Wakefield of structured dense bunches with diverse charges in a one-dimensional plasma model

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Three interesting and challenging directions of plasma usage in accelerator physics

Plasma lens

Radially symmetric focusing gradients equivalent to **a quadrupole lens** gradient of the order **1 MT/m**, which exceeds the strength of conventional devices by many orders of magnitude (see e.g. [M.C. Thompson et al., _Underdense_fermilab-conf-07-330-apc.pdf])

Beam energy converter into radiation

G. G. Oksuzyan, M. I. Ivanyan, A. S.Vardanyan, Coherent Interaction of a Relativistic Electron Beam with a Plasma, Plasma Physics Reports, Vol. 27, No. 6, 2001, pp. 507–510 [_oksuzyan2001.pdf]

Brilliant idea: plasma is generated by the same frequency as a bunch structure RF frequency

Plasma as resonator for beam acceleration …

… Plasma as resonator for beam acceleration

Waves in plasma with speed less than speed of light can be used for synchronous acceleration of charges

Waves can be generated by laser or charged beams driven in plasma

PLASMA WAKEFIELD RECENT ACTIVITY and some PUBLICATIONS (2010-19)

K V Lotov, V I Maslov, I N Onishchenko, E N Svistun, Resonant excitation of plasma wakefields by a non-resonant train of short electron bunches [_lotov2010.pdf]

E. Adli, P. Muggli, **Proton-Beam-Driven Plasma Acceleration,** Reviews of Accelerator Science and Technology Vol. 9 (2016) 85–104 [_adli2016.pdf]

M. Chung *et al.*, Studies of the self-modulation and other instabilities in proton beam–driven plasma wakefield accelerators. URSI Asia-Pacific Radio Science Conference, Seoul, South Korea (2016).

Analytical model [Y. Golian, D. Dorranian, Proton driven plasma wakefield generation in a parabolic plasma channel, J Theor Appl Phys (2017) 11:27–35, _Golian-Dorranian2017_Article_ProtonDrivenPlasmaWakefieldGen.pdf]

In 2016 at CERN created AWAKE Collaboration which has been formed in order to demonstrate proton-driven plasma wakefield acceleration for the first time [C. Bracco et al., AWAKE: A Proton-Driven Plasma Wakefield Acceleration Experiment at CERN, Nuclear and Particle Physics Proceedings 273–275 (2016) 175–180, _AWAKE-10.1016_j.nuclphysbps.2015.09.022.pdf]

PLASMA WAKEFIELD RECENT ACTIVITY and some PUBLICATIONS (2015-19)

M. Turner et al., Experimental Observation of Plasma Wakefield Growth Driven by the Seeded Self-Modulation of a Proton Bunch, PHYSICAL REVIEW LETTERS 122, 054801 (2019);
E. Adli et al., Experimental Observation of Proton Bunch Modulation in a Plasma at Varying Plasma Densities, PHYSICAL REVIEW LETTERS 122, 054802 (2019)

"We measure the effects of transverse wakefields driven by a relativistic proton bunch in plasma with densities of 2.1×10^{14} and 7.7×10^{14} electrons/cm^3. We show that these wakefields periodically defocus the proton bunch itself, consistently with the development of the seeded self-modulation process. We evaluate the transverse wakefield amplitudes and show that they exceed their seed value (< 15 MV/m) and reach over 300 MV/m. All these results confirm the development of the seeded self-modulation process, a necessary condition for external injection of low energy and acceleration of electrons to multi-GeV energy levels."

"…observation of the full transverse self-modulation of a long, relativistic proton bunch propagating through a dense plasma. The bunch exits the plasma with a periodic density modulation resulting from radial wakefield effects…"

"The accelerating field in a plasma with electron density n_e can reach a significant fraction of the wave breaking field

$$E[V/m] \approx 100\sqrt{n_e[cm^{-3}]}$$

This field is > 1 GV/m **for** plasma densities >10^{14} cm^{-3} which makes plasma a promising candidate as a medium for high-gradient acceleration."

PLASMA WAKEFIELD

_dawson1959.pdf

Amatuni …Magomedov 1977 itd [_Amatuni_Magomedov_EFI2433677.pdf]

P.Chen, J.M.Dawson, The Plasma Wake-Field Accelerator. SLAC-PUB-3601, 1985.

[_rosenzweig1987.pdf]

[_Amatuni_Elbakyan_Sechposyan_HEACC86_I_177-182.pdf]

Schutt, P., T. Weiland and V. M. Tsakanov, "*On the wakefield acceleration using a sequence of driving bunches*",DESY-M-88-13 (1988).

T. Tajima and J. M. Dawson, Laser electron accelerator, *Phys. Rev. Lett.* **43**, 267 (1979).

[_Amatunin...EPAC1988_0499.PDF]

1997 T. Katsouleas, S. Lee et al., A Proposal for a 1 GeV Plasma-Wakefield Acceleration Experiment at SLAC, [_Katsouleas_fc289a4462d83eae13d255cb9804cef5d642.pdf])

P. Chen, J. Dawson, R. Huff and T. Katsouleas, Acceleration of electrons by the interaction of a bunched electron beam with a plasma, *Phys. Rev. Lett.* **54**, 693 (1985).

Yerevan accelerator….

Э.М. ЛАЗИЕВ и др., РАБОТЫ ПОСЛЕДНИХ ЛЕТ И ОСНОВНЫЕ НАПРАВЛЕНИЯ ИССЛЕДОВАНИЙ В ОБЛАСТИ ФИЗИКИ ПУЧКОВ И УСКОРИТЕЛЬНОЙ ТЕХНИКИ В ЕРФИ, Известия НАН Армении, Физика, т.44, No5, с.363-372 (2009)

P.Chen, J.M.Dawson P.Chen, J.M.DawsonP.Chen, J.M.DawsonP.Chen, J.M.Dawson, The Plasma Wake-Field Accelerator. SLAC-PUB-3601, March 1985.

A.Ts.Amatunl, R.O.Avakyan, A.Z.Babaian, A.I.Baryshev, H.A.Vartapetyan, N.A.Zapolsk:y, I.P.Karabekov, E.M.Lasyev, R.O.Manoukyan, H.A.Martirosyan, V,Ts.Nikogosyan, G.G.Oksuzyan K.A.Sadoyan, Kh.R.Simonyan, V,M.Tsakanov, A.R.Toumanyan, V.P.Belov, V.P.Goncharenko, A.A.Makarov, D.S.Efremov, W.Nothe, J.Rossbach, J.Rummler, K.Steffen, THE YEREVAN ELECTRON ACCELERATOR: STATUS AND DEVELOPMENT, EPAC 1990, pp. 406-408.

Simplest model for analytical estimations:

1D model – remove main instabilities of plasma, remove necessity to calculate magnetic field

Cold collisionless plasma

Rigid ions

Rigid beam – co-propagation of wakefield with beam, allow to set instead of time and longitudinal coordinate only one concomitante coordinate

$$\frac{\partial^2 \varphi}{\partial z^2} = -4\pi e(n_e + \frac{q}{e} n_b - n_0)$$

Poisson equation

$$\frac{\partial^2 \varphi}{\partial \hat{z}^2} = -4\pi e(n_e + \frac{q}{e} n_b - n_0)$$

$$\frac{\partial n_e}{c\partial t} + \frac{\partial(\beta_e n_e)}{\partial z} = 0$$

Continuity equation for electrons of plasma

$$-\beta_0 \frac{\partial n_e}{\partial \hat{z}} + \frac{\partial(\beta_e n_e)}{\partial \hat{z}} = 0$$

$$\frac{\partial(\beta_e \gamma_e)}{c\partial t} + \frac{\partial(\beta_e \gamma_e)}{\partial z} = -\frac{e}{mc^2}\frac{\partial \varphi}{\partial z}$$

plasma electrons motion equation

$$(\beta_e - \beta_0)\frac{\partial(\beta_e \gamma_e)}{\partial \hat{z}} = -\frac{e}{mc^2}\frac{\partial \varphi}{\partial \hat{z}}$$

$$\hat{z} = z - \beta_0 ct$$

$$n_e(\beta_0 - \beta_e) = n_0 \beta_0$$

$$\beta_e < \beta_0$$

$$\beta_e = \frac{\beta_0 - \Phi\sqrt{\Phi^2 - \gamma_0^{-2}}}{\Phi^2 + \beta_0^2}$$

$$\frac{1 - \beta_0 \beta_e}{\sqrt{1 - \beta_e^2}} = 1 - \frac{e}{mc^2}\varphi \equiv \Phi$$

$$\gamma_0^{-1} < \Phi$$

$$n_e = -n_0\left(\beta_0^2 \gamma_0^2 - \frac{\beta_0 \gamma_0^2 \Phi}{\sqrt{\Phi^2 - \gamma_0^{-2}}}\right)$$

Main equation

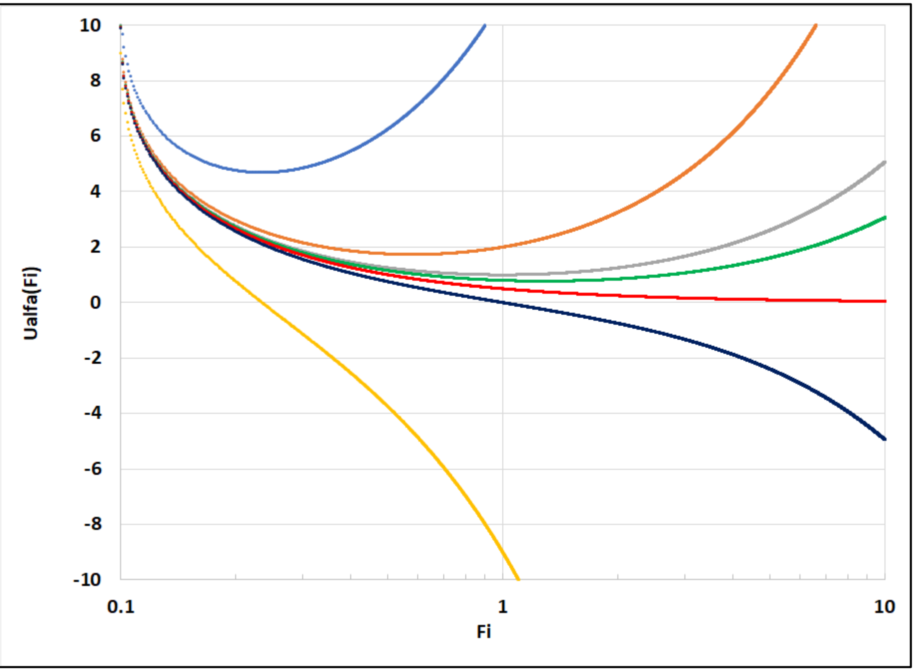
$$\frac{\partial^2\Phi}{\partial^2\xi} = \alpha - \gamma_0^2 + \frac{\beta_0\gamma_0^2\Phi}{\sqrt{\Phi^2 - \gamma_0^{-2}}} \qquad \alpha = (q/e)(n_b/n_0)$$

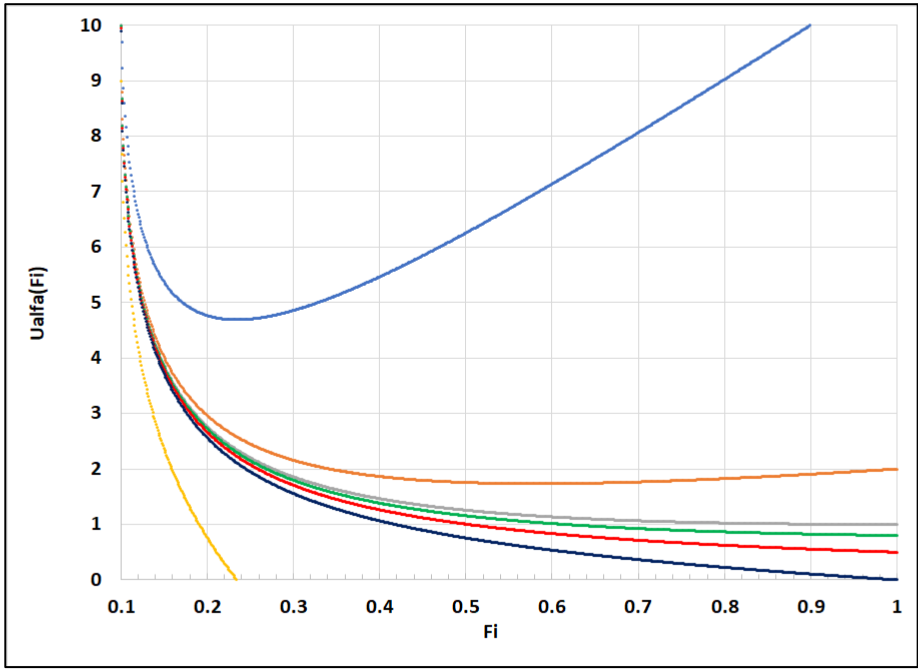
Phase point energy

$$\varepsilon = \Phi'^2/2 + \gamma_0^2\left(\Phi - \beta_0\sqrt{\Phi^2 - \gamma_0^{-2}}\right) - \alpha\Phi$$

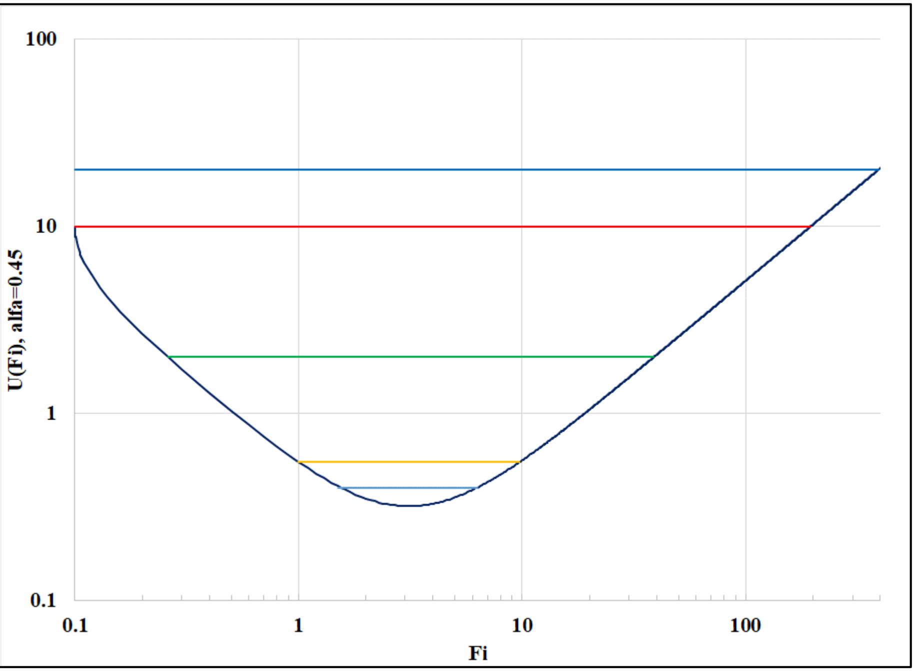
Potential well

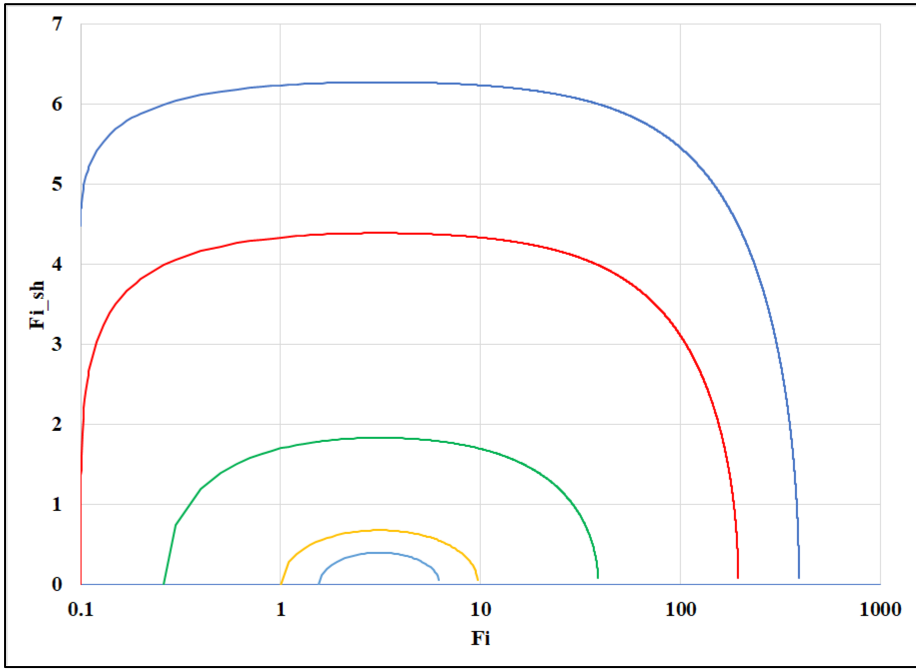
$$U_\alpha = \gamma_0^2\left(\Phi - \beta_0\sqrt{\Phi^2 - \gamma_0^{-2}}\right) - \alpha\Phi$$



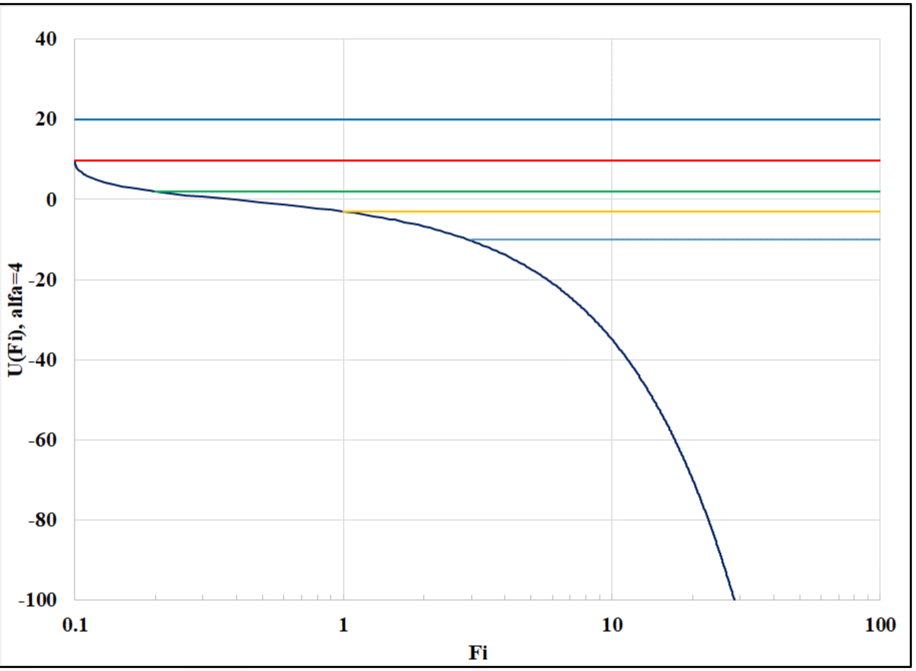


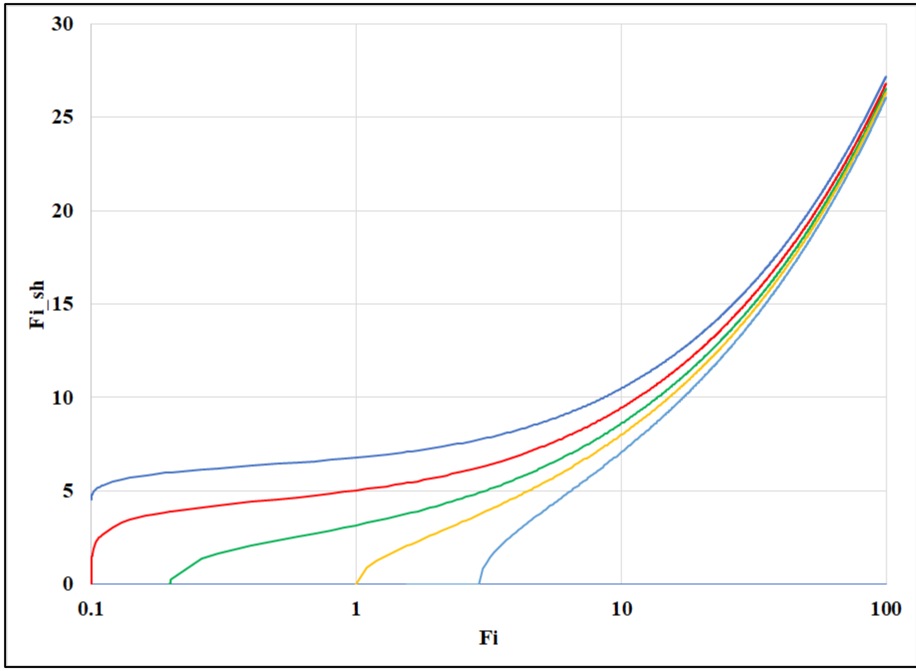
Potential wells U_α for $\gamma_0 = 10$ and different values of α in different scales of axis Φ. Curves from top to bottom correspond to following values of parameter α: -10; -1; 0; 0.2; $1/(1+\beta_0) = 0.501$; 1; 10. The curve of critical value $\alpha = 1/(1+\beta_0)$ that separates the type of curves depicted in red color



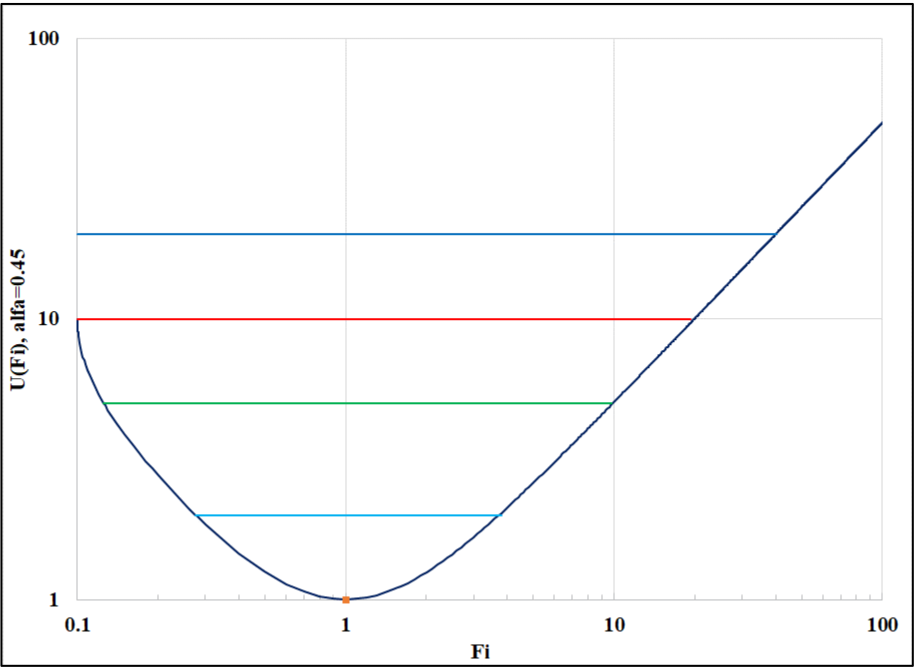


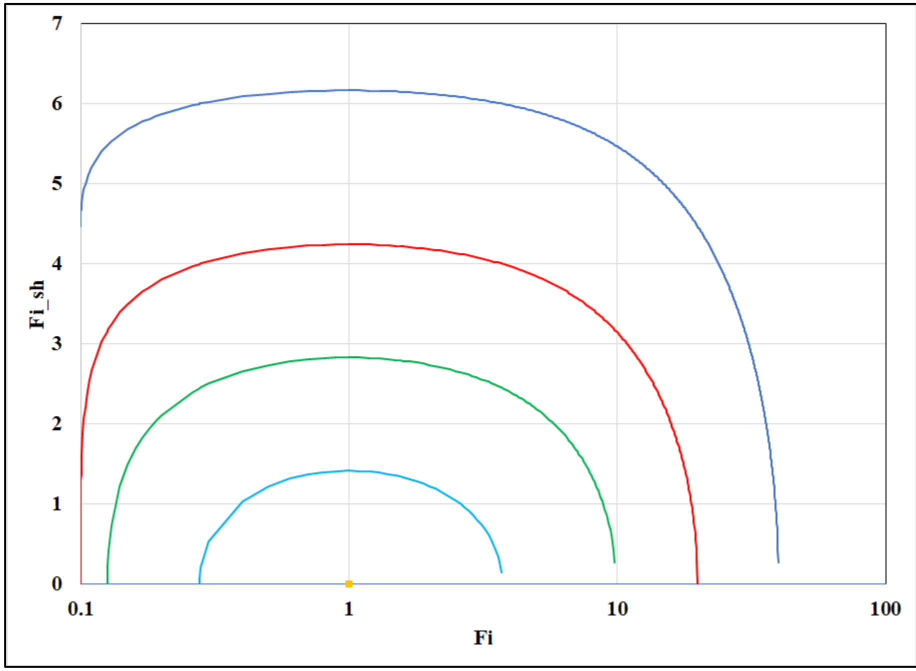
Phase point motion in case of bunch density $\alpha = 0.45$, left - potential well, right - phase trajectories at different energies. Energy from top to bottom: 20; 9.955; 2; 0.55; 0.4. Energy 9.955 corresponds to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$ (marked by red); energy 0.55 corresponds to $U_\alpha(1) = 1 - \alpha$ (marked by yellow).



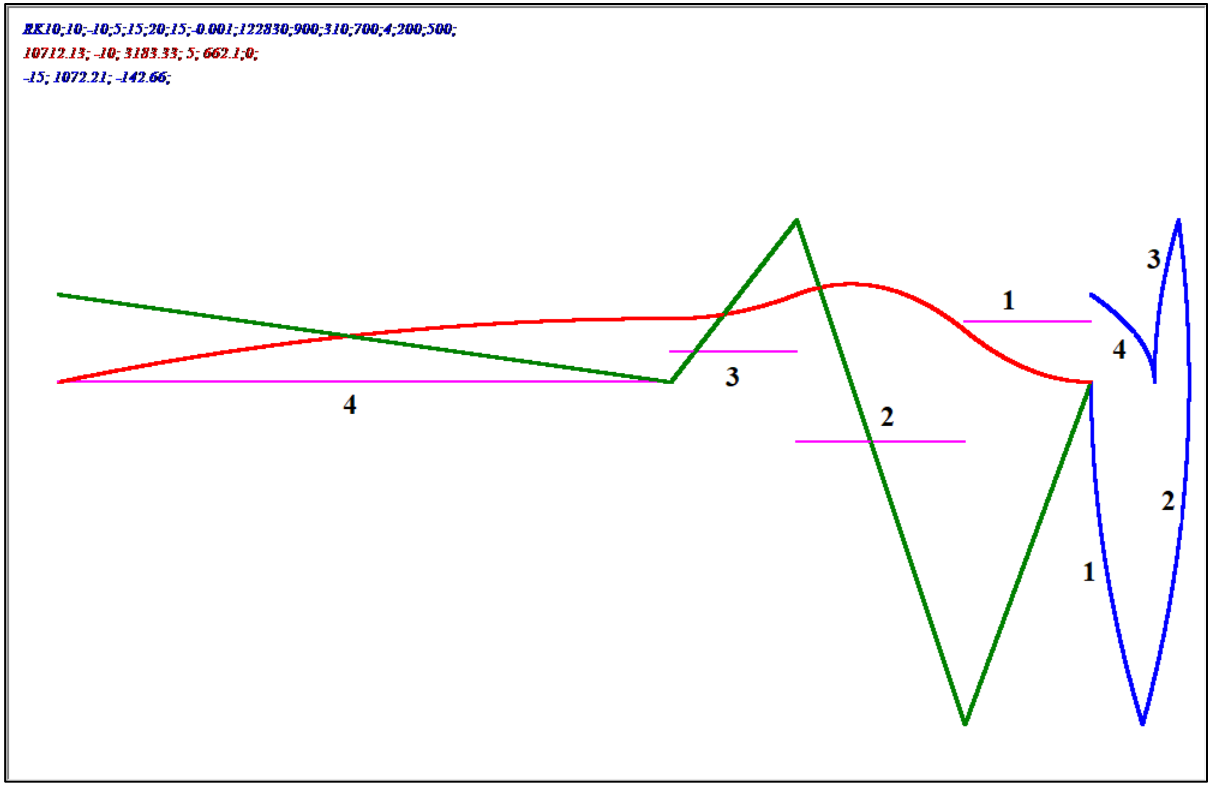


Phase point motion in case of bunch density $\alpha = 4$, left - potential well, right - phase trajectories at different energies. Energy from top to bottom: 20; 9.6; 0; -3; -10. Energy 9.6 corresponds to $U_\alpha(\gamma_0^{-1}) = \gamma_0 - \alpha\gamma_0^{-1}$ (marked by red); energy -3 corresponds to $U_\alpha(1) = 1 - \alpha$ (marked by yellow).

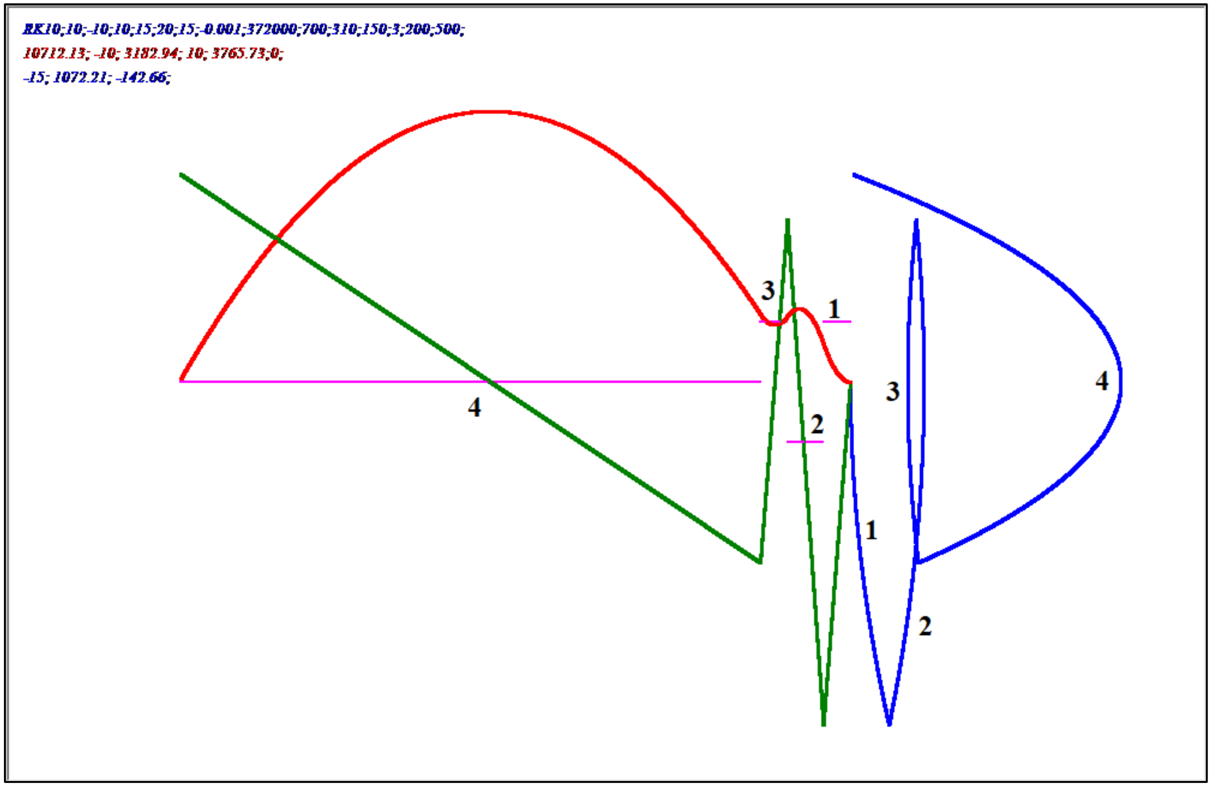




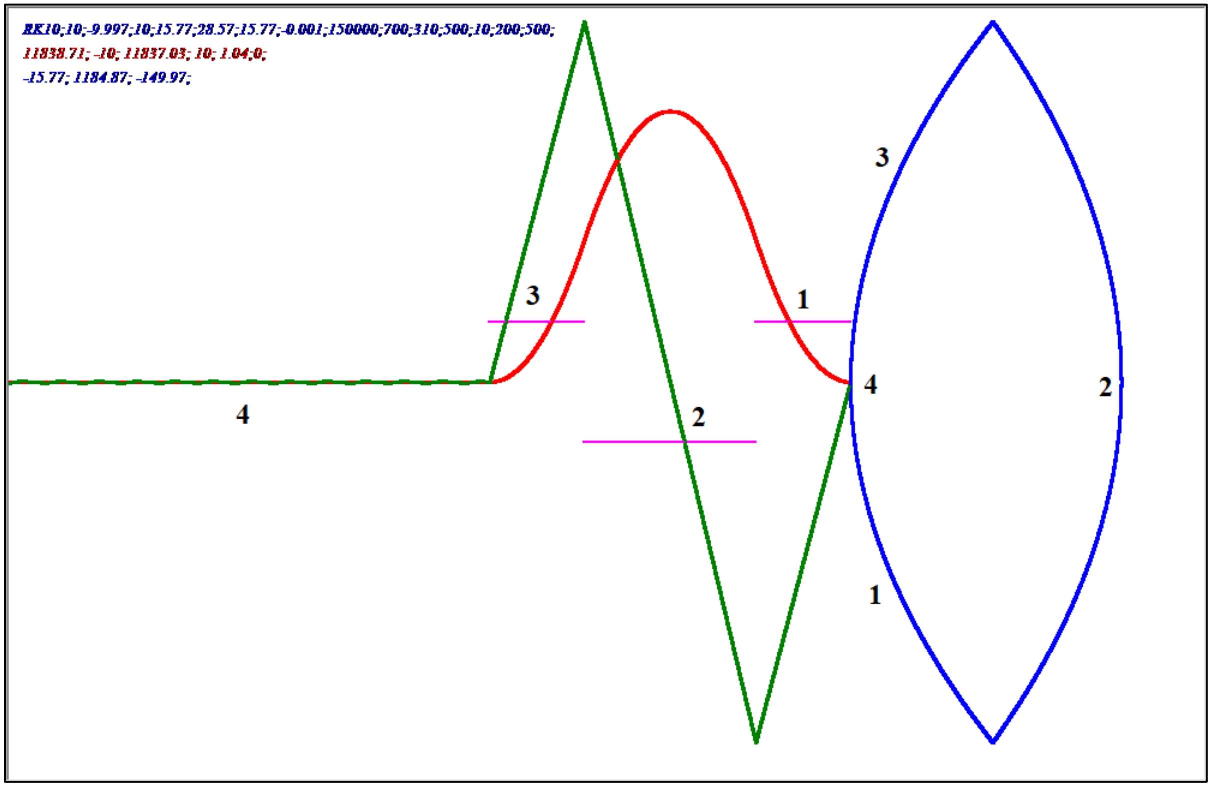
Phase point motion in case of bunch density $\alpha = 0$, left - potential well, right - phase trajectories at different energies. Energy from top to bottom: 20; 10; 5; 2; 1. Energy 10 corresponds to $U_{\alpha=0}(\gamma_0^{-1}) = \gamma_0$ (marked by red); energy 1 corresponds to $U_\alpha(1) = 1$ (marked by yellow).



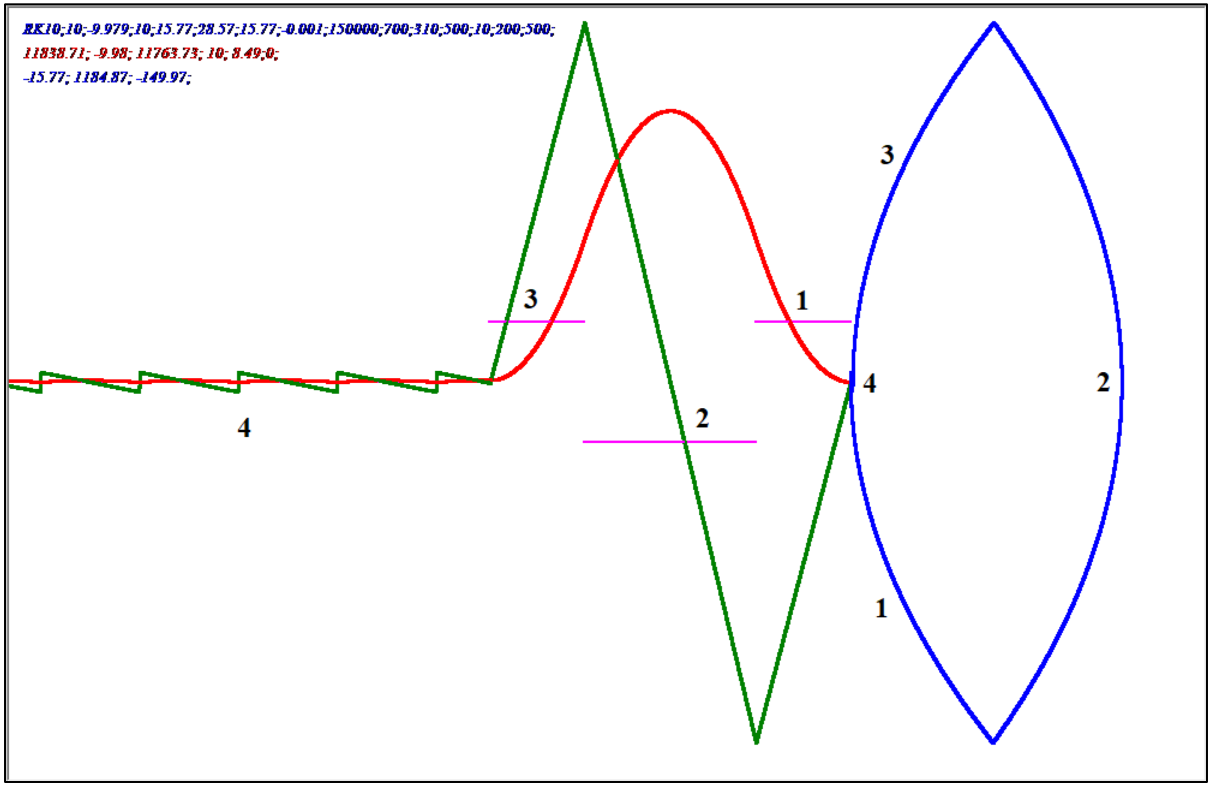
Wakefield of stru8ctured bunch consists of 3 sub-bunches with following parameters: $\alpha_1 = 10, d_1 = 15$; $\alpha_2 = -10, d_2 = 20$; $\alpha_3 = 5, d_3 = 15$. Sub-bunches marked in magenta and denoted as 1, 2, 3. By 4 is denoted free plasma behind the bunch. Bunch movement direction is from left to right. In red marked function Φ depending on ξ. In green marked function Φ' depending on ξ. The space right from the front point of bunch is used for presenting of phase point trajectory marked in blue (axis Φ is horizontal directed to right and axis Φ' is vertical directed to up). Numbers correspond to presented bunch structure. All values are presented in arbitrary units.



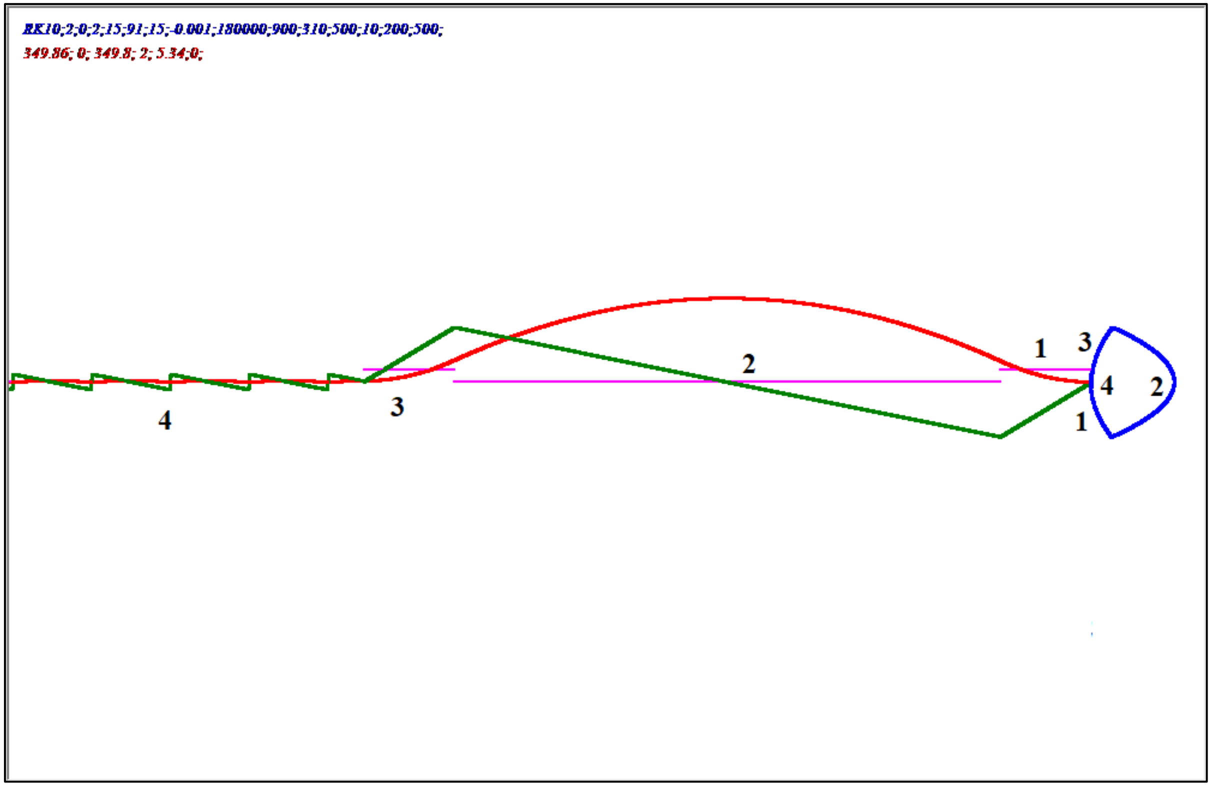
Wakefield of structured bunch consists of 3 sub-bunches with following parameters: $\alpha_1 = 10, d_1 = 15$; $\alpha_2 = -10, d_2 = 20$; $\alpha_3 = 10, d_3 = 15$. Other notifications are the same as in previous slide



Ideal entrance of phase point into free plasma behind the bunch
(entrance energy is 1.04, the density of the second bunch is -0.997)



Entrance into free plasma on the level near to the plasma wave breakdown.



As the second bunch can be used also free plasma (first and the last bunches density are 2 with length 15, distance between bunches is 91).

Wakefield arising time $2\pi / \omega_p$ $\omega_p = \sqrt{4\pi e^2 n_0 / m}$

Characteristic time of bunch pulse change p_b / qE $E = -\frac{mc^2}{e}\frac{\omega_p}{c}\Phi'$

n0, 1/cm3	omega_p, s-1	tau_e	lambda_p, cm	F_sh	eE_max, eV/cm	tau_b_e, s	tau_b_p, s
2.10E+14	8.18E+11	1.22E-12	2.30E-01	100	1.39E+09	1.22E-13	2.23E-10
7.70E+14	1.57E+12	6.39E-13	1.20E-01	100	2.67E+09	6.36E-14	1.17E-10
1.00E+15	1.78E+12	5.61E-13	1.06E-01	100	3.04E+09	5.58E-14	1.02E-10
2.10E+14	8.18E+11	1.22E-12	2.30E-01	100	1.39E+09	1.22E-13	2.23E-10

Here we follow [A.Ts. Amatuni, SELFACCELERATION OF ELECTRONS IN ONE-DIMENSIONAL BUNCHES,MOVING IN COLD PLASMA, Preprint YERPHI-1473(10)-96, [_Amatuni-9609003.pdf]

$$\frac{\partial(\beta_e\gamma_e)}{\partial\tau}+\frac{\partial(\beta_e\gamma_e)}{\partial\xi}=\frac{\partial\Phi}{\partial\xi}$$

$$\frac{\partial(\beta_b\gamma_b)}{\partial\tau}+\frac{\partial(\beta_b\gamma_b)}{\partial\xi}=\frac{q}{e}\frac{\partial\Phi}{\partial\xi}$$

$$\frac{\partial\alpha_e}{\partial\tau}+\frac{\partial(\beta_e\alpha_e)}{\partial\xi}=0$$

$$\frac{\partial\alpha_b}{\partial\tau}+\frac{\partial(\beta_b\alpha_b)}{\partial\xi}=0$$

$$\frac{\partial^2\Phi}{\partial\xi^2}=\alpha_e+\alpha_b-1$$

$$z=\frac{c}{\omega_p}\xi \qquad t=\frac{1}{\omega_p}\tau$$

$$\varphi=\frac{mc^2}{e}(1-\Phi)$$

$$n_e=\alpha_e n_0 \qquad n_b=(q/e)\alpha_b n_0$$

Conclusion

Analytical 1D model is a good instrument to set and check the desirable configuration before starting the simulations on complicated software.

E.g. in AWAKE2 the maximum accelerating field depending on the plasma electron density n_e was estimated from the cold plasma wave breaking field

$$E = \frac{mc^2}{e}\frac{\omega_p}{c}$$

In our 1D model we predict much more intensity for specially structured bunches with densities more than plasma density.

$$E = -\frac{mc^2}{e}\frac{\omega_p}{c}\Phi', \Phi' \gg 1$$

Numerical simulations and experiments on more realistic model can confirm or disprove this statement, but in any case it can serve the purpose of the further study

Thank You