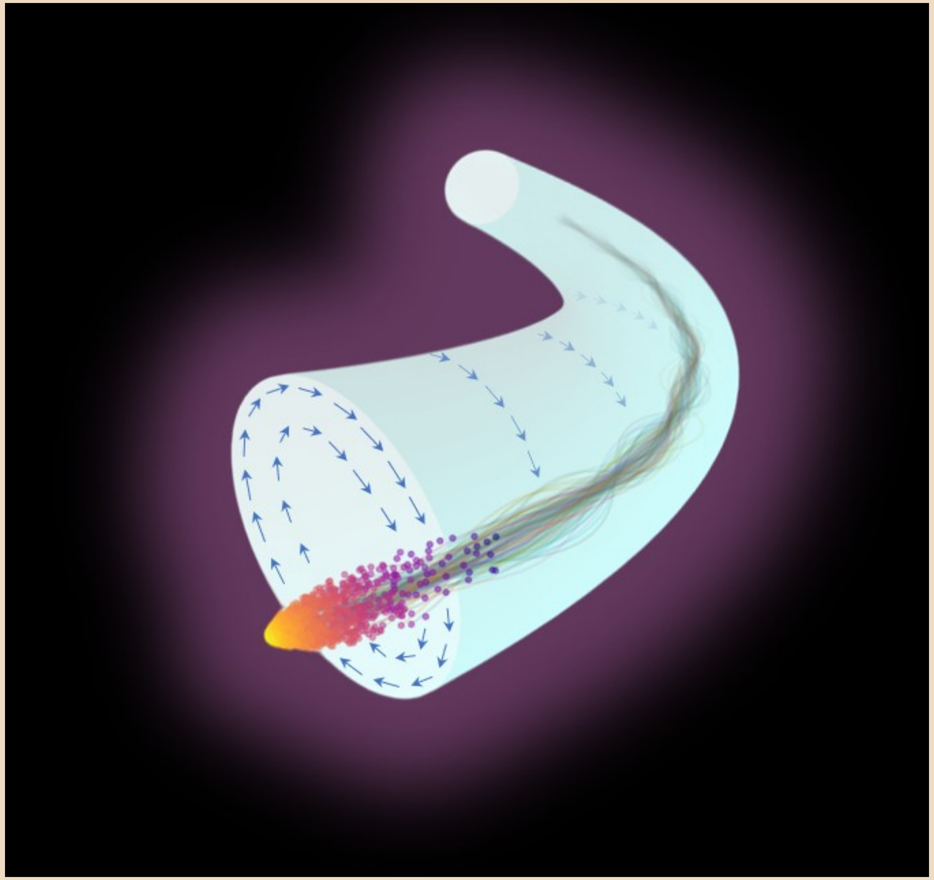
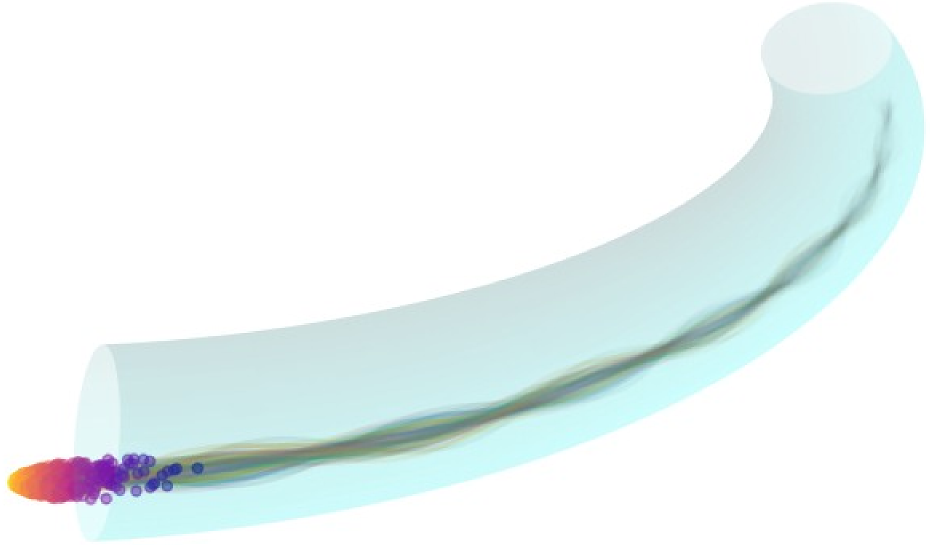
Active Plasma Bending

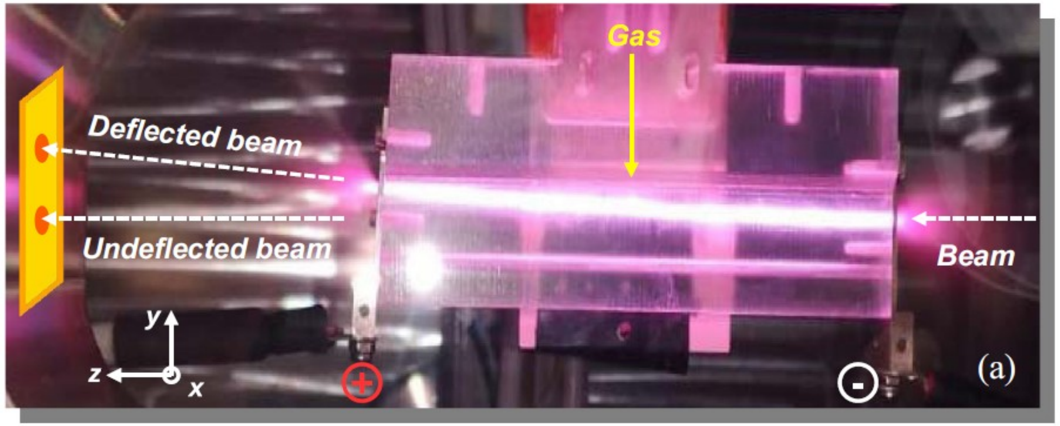
A novel technique for beam bending and focusing



Outline

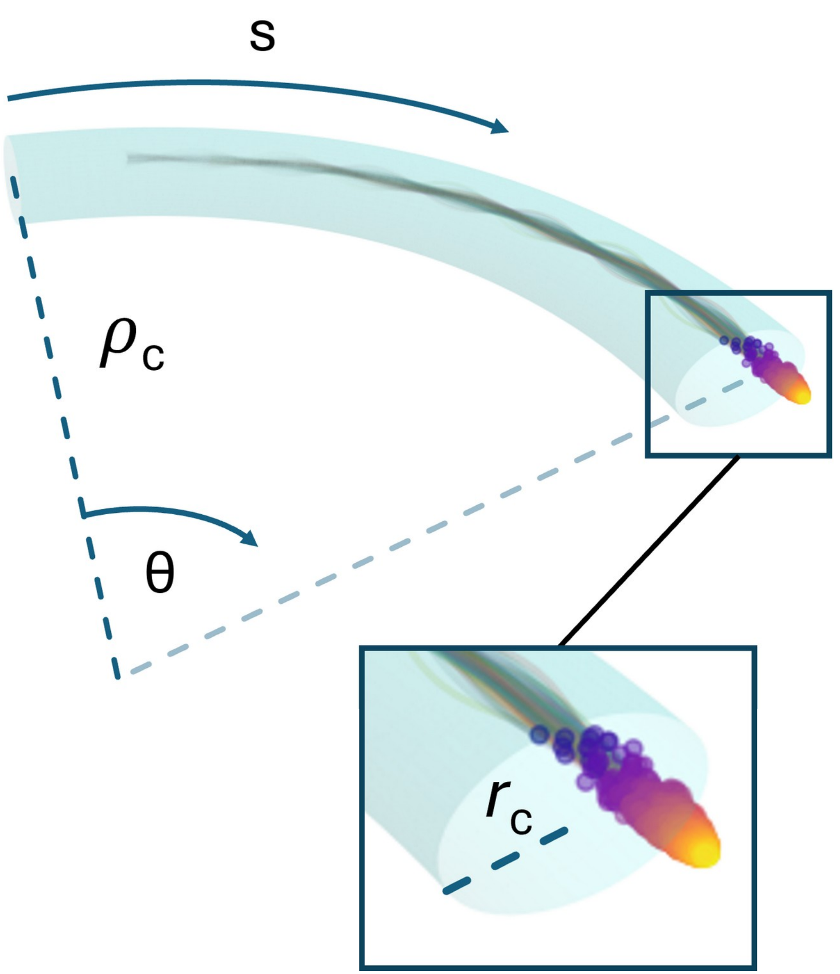
- Overview and motivation
- Principles
  - Bending and focusing
  - Minimum current
- Properties
  - Dispersion and transition
  - Long term behavior
- What has been done and what comes next





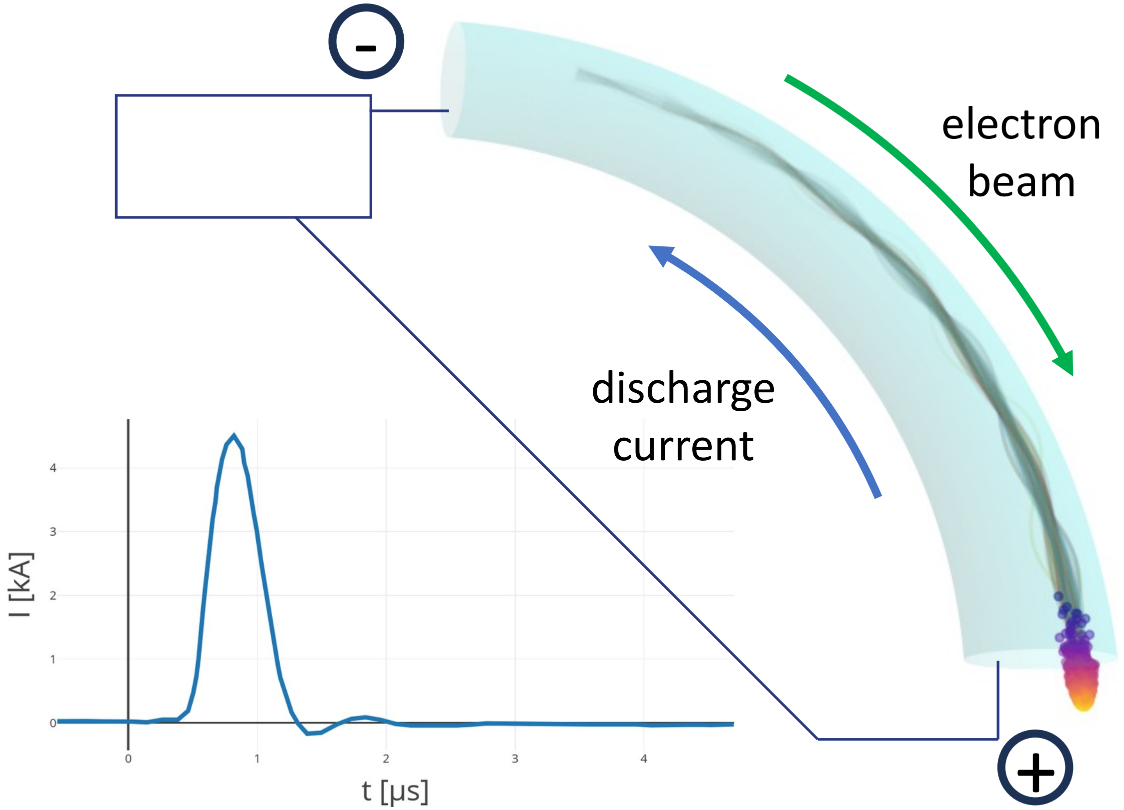
Overview and motivation

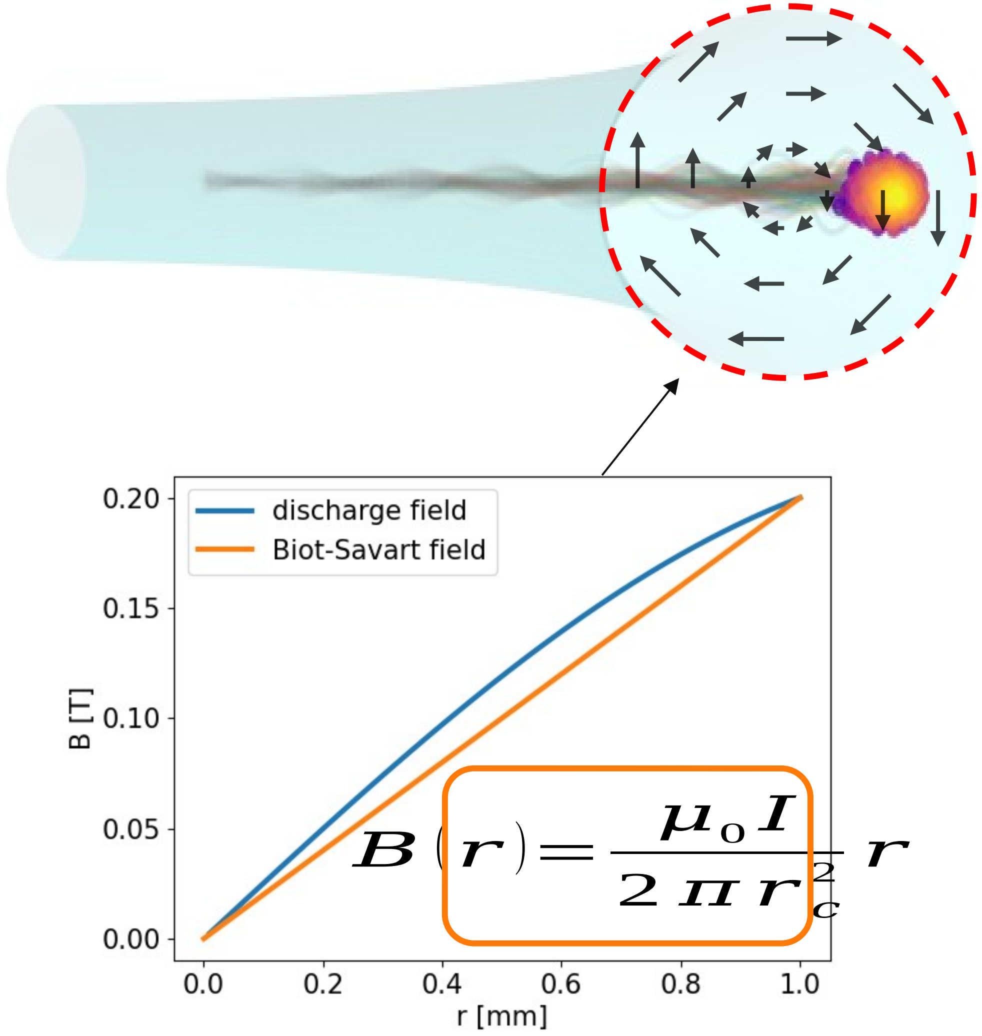
- Curved active plasma lens
- Unique **bending** properties: **focusing** effects + non trivial chromatic **dispersion**
- Novel beam bending technique **tested at SPARC_LAB** in October 2023
- Opens the way to **full plasma-based beamline**



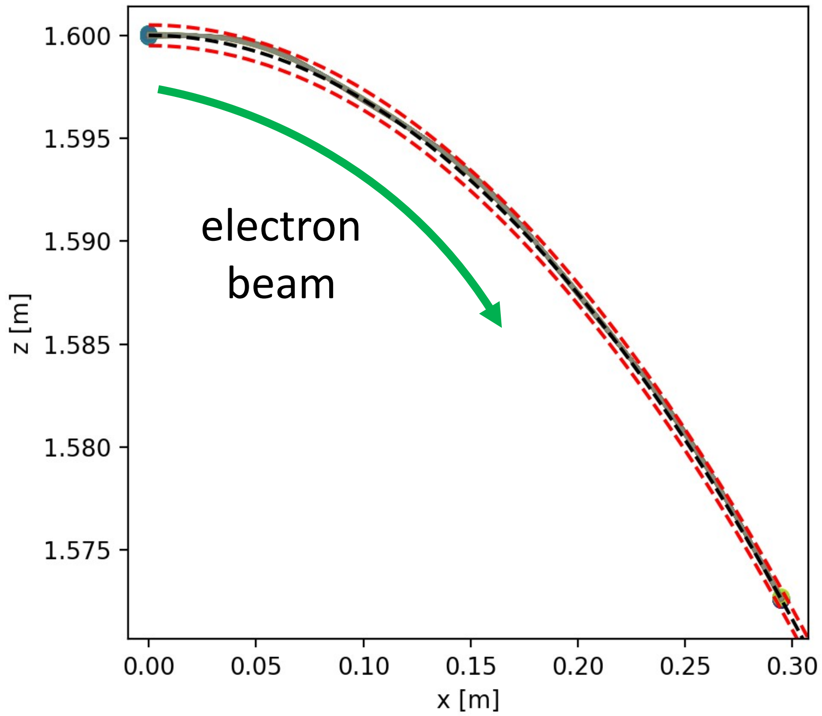
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
EuPRAXIA Advanced Photon Sources

Principles: bending and focusing

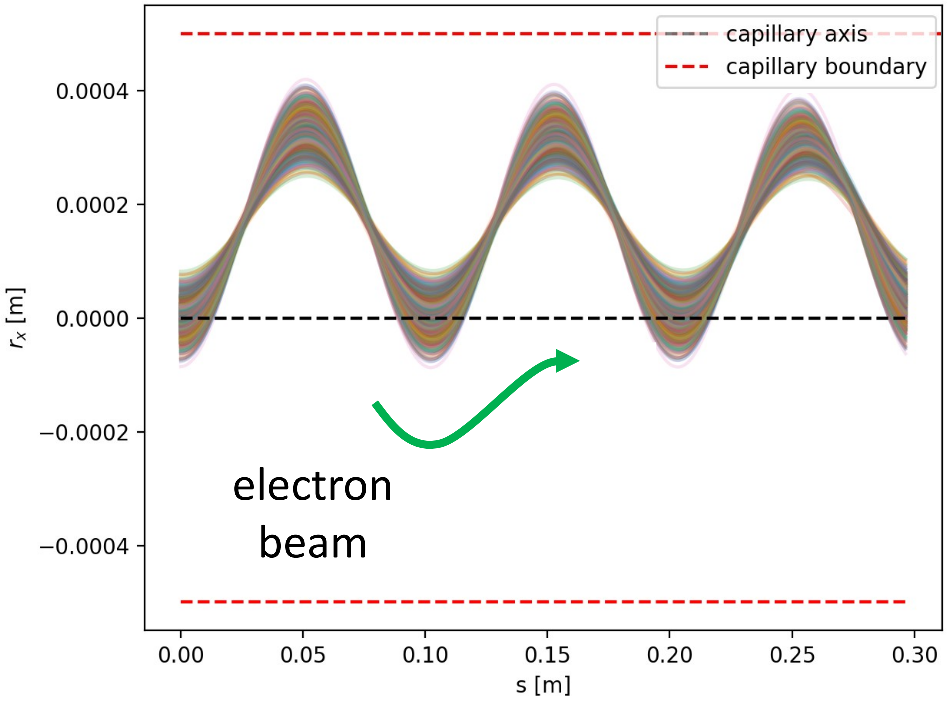




$$B(r) = \frac{\mu_0 I}{2\pi r_c^2} r$$

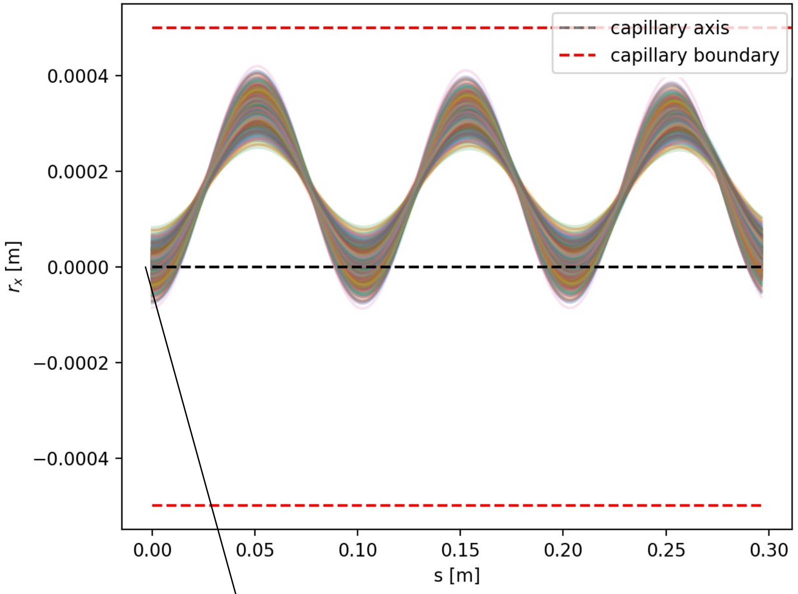


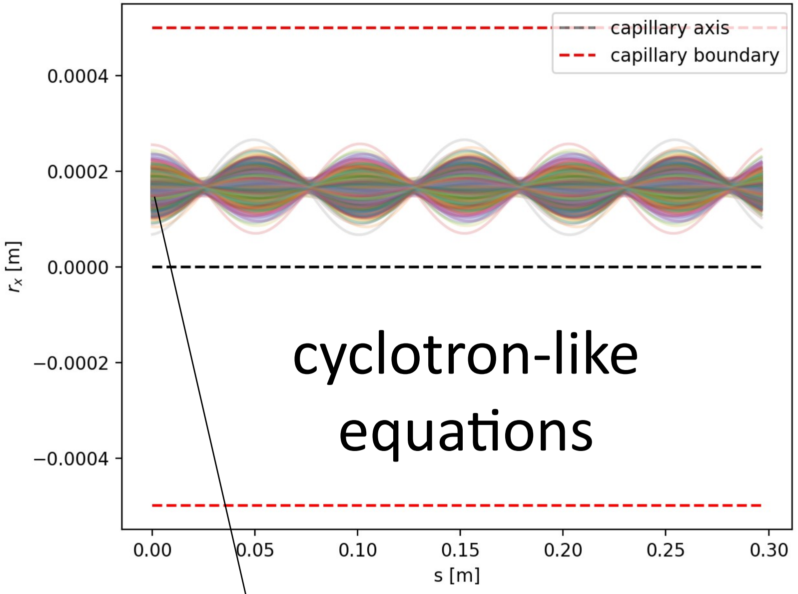
trajectory plane



rectified trajectory plane

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
EuPRAXIA Advanced Photon Sources

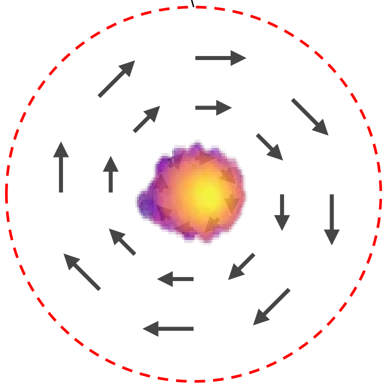




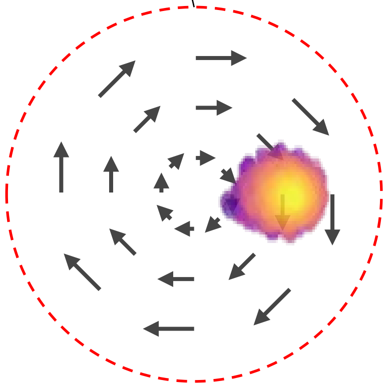
cyclotron-like equations

equilibrium offset

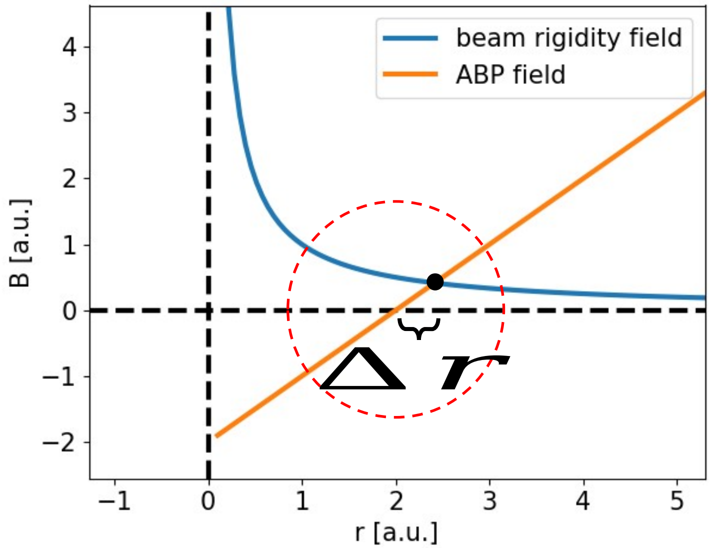
$$\Delta r \approx \tilde{I}_0 \frac{\gamma}{I} \frac{r_c^2}{\rho_c}$$



off equilibrium:
collective betatron motion

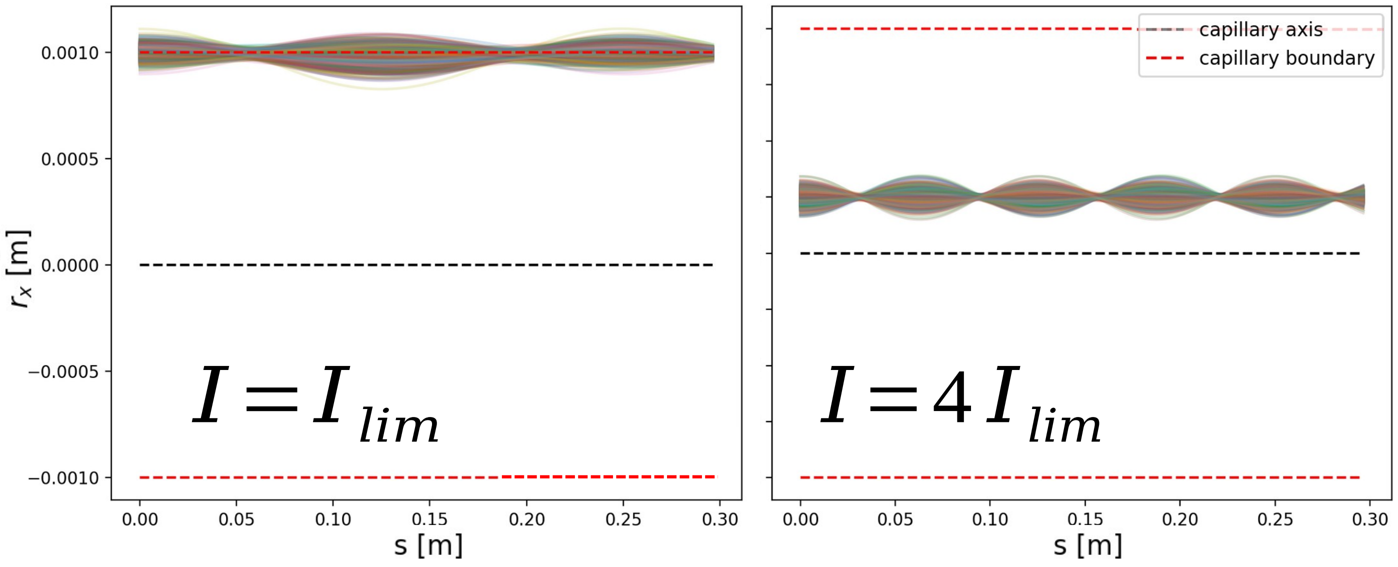


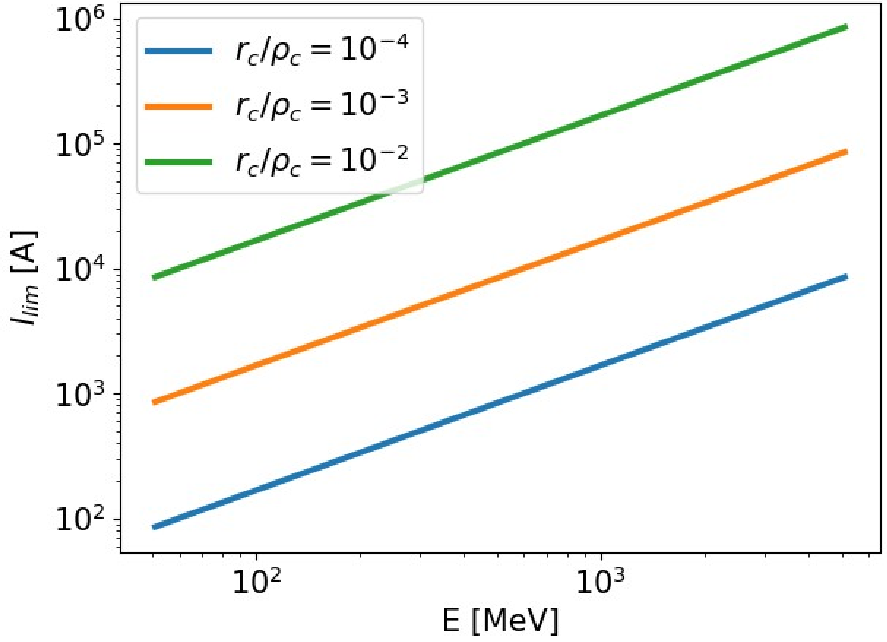
on equilibrium:
envelope oscillations only



Principles: minimum usage current

$$I > \tilde{I}_0 \gamma \frac{r_c}{\rho_c} := I_{lim}$$





Dispersion: CBM vs ABP

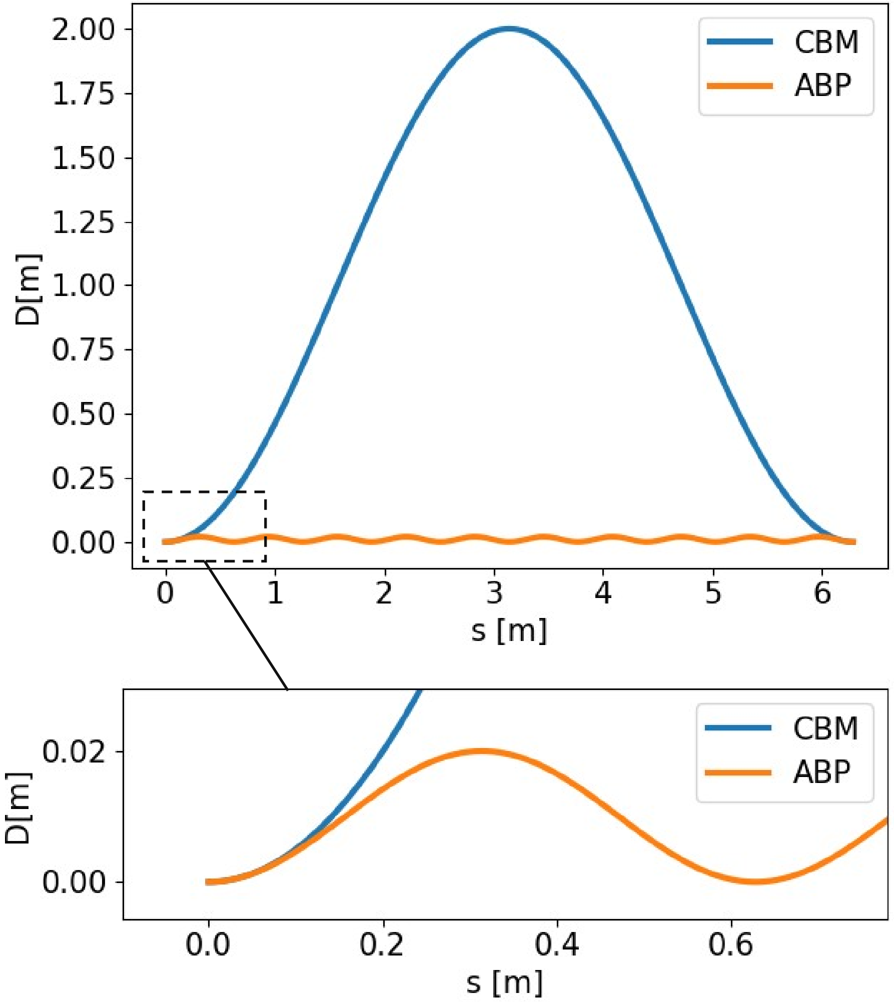
$$k_b = \sqrt{\frac{I}{\tilde{I}_0 \gamma r_c^2} + \frac{1}{\rho_c^2}}$$

$$D_{CBM} = \rho_0 (1 - \cos(s/\rho_0))$$

high amplitude
low frequency

$$D_{ABP} = \frac{1}{\rho_0 k_b^2} (1 - \cos(k_b\ s))$$

low amplitude
high frequency



Transition energy

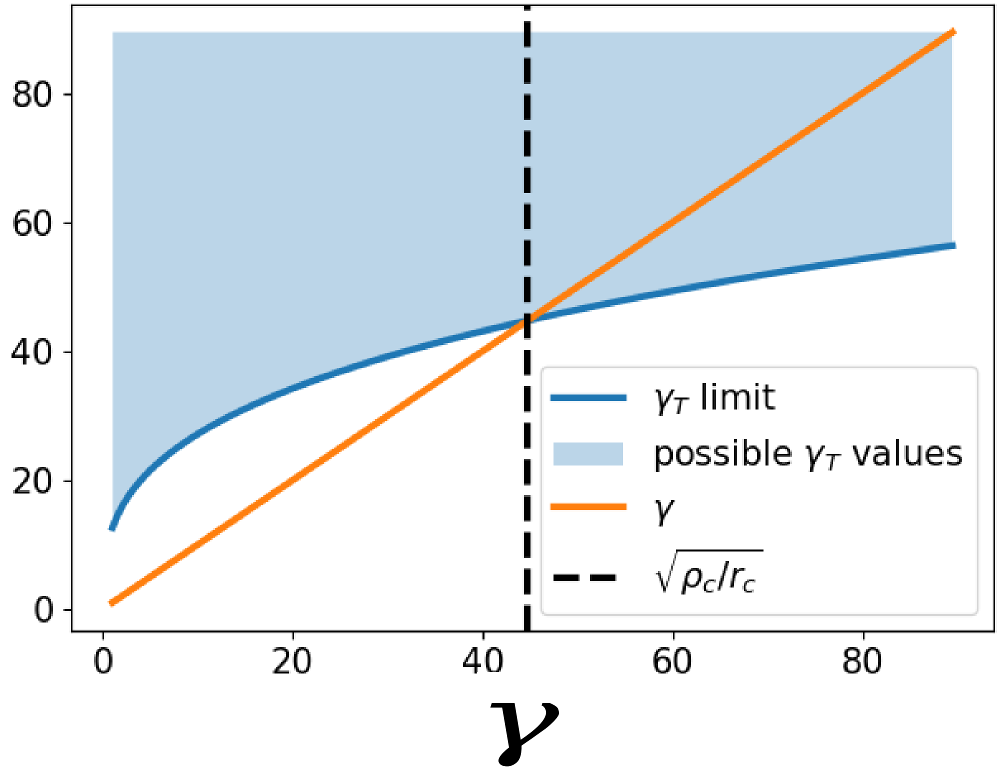
$$\gamma_T = \sqrt{\frac{I\rho_c^2}{\beta \tilde{I}_0 r_c^2}}$$

$$\gamma_T(I_{lim}) = \sqrt{\frac{\rho_c \gamma}{\beta r_c}} := \gamma_{T,lim}$$

- For the beam will be **bounded below transition**

below transition only

both above and below transition

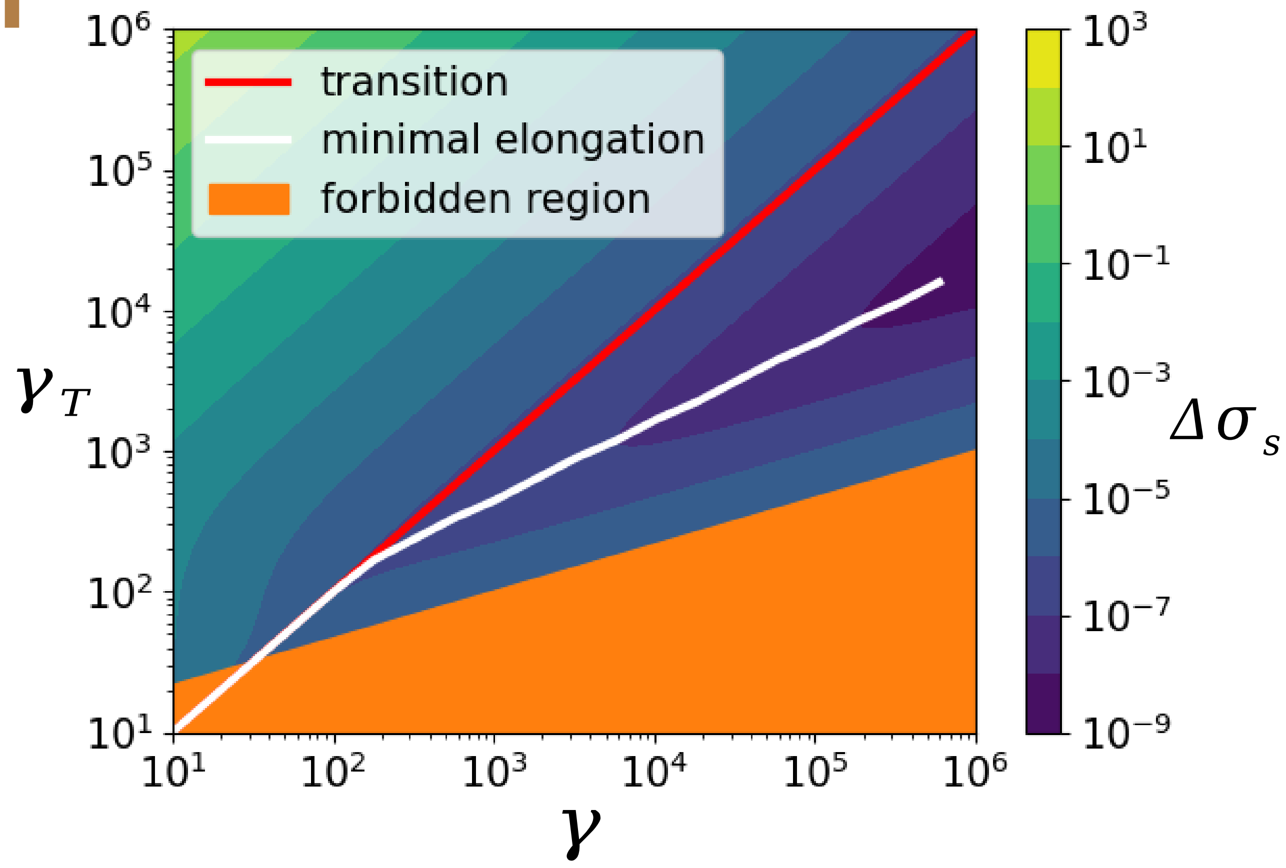


Beam rms elongation

- dominant **above and below transition** for **low transition energy**

$$\sigma_s^2 = \sigma_{s,0}^2 + s^2 \left[\frac{\epsilon_{rms}}{4\rho_0} \sqrt{\frac{\gamma}{\gamma_T^3}} + \gamma^2 \left(\frac{1}{\gamma^3} - \frac{1}{\gamma_T^3} \right)^2 \right] \sigma_{\Delta\gamma/\gamma}^2 + $$
$$+ s^2 \frac{\epsilon_{rms}^2}{4\rho_0^2} \frac{\gamma_T^3}{\gamma} \left(1 + k_x^4 \rho_0^4 \frac{\gamma^2}{\gamma_T^6} \right)$$

- dominant **below transition** for **high transition energy**
- depends on



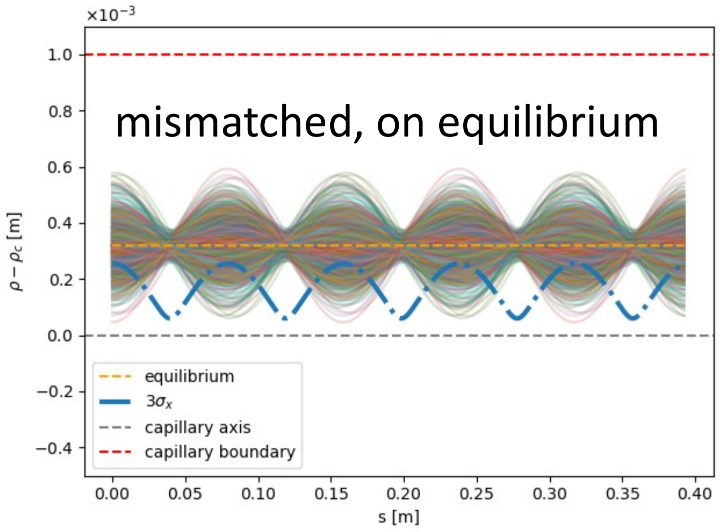
Transverse rms description

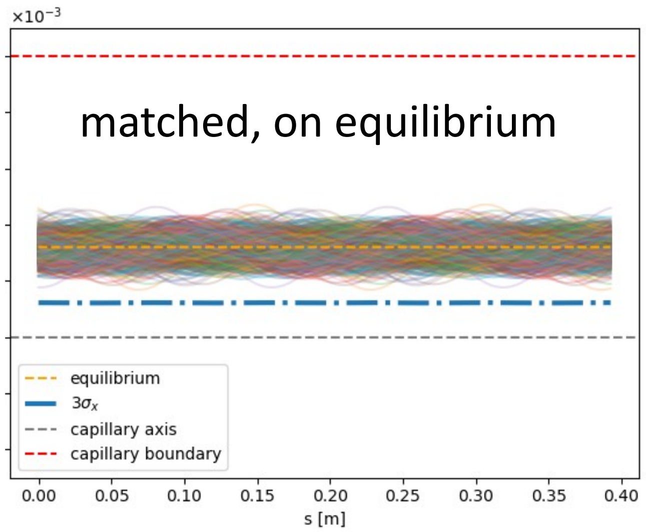
- Curved **envelope equatio**

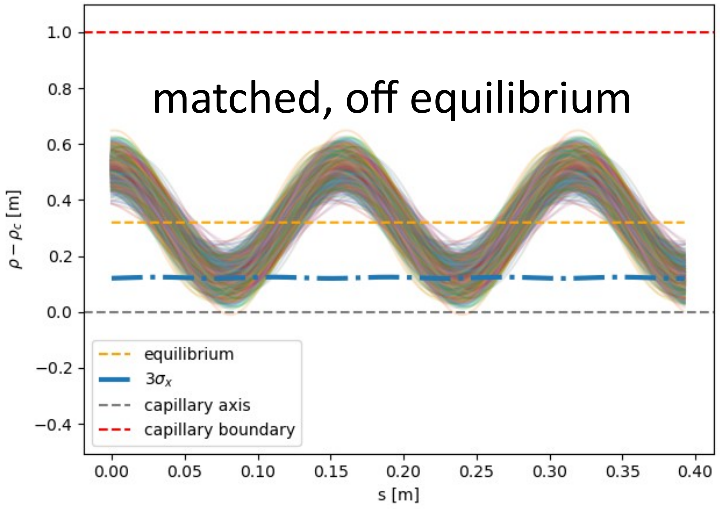
$$\frac{\partial^2 \sigma_x}{\langle \rho \rangle^2 \partial \theta^2} - \frac{\langle \rho' \rangle}{\langle \rho \rangle} \frac{\partial \sigma_x}{\langle \rho \rangle \partial \theta} + k_x^2 \sigma_x = \frac{\epsilon_{rms}^2}{\sigma_x^3}$$

  - Gives conventional **matching condition**

$$\sigma_{xy,M} = \sqrt{\frac{\epsilon_{rms}}{k_{xy}}}$$







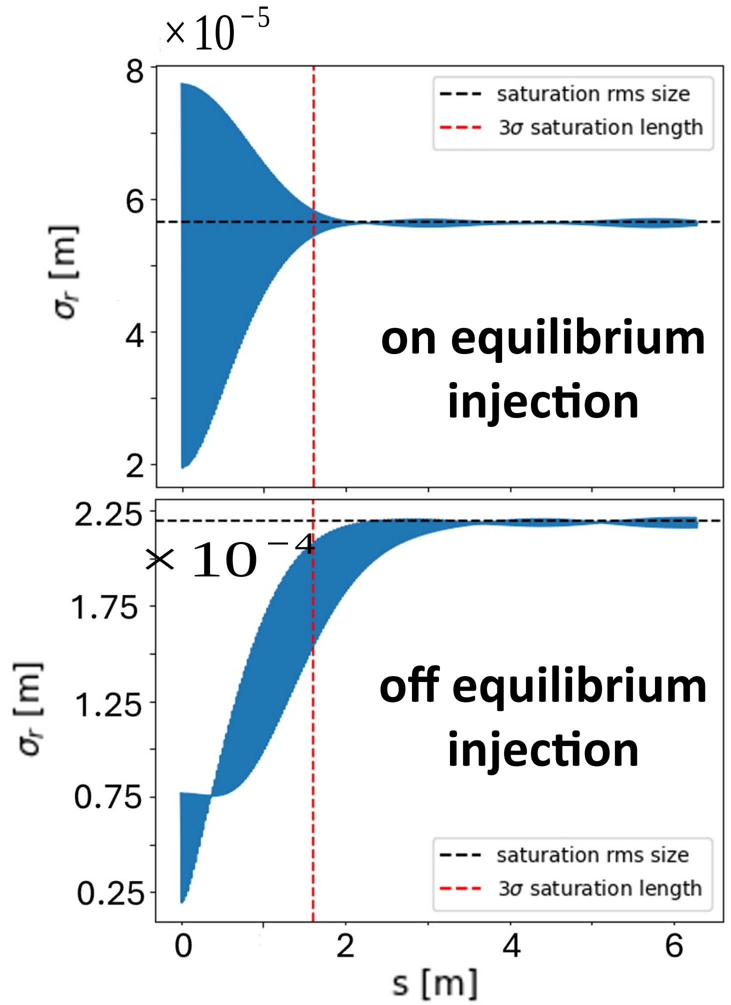
Long term behaviour

- Beam energy spread gives **equilibrium position and betatron wavelength spread**

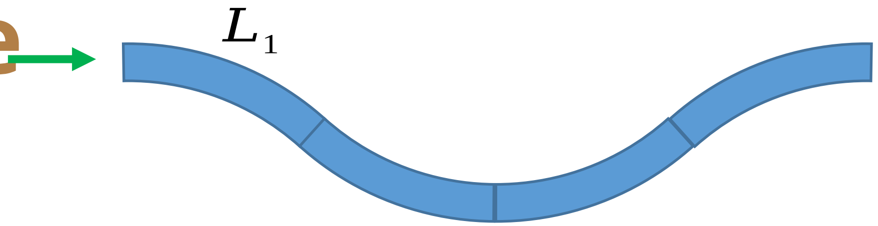
- The **phase mixing** process increases beam transverse spot, even for matched beams

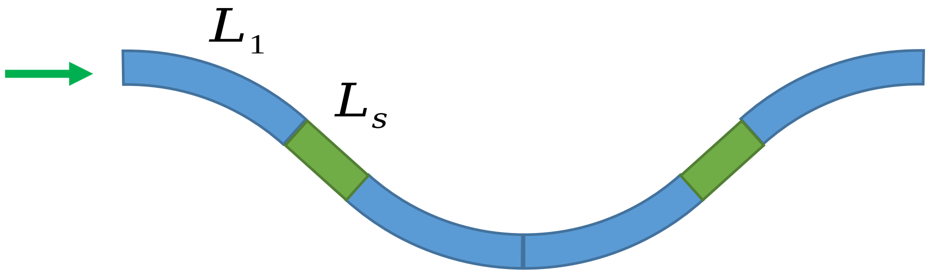
$$\sigma^2_{sat} = \frac{\Delta x^2_{inj}}{2} + \frac{\epsilon_{rms}}{k_b} + \frac{3}{2}\frac{\sigma^2_{\Delta\gamma/\gamma}}{\rho_c^2 k_b^4}$$

$$L_{sat} = \frac{2\pi\gamma k_b \rho_c^2}{\left(I\rho_c^2/\tilde{I}_0 r_c^2 - 2\gamma\right)} \frac{1}{3\sigma_{\Delta\gamma/\gamma}}$$

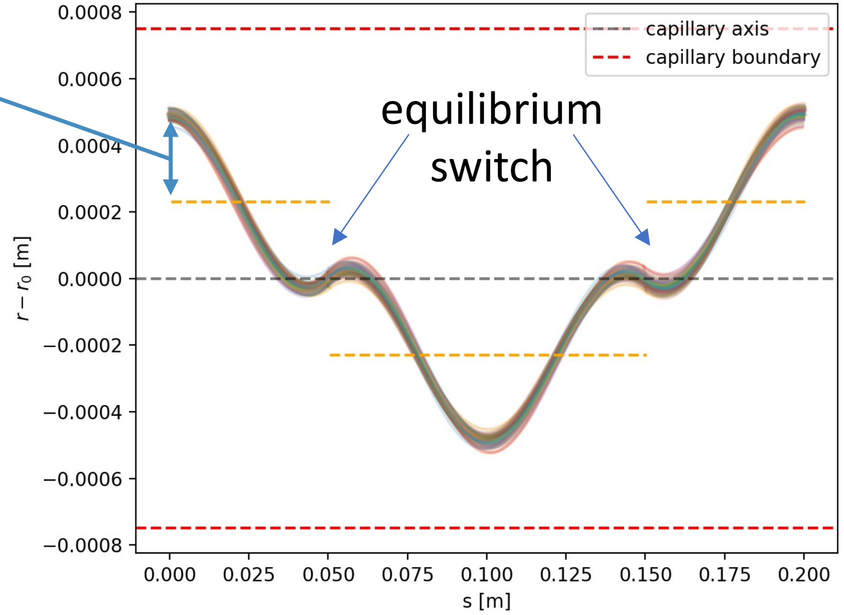


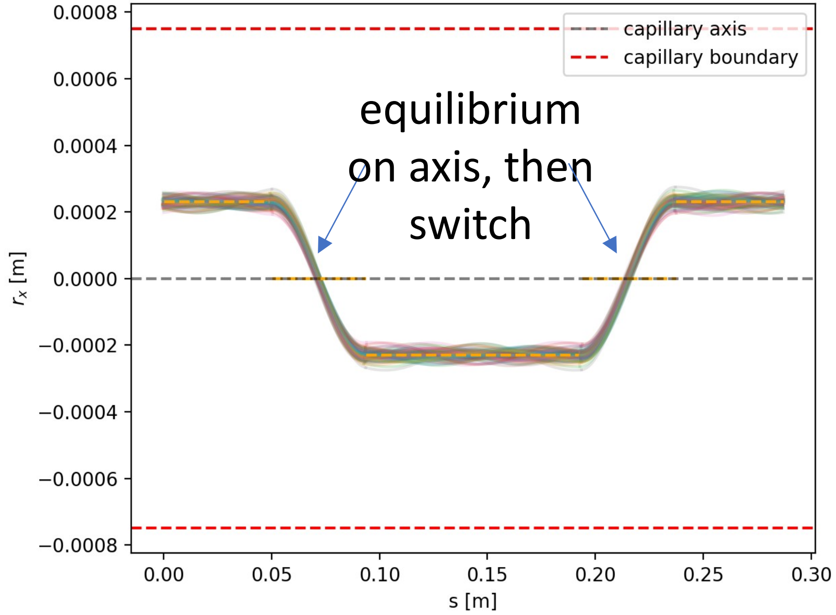
Moving forward: ABP chicane





$$\Delta r_{off} = -\frac{r_{off}}{\cos(L_1\omega_\beta/c)}$$

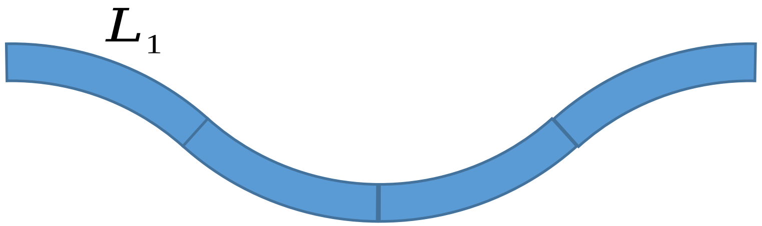




$$I = \gamma\frac{2\pi^3 m_e c r_c^2}{e\mu_0 L_s^2}$$

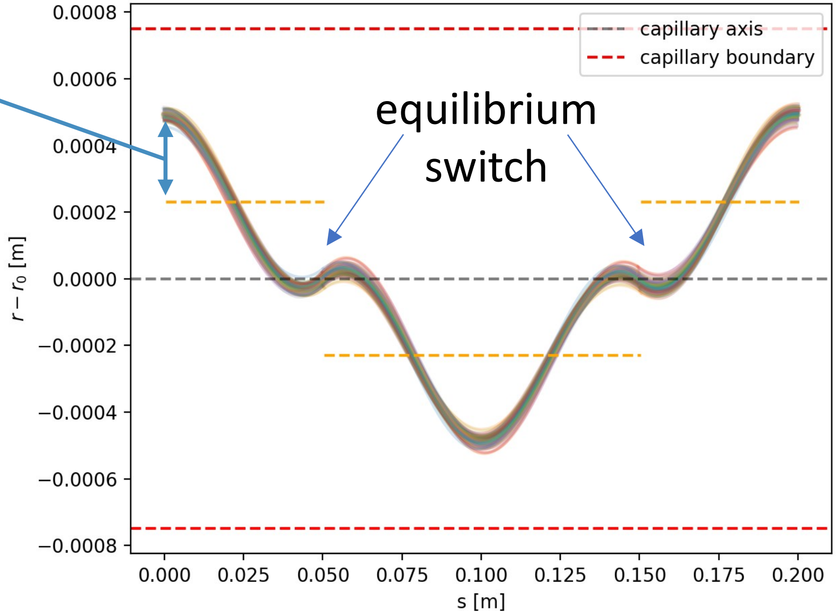
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
EuPRAXIA Advanced Photon Sources

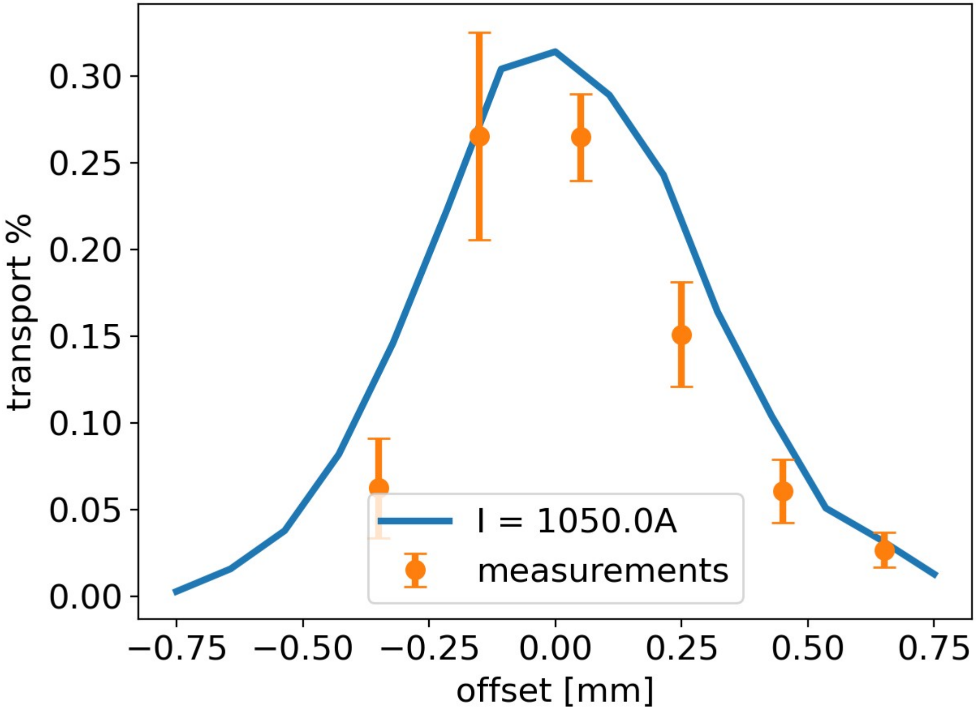
Moving forward: ABP chicane



$$\Delta r_{off} = -\frac{r_{off}}{\cos(L_1 \omega_\beta / c)}$$

$$I = \gamma \frac{2\pi^3 m_e c r_c^2}{e \mu_0 L_s^2}$$





Next steps

- **Beam-plasma interaction** (wakefields) studies to check for corrections or operational regimes (charge, aspect ratio)
- **Nonlinear discharge field** corrections
- **Synchrotron-betatron radiation** studies, possibly for diagnostics (off equilibrium injection)