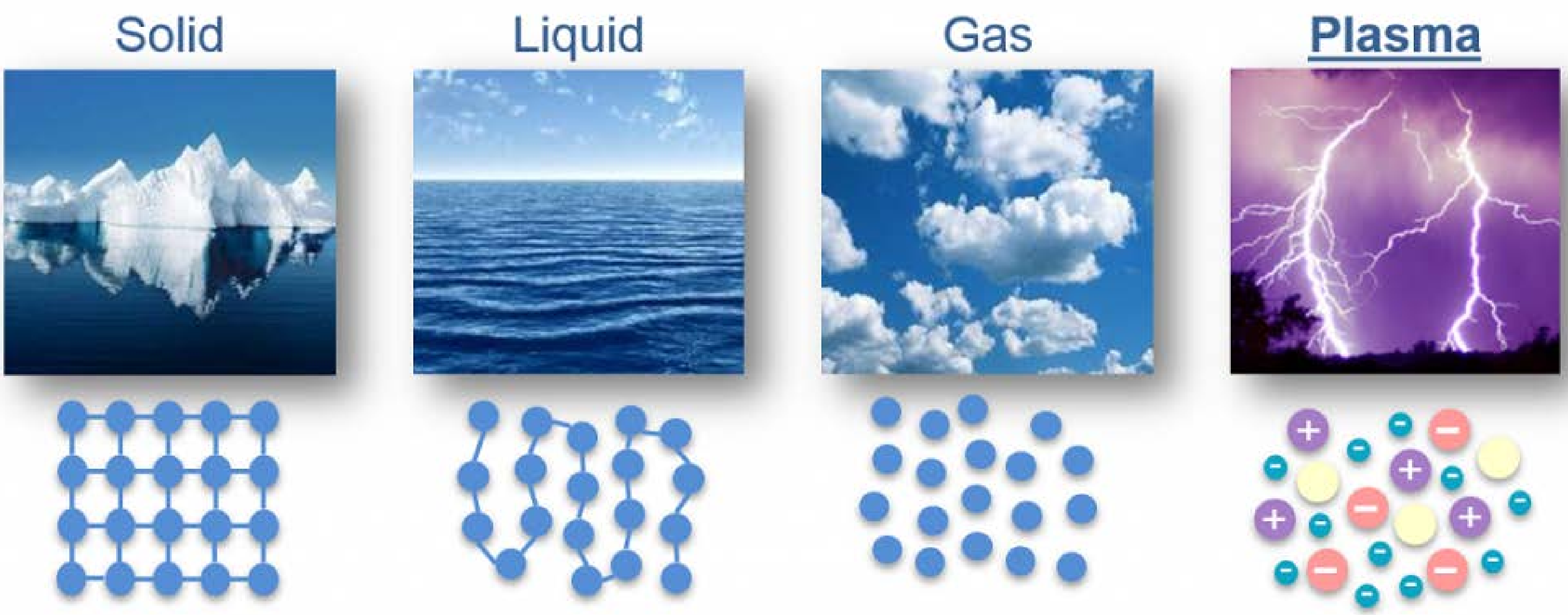
Plasma

✓Plasma description
✓Plasma general characteristic



Where the plasma can be found?

- Plasma in Universe

❖ Natural Plasma

✓ AstroPlasma

➢ *Interstellar medium*

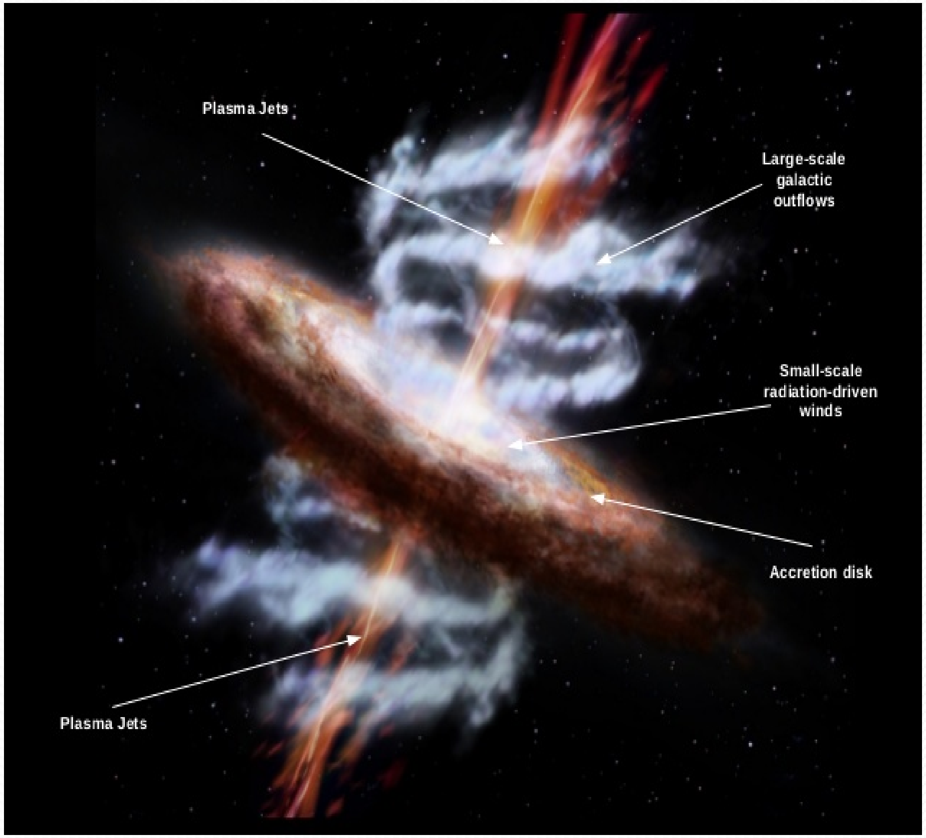
➢ *Intergalactic space*

✓ Earth Plasma

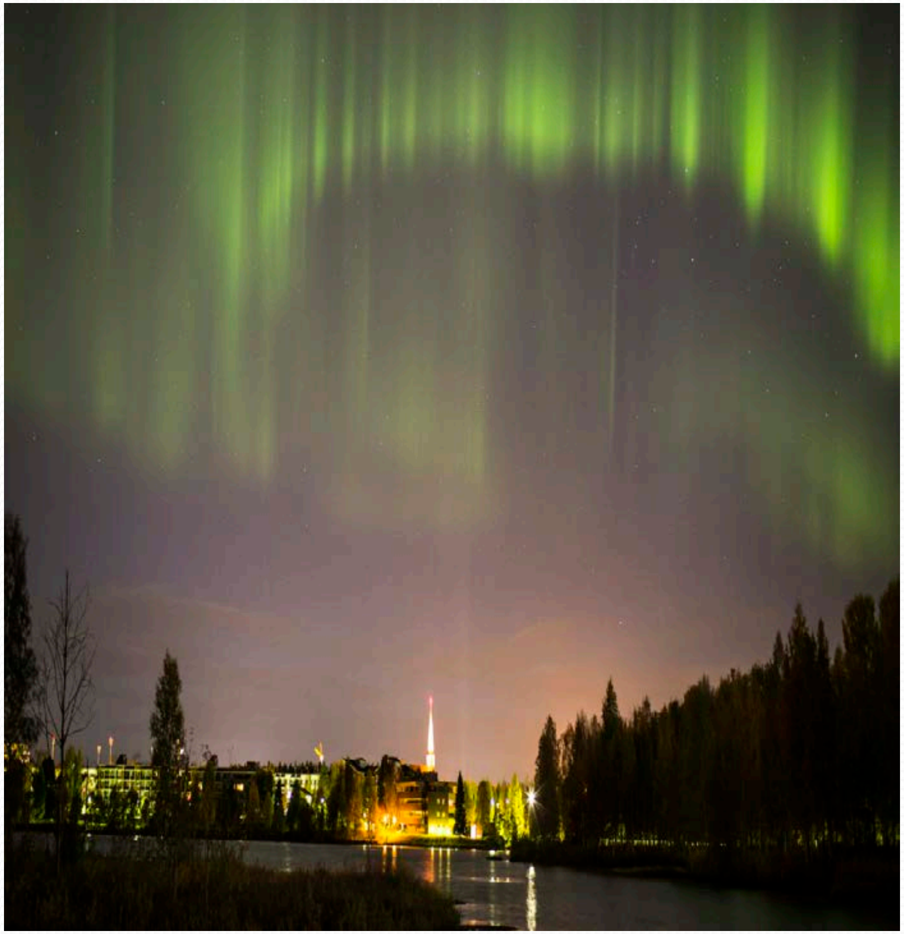
➢ *Lightning*

➢ *Polar aururate*

AstroPlasma



Earth Plasma



Shear Plasma systems

- ✓ layer astrophysical Plasma systems

- Accretion disks
- Active Galactic Nuclei
- Currents in the Sun's atmosphere

- ✓ Layer astrophysical flow plasma systems

- Solar winds
- Extragalactic and Galactic jets

Dusty Plasma systems

The characteristics of the dusty plasma systems

- Coulomb coupling parameter

- $$\Gamma = \frac{z_d^2 e^2}{k_B T_d\, a}$$

where a is the average inter particle distance and z_d , T_d , K_d are the charge on the dust particle, the temperature of dust component and the Boltzmann constant respectively.

- The weakly coupled dusty plasma system ($\Gamma \ll 1$
- The strongly coupled dusty plasma system ($\Gamma \gg 1$

Dispersion relation of the systems

Conditions of under study layer dusty plasma system

- A viscous, collisionless, incompressible, and magnetized rotating flow system of three-component plasma comprising electrons, ions, and negative dust grains has been considered.
- An external helical magnetic field consisting of an axial and azimuthal magnetic field assumes for this configuration, which it components are

$$\overrightarrow{B_0} = B_{0\varphi}(r)\boldsymbol{e}_{\varphi} + B_{0z}\boldsymbol{e}_z$$

Where $B_{0\varphi}(r) \propto \frac{B_0'}{r}$ and $B_{0z} = B_0$.

Equations

- Maxwell's equations coupled to the standard set of the hydrodynamic equations as

$$\frac{\partial}{\partial t} n_j + \boldsymbol{\nabla}.(n_j \boldsymbol{v}_j) = 0,$$

$$\rho_j \left(\frac{\partial}{\partial t} + \boldsymbol{v}_j . \nabla \right) \boldsymbol{v}_j = -\nabla p_j + q_j n_j (\boldsymbol{E} + \boldsymbol{v}_j \times \boldsymbol{B}) + \rho_j \boldsymbol{g} + \rho_j \eta \nabla^2 \boldsymbol{v}_j,$$

Dispersion relation of the system

The linearized momentum equation for the electron species has been obtained in r, ϕ, and z components.

$$\rho_e\left(-i\omega\widetilde{\boldsymbol{v}_e} - 2\Omega\widetilde{v_{e\theta}}\boldsymbol{e_r} + \frac{\kappa^2}{2\Omega}\widetilde{v_{er}}\boldsymbol{e_\varphi}\right) = -\nabla\widetilde{p_e} - e\,n_e\left(\widetilde{\boldsymbol{E}} + \widetilde{\boldsymbol{v}_e}\times\left(B_0\boldsymbol{e_z} + B_{0\varphi}(r)\boldsymbol{e_z}\right) + \boldsymbol{v_{e0}}\times\widetilde{\boldsymbol{B}}\right) + \rho_e\eta\left[\left(\frac{\partial^2\widetilde{v_{er}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{er}}}{\partial r} + \frac{\partial^2\widetilde{v_{er}}}{\partial z^2} - \frac{\widetilde{v_{er}}}{r^2}\right)\boldsymbol{e_r} + \left(\frac{\partial^2\widetilde{v_{e\varphi}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{e\varphi}}}{\partial r} + \frac{\partial^2\widetilde{v_{e\varphi}}}{\partial z^2} - \frac{\widetilde{v_{e\varphi}}}{r^2}\right)\boldsymbol{e_\varphi} + \left(\frac{\partial^2\widetilde{v_{ez}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{ez}}}{\partial r} + \frac{\partial^2\widetilde{v_{ez}}}{\partial z^2}\right)\boldsymbol{e_z}\right]$$

components

$$\rho_e\left(-i\omega\widetilde{v_{er}} - 2\,\Omega\widetilde{v_{e\varphi}}\right) = -e\,n_e\left(\widetilde{E_r} + r\Omega\widetilde{B_z} + B_0\widetilde{v_{e\varphi}} - B_{0\varphi}(r)\widetilde{v_{ez}}\right) - ik_r\widetilde{p_e} + \rho_e\eta\left(\frac{\partial^2\widetilde{v_{er}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{er}}}{\partial r} + \frac{\partial^2\widetilde{v_{er}}}{\partial z^2} - \frac{\widetilde{v_{er}}}{r^2}\right)$$

$$\rho_e\left(-i\omega\widetilde{v_{e\varphi}} + \frac{\kappa^2}{2\Omega}\widetilde{v_{er}}\right) = -e\,n_e\left(\widetilde{E_\varphi} - B_0\widetilde{v_{er}}\right) + \rho_e\eta\left(\frac{\partial^2\widetilde{v_{e\varphi}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{e\varphi}}}{\partial r} + \frac{\partial^2\widetilde{v_{e\varphi}}}{\partial z^2} - \frac{\widetilde{v_{e\varphi}}}{r^2}\right)$$

$$-i\omega\rho_e\widetilde{v_{ez}} = -ik_z\widetilde{p_e} - e\,n_e\left(\widetilde{E_z} - r\Omega\widetilde{B_r} + B_{0\varphi}(r)\widetilde{v_{er}}\right) + \rho_e\eta\left(\frac{\partial^2\widetilde{v_{ez}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{ez}}}{\partial r} + \frac{\partial^2\widetilde{v_{ez}}}{\partial z^2}\right)$$

$$\widetilde{p_e} = \frac{\omega}{k_z}\rho_e\widetilde{v_{ez}} - \frac{en_e}{ik_z}\left(\widetilde{E_z} - r\Omega\widetilde{B_r} + B_{0\varphi}(r)\widetilde{v_{er}}\right) + \frac{\rho_e\eta}{ik_z}\left(\frac{\partial^2\widetilde{v_{ez}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{ez}}}{\partial r} + \frac{\partial^2\widetilde{v_{ez}}}{\partial z^2}\right)$$

Substituting $\widetilde{p_e}$ into previous equations yields,

$$-i\omega\widetilde{v_{e\varphi}} + \left(\frac{\varkappa^2}{2\Omega} - \Omega_e\right)\widetilde{v_{er}} = \frac{\omega\Omega_e}{k_z B_0}\widetilde{B_r} + \eta\left(-k_z{}^2\widetilde{v_{e\varphi}} + \frac{\partial^2\widetilde{v_{e\varphi}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{e\varphi}}}{\partial r} - \frac{\widetilde{v_{e\varphi}}}{r^2}\right)$$

$$\begin{aligned}-i\omega\widetilde{v_{er}} - (2\,\Omega - \Omega_e)\widetilde{v_{e\varphi}} \\ &= -\frac{i\omega}{k_z{}^2}\frac{\partial}{\partial r}\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\widetilde{v_{er}} - \frac{1}{ik_z}\eta\frac{\partial}{\partial r}\left(-k_z{}^2\widetilde{v_{ez}} + \frac{\partial^2\widetilde{v_{ez}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{ez}}}{\partial r}\right) \\ &+ \eta\left(-k_z{}^2\widetilde{v_{er}} + \frac{\partial^2\widetilde{v_{er}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{er}}}{\partial r} - \frac{\widetilde{v_{er}}}{r^2}\right) - \frac{\omega\Omega_e}{k_z B_0}\widetilde{B_\varphi} + \frac{i\Omega_e}{k_z B_0}\frac{d\Omega}{d\ln r}\widetilde{B_r} + i\Omega_e d\,\widetilde{v_{er}}\end{aligned}$$

From the same process, for ions will be obtained as follow,

$$-i\omega\widetilde{v_{i\varphi}} + \left(\frac{\varkappa^2}{2\Omega} + \Omega_i\right)\widetilde{v_{ir}} = -\frac{\omega\Omega_i}{k_z B_0}\widetilde{B_r} + \eta\left(-k_z{}^2\widetilde{v_{i\varphi}} + \frac{\partial^2\widetilde{v_{i\varphi}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{i\varphi}}}{\partial r} - \frac{\widetilde{v_{i\varphi}}}{r^2}\right)$$

$$-i\omega\widetilde{v_{ir}} - (2\,\Omega + \Omega_i)\widetilde{v_{i\varphi}} = -\frac{i\omega}{k_z{}^2}\frac{\partial}{\partial r}\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\widetilde{v_{ir}} - \frac{1}{ik_z}\eta\frac{\partial}{\partial r}\left(-k_z{}^2\widetilde{v_{iz}} + \frac{\partial^2\widetilde{v_{iz}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{iz}}}{\partial r}\right) +$$
$$\eta\left(-k_z{}^2\widetilde{v_{ir}} + \frac{\partial^2\widetilde{v_{ir}}}{\partial r^2} + \frac{1}{r}\frac{\partial\widetilde{v_{ir}}}{\partial r} - \frac{\widetilde{v_{ir}}}{r^2}\right) + \frac{\omega\Omega_i}{k_z B_0}\widetilde{B_\varphi} - \frac{i\Omega_i}{k_z B_0}\frac{d\Omega}{d\ln r}\widetilde{B_r} + i\Omega_i d\widetilde{v_{ir}}$$

And for the dust grains yields,

$$-i\omega \widetilde{v_{d\varphi}} + \left(\frac{\varkappa^2}{2\Omega} - \Omega_d\right)\widetilde{v_{dr}} = \frac{\omega\Omega_d}{k_z B_0}\widetilde{B_r} + \eta(-k_z{}^2\widetilde{v_{d\varphi}} + \frac{\partial^2 \widetilde{v_{d\varphi}}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{v_{dr}}}{\partial r} - \frac{\widetilde{v_{d\varphi}}}{r^2})$$

$$-i\omega \widetilde{v_{dr}} - (2\,\Omega - \Omega_d)\widetilde{v_{d\varphi}} = -\frac{i\omega}{k_z{}^2}\frac{\partial}{\partial r}\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)\widetilde{v_{dr}} - \frac{1}{ik_z}\eta\frac{\partial}{\partial r}\left(-k_z{}^2\widetilde{v_{dz}} + \frac{\partial^2 \widetilde{v_{dz}}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{v_{dz}}}{\partial r}\right) + \eta\left(-k_z{}^2\widetilde{v_{dr}} + \frac{\partial^2 \widetilde{v_{dr}}}{\partial r^2} + \frac{1}{r}\frac{\partial \widetilde{v_{dr}}}{\partial r} - \frac{\widetilde{v_{dr}}}{r^2}\right) - \frac{\omega\Omega_d}{k_z B_0}\widetilde{B_\varphi} + \frac{i\Omega_d}{k_z B_0}\frac{d\Omega}{d\ln r}\widetilde{B_r} + i\Omega_d d\,\widetilde{v_{dr}}$$

Use is made of local approximation coupled with obtained relations, after a lot algebra, the dispersion relation of the layer viscose dusty plasma system obtains as,

General Dispersion Relation

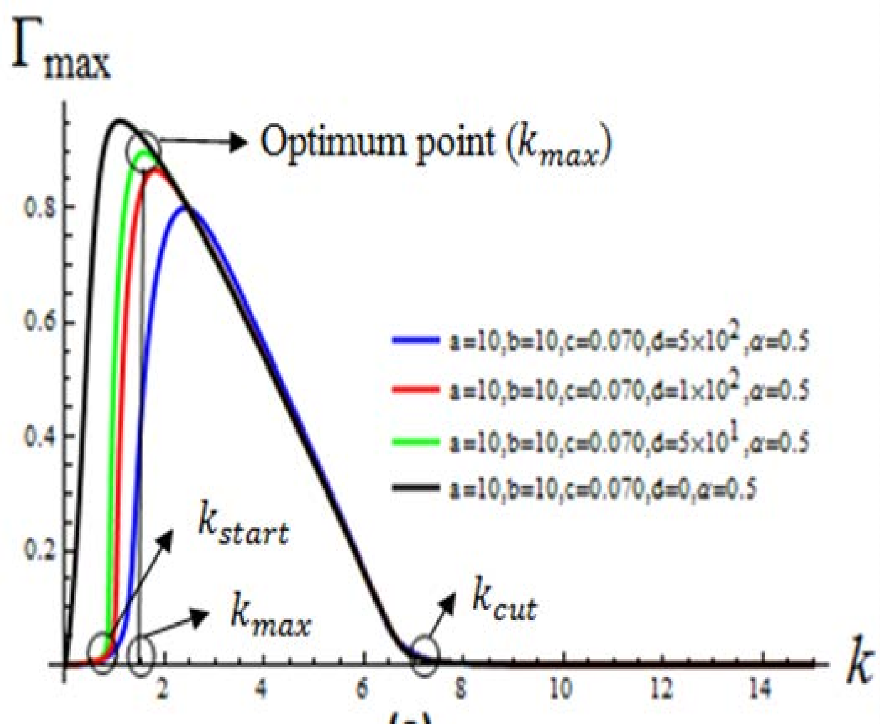
$$
\begin{aligned}
&(\eta k^2 - i\omega)^2 \frac{k^2}{k_z^2}\{ik^2 v_{Ai}^2[\eta k^2 - i\omega + \frac{k_z^2}{k^2}(\frac{\kappa^2}{2\Omega}+\Omega_i)\frac{i}{\omega}\frac{d\Omega}{d\ln r}] + (\omega\Omega_i^2 + \omega\Omega_e^2\frac{v_{Ai}^2}{v_{Ae}^2}) + \\
&\frac{ik_z^2 v_{Ai}^2}{(\eta k^2 - i\omega)}(2\Omega-\Omega_e)(\frac{\kappa^2}{2\Omega}+\Omega_i) + A[(\eta k^2 - i\omega)(\eta k^2 - i\omega - \frac{k_z^2}{k^2} i\Omega_d d)(\frac{k^2}{k_z^2}) + (\Omega_e - \Omega_d)(\frac{\kappa^2}{2\Omega}+ \\
&\Omega_i) + (2\Omega-\Omega_e)(\frac{\kappa^2}{2\Omega}-\Omega_d)]\} \times \{ik_z^2 v_{Ai}^2[\frac{-i}{\omega}\frac{d\Omega}{d\ln r}\frac{(\frac{\kappa^2}{2\Omega}-\Omega_e)}{(\eta k^2-i\omega)}(\eta k^2 - i\omega - \frac{k_z^2}{k^2} i\Omega_i d) + \\
&\frac{k^2}{k_z^2(\eta k^2 - i\omega)}(\eta k^2 - i\omega - \frac{k_z^2}{k^2}i\Omega_e d)(\eta k^2 - i\omega - \frac{k_z^2}{k^2} i\Omega_i d)] - \frac{(2\Omega+\Omega_i)ik^2 v_{Ai}^2}{(\eta k^2-i\omega)}(\frac{\kappa^2}{2\Omega}-\Omega_e) + \\
&[\frac{v_{Ai}^2(\omega\Omega_e^2)}{v_{Ae}^2(\eta k^2-i\omega)}(\eta k^2 - i\omega - \frac{k_z^2}{k^2}i\Omega_i d) + \frac{(\omega\Omega_i^2)}{(\eta k^2-i\omega)}(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_e d)] + A[\frac{(2\Omega-\Omega_d)}{(\eta k^2-i\omega)}(\frac{\kappa^2}{2\Omega}- \\
&\Omega_e)(\eta k^2 - i\omega - \frac{k_z^2}{k^2}i\Omega_i d) + (\frac{k^2}{k_z^2})(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_e d)(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_i d) - (2\Omega+ \\
&\Omega_i)[\frac{(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_d d)}{(\eta k^2-i\omega)}(\frac{\kappa^2}{2\Omega}-\Omega_e) - \frac{(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_e d)}{(\eta k^2-i\omega)}(\frac{\kappa^2}{2\Omega}-\Omega_d)]] \\
&= \\
&\{(\frac{\kappa^2}{2\Omega}-\Omega_e)((\eta k^2-i\omega)ik^2v_{Ai}^2+\omega\Omega_i^2) + (\frac{\kappa^2}{2\Omega}+\Omega_i)[ik_z^2v_{Ai}^2(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_e d)\frac{k^2}{k_z^2} + \\
&\frac{k_z^2 v_{Ai}^2}{\omega}\frac{d\Omega}{d\ln r}(\frac{\kappa^2}{2\Omega}-\Omega_e) + \omega\Omega_e^2\frac{v_{Ai}^2}{v_{Ae}^2}] + A[(2\Omega-\Omega_d)(\frac{\kappa^2}{2\Omega}-\Omega_e)(\frac{\kappa^2}{2\Omega}+\Omega_i) + (\frac{k^2}{k_z^2})(\eta k^2 - \\
&i\omega)[\Omega_d(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_e d) - \Omega_e(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_d d) + (\frac{\kappa^2}{2\Omega}+\Omega_i)(\eta k^2-i\omega- \\
&\frac{k_z^2}{k^2}i\Omega_e d)]]\} \times \{ik_z^2 v_{Ai}^2(\eta k^2-i\omega)(\frac{k^2}{k_z^2})(\frac{-i}{\omega}\frac{d\Omega}{d\ln r}(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_i d) - (2\Omega-\Omega_e)) - \\
&\omega\Omega_i^2(2\Omega-\Omega_e) - (2\Omega+\Omega_i)(ik^2v_{Ai}^2(\eta k^2-i\omega)+\omega\Omega_e^2\frac{v_{Ai}^2}{v_{Ae}^2}) - A[\frac{\Omega_e}{\Omega_d}(2\Omega+\Omega_i)(2\Omega- \\
&\Omega_e)(\frac{\kappa^2}{2\Omega}-\Omega_d) + (\frac{k^2}{k_z^2})(\eta k^2-i\omega)((\Omega_d-\Omega_e)(\eta k^2-i\omega-\frac{k_z^2}{k^2}i\Omega_i d) + \frac{\Omega_e}{\Omega_d}(2\Omega+\Omega_i)(\eta k^2- \\
&i\omega-\frac{k_z^2}{k^2}i\Omega_d d))]\}.
\end{aligned}
$$

The obtained dispersion relation in the high-frequency approximation $(\Omega_e \gg \omega \gg \Omega \gg \Omega_i \gg \Omega_d)$

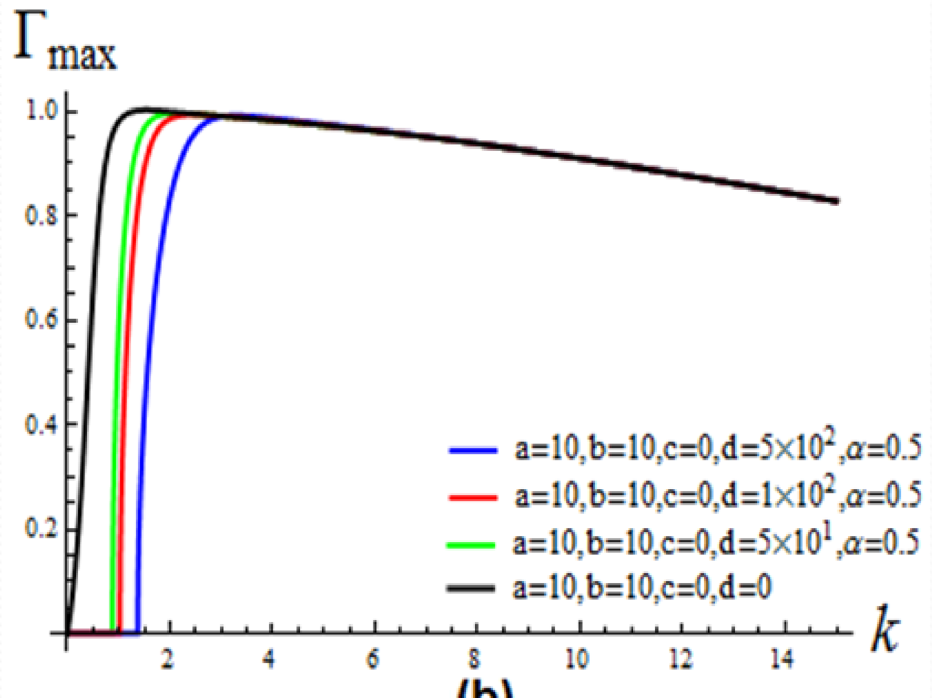
$$
\begin{aligned}
&(cK^2 - i\omega')^2 aK^2 (iK^2(cK - i\omega') + \omega' + \omega' \frac{\mathrm{mi}}{\mathrm{me}}(1-\alpha) - i\frac{b}{2}\frac{1}{a(cK^2 - i\omega')}\frac{\mathrm{mi}}{\mathrm{me}} \\
&\quad + \omega' \frac{\mathrm{mi}}{\mathrm{md}}\alpha)(iK^2((cK^2 - \frac{i}{aK^2}\frac{\mathrm{mi}}{\mathrm{me}}d) + \frac{i}{\omega'}\frac{3b}{2}\frac{1}{aK^2}\frac{\mathrm{mi}}{\mathrm{me}} + \frac{\mathrm{mi}}{\mathrm{me}}\frac{2b}{(cK^2 - i\omega')}) \\
&\quad + \omega' \frac{\mathrm{mi}}{\mathrm{me}}(1-\alpha) + \frac{(cK^2 - \frac{i}{aK^2}\frac{\mathrm{mi}}{\mathrm{me}}d)}{(cK^2 - i\omega')}(\omega' + \omega' \frac{\mathrm{mi}}{\mathrm{md}}\alpha)) - (iK^2\frac{b}{2}(cK^2 \\
&\quad - \frac{i}{aK^2}\frac{\mathrm{mi}}{\mathrm{me}}d) - iK^2\frac{\mathrm{mi}}{\mathrm{me}}(cK^2 - i\omega') + \frac{b}{2}\omega' \frac{\mathrm{mi}}{\mathrm{me}}(1-\alpha) - \omega' \frac{\mathrm{mi}}{\mathrm{me}} \\
&\quad - \omega' \frac{\mathrm{mi}}{\mathrm{me}}\frac{\mathrm{mi}}{\mathrm{md}}\alpha)(iK^2(cK^2 - i\omega')(\frac{\mathrm{mi}}{\mathrm{me}} - \frac{i}{\omega}\frac{3b}{2}(cK^2 - i\omega')) + \frac{\mathrm{mi}}{\mathrm{me}}\omega'(\frac{\mathrm{mi}}{\mathrm{md}}\alpha + 1) \\
&\quad - 2b(\omega' \frac{\mathrm{mi}}{\mathrm{me}}(1-\alpha) + iK^2(cK^2 - i\omega'))) = 0
\end{aligned}
$$

Discussions

The effect of helicity on the stability of the system

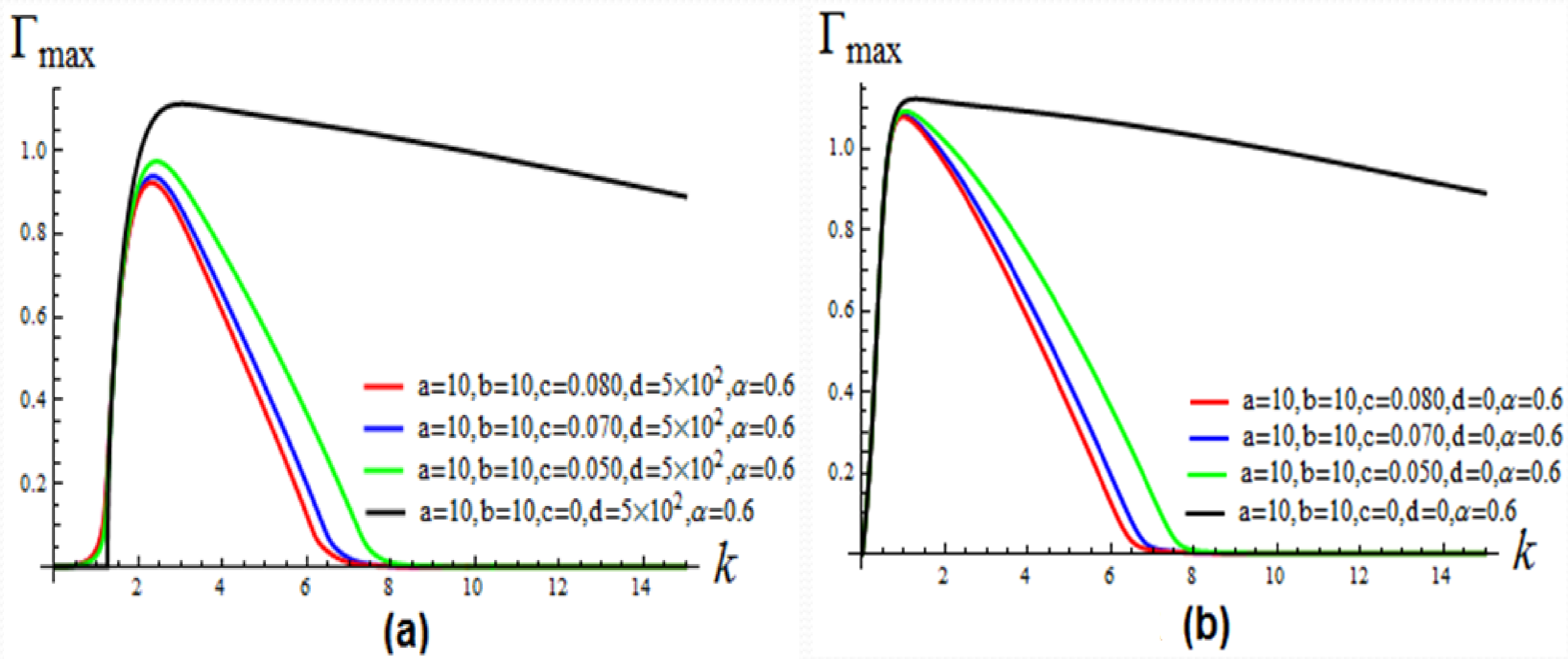


(a)

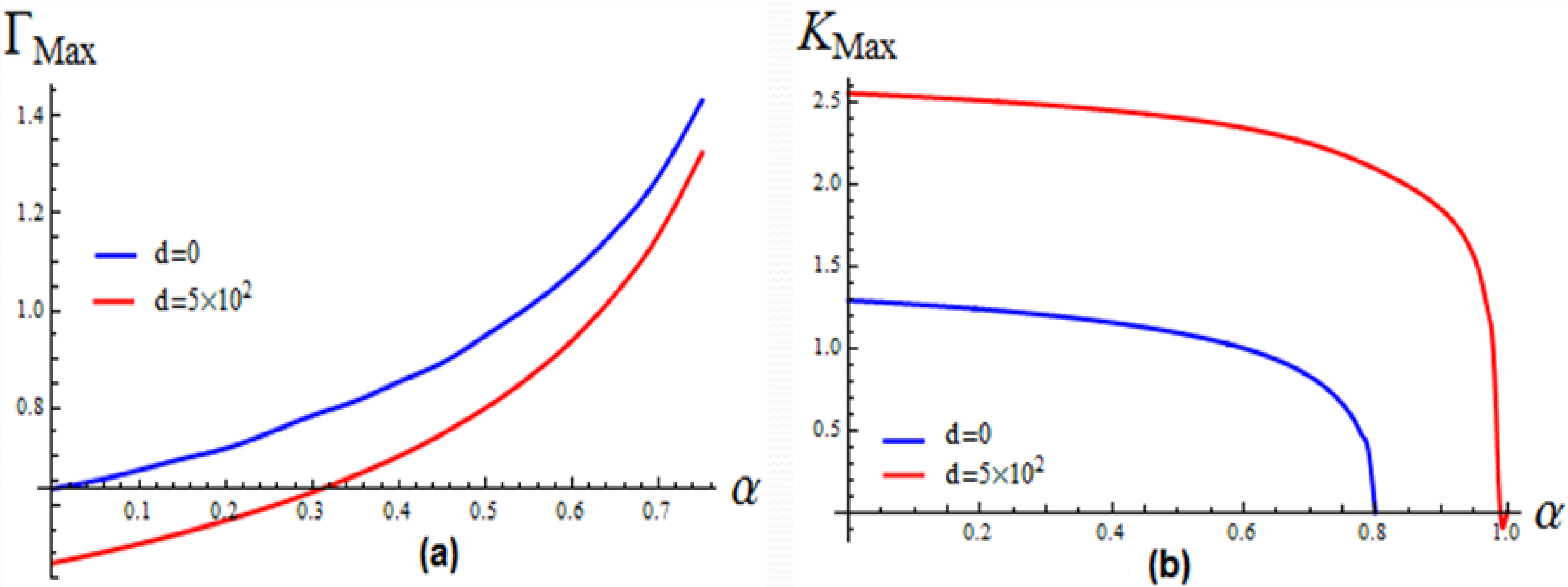


(b)

The effect of viscosity on the stability of the system



(a) graph of maximum growth rate versus α (density ratio) and (b) graph of maximum normalized wave number versus α



Conclusions

(1) The magnetic field helicity has a stabilization role against the magnetorotational instability of the Keplerian rotating flow system due to contraction of the unstable wavelength region and decreasing the maximum growth rate of the instability.

(2) Transport of angular momentum in viscose Keplerian disks in the presence of azimuthal magnetic field component due to HMRI is more rapidly than those in the corresponding non-viscose case.

(3) The viscosity term has a more considerable role on the HMRI than those on the corresponding MRI, especially on the start-up wave number and maximum growth rate of instability.

(4) The instability of the Keplerian rotational dusty plasma flow in the presence of azimuthal magnetic field component (HMRI) can be obtained in the all possible value of particle density ratios in contrast with it for axial magnetic field structure (MRI).

Thanks for your attention

And the learning process continues ...